

A
COMPENDIOUS SYSTEM
OF
Natural Philosophy.

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With NOTES,
Containing the
MATHEMATICAL DEMONSTRATIONS,
AND
Some Occasional REMARKS.

In Four PARTS.

VOL. II.

By J. ROWNING, M. A.

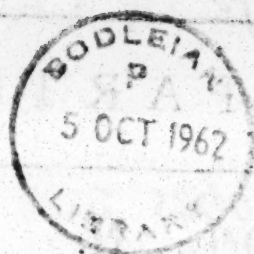
Rector of ANDERBY in LINCOLNSHIRE, and late
Fellow of MAGDALEN College in CAMBRIDGE.

L O N D O N,

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Place this Title before Page 97, in Part 3d. continued.

COMPREHENSIVE SYSTEM
OF
Natural Philosophy





PART III. continued.

To which are added,

DISSERTATIONS on the following Subjects,

Viz.

Of the *Cause* of the *Reflection*
of *Light*.

Of *Microscopes* and *Telescopes*.

A N D

Of the Phænomenon of the
Rainbow.



(Price Two Shillings.)

Part III. continued.

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(Two Shillings)

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OF
Natural Philosophy.

With NOTES
Containing the MATHEMATICAL DEMONSTRATIONS, and some occasional REMARKS.

PART III. continued.
CONTAINING
CATOPTRICS,
AND THE
Doctrine of LIGHT and COLOURS.

To this are added,
DISSERTATIONS on the following Subjects,
Viz.

Of the *Cause* of the *Reflection* of *Light*.
Of *Microscopes* and *Telescopes*. And
Of the Phænomenon of the *Rainbow*.

The THIRD EDITION.

By J. ROWNING, M. A.
Rector of ANDERBY in LINCOLNSHIRE, and late
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C H A P. VIII.

Of the Manner wherein Light is reflected.

THE *Refraction* of Light has been considered, and explained: Its *Reflection* is now to be inquired into (a). And first, for the Manner wherein it is performed.

When a Ray of Light falls upon a Body not transparent, *Part* or *all* of it is *reflected*; if any enters, it is suffocated and lost within the Body: When it falls upon a *transparent* Body, *Part* of it is *reflected*, and *Part* enters; of what enters, some is also suffocated and lost in the Body; the rest, when it arrives at the *other Side*, some of it is reflected there, the Remainder goes on and leaves the Body, unless its Inclination to *that Side* exceeds a *certain Degree*; which if it does, it is *all reflected* there. And the Power whereby a Ray is reflected at *this other Side* of a Body (which for Distinction sake, I shall hereafter call *the second Surface*) is stronger than that by which it would be reflected by the same Surface, were it about to enter the Body there with an equal Degree of Obliquity.

(a) The Subject of this *Inquiry* is distinguished from that of the former, by the Name of *Catoptrics*, as tending to explain the Manner in which Objects appear, when seen by *reflected* Light.

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The *Degree of Inclination* necessary to cause a *total Reflection* of a Ray at the second Surface of a *Medium*, is that which requires that the refracted Angle (was the Ray supposed to pass out there) should be equal to, or greater than a right one; and consequently it depends on the refractive Power of the *Medium* through which the Ray passes; and is therefore different in different *Media*. When a Ray passes through Glass surrounded with Air, and is inclined to its second Surface under an Angle of 42 *Degrees or more*, it will be *wholly reflected* there (b). For as 11 is to 17,

(b) From hence it follows, that when a Ray of Light arrives at the second Surface of a transparent Substance with as great or greater Degree of Obliquity, than that which is necessary to make a total Reflection, it will there be all returned back to the first; and if it proceeds towards that with as great an Obliquity as it did towards the other (which it will do, if the Surfaces of the *Medium* be parallel to each other) it will there be all reflected again, &c. and will therefore never get out, but pass from Side to Side, till it be wholly suffocated and lost within the Body.

From hence may arise an obvious Inquiry, how it comes to pass that Light, falling very obliquely upon a Glass Window from without, should be transmitted into the Room? In Answer to this, it must be considered, that however obliquely a Ray falls upon the first Surface of any *Medium* whose Sides are parallel (as those of the Glass in a Window are) it will suffer such a Degree of Refraction in entering there, that it shall fall upon the second with a less Obliquity than that which is necessary to cause a total Reflection. For Instance, let the *Medium* be Glass, as supposed in the present Case, then as 17 is to 11 (the *Ratio* of Refraction out of Air into Glass) so is the Sine of the largest Angle of Incidence with which a Ray can fall upon any Surface, to the Sine of a less Angle than that of total Reflection. And therefore, if the Sides of a Glass be parallel, the Obliquity, with which a Ray falls upon the first Surface, cannot be so great, but that it shall pass the second without suffering a total Reflection there.

(the

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(the *Ratio* of Refraction out of Glass into Air) so is the Sine of an Angle of 42 Degrees to a fourth Number, that will exceed the Sine of a right Angle.

When Light passes out of a denser into a rarer *Medium*, the nearer the second *Medium* approaches the first in Density (or more properly in its refractive Power) the less of it will be reflected in passing from one to the other; and when their refracting Powers are equal, all of it will pass into the second *Medium*.

Whether Light be reflected from the first or second Surface of a Body, the Law it observes is this, viz. *That the Angle of Reflection of each Ray shall be equal to the Angle of Incidence of the same.*

By the *Angle of Reflection* is meant the Angle comprehended between a Perpendicular to the Surface at the Point where the Reflection is made, and the reflected Ray.

These are all the Circumstances attending the Reflection of Light necessary to be taken Notice of at present: There are others, but they respect the *Doctrine of Light and Colours* not yet explained; we shall therefore pass them by till we treat of that Subject, and in the mean Time proceed to consider the Reflection of Light from plain and spherical Surfaces.

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*Of the Reflection of Light from plain
and spherical Surfaces.*

IT was observed in the foregoing Chapter, that the Law of Reflection is such, that the Angle of Reflection of each Ray shall be equal to the Angle of Incidence of the same. From whence the seven following Propositions relating to the Reflection of Light from plain and spherical Surfaces may be deduced.

I. Rays of Light reflected from a plain Surface have the same Degree of Inclination to each other that their respective incident ones have.

For the Angle of Reflection of each Ray being equal to that of its respective incident one, it is evident that each reflected Ray will have the same Degree of Inclination to that Portion of the Surface from whence it is reflected, that its incident one has; but it is here supposed that all those Portions of Surface from whence the Rays are reflected are situated in the same Plain; consequently the reflected Rays will have the same Degree of Inclination to each other that their incident ones have, from whatever Part of the Surface they are reflected.

See

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See this and the following Propositions expressed more determinately, and demonstrated in the Note below (a).

II. Parallel Rays, reflected from a concave Surface, are render'd converging.

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To

(a) Proposition I.

Of the Reflection of Rays from a plain Surface.

When Rays fall upon a plain Surface, if they diverge, the Focus of the reflected Rays will be at the same Distance behind the Surface, that the Radiant Point is before it: If they converge, it will be at the same Distance before the Surface, that the imaginary Focus of the incident Rays is behind it.

This Proposition admits of two Cases.

Case I. Of diverging Rays.

Dem. Let AB, AC (Fig. 45) be two diverging Rays incident in the plain Surface DE, the one perpendicularly, the other obliquely; the perpendicular one AB will be reflected to A proceeding as from some Point in the Line AB produced; the oblique one AC will be reflected into some Line as CF, such that the Point G, where the Line FC produced intersects the Line AB produced also, shall be at an equal Distance from the Surface DE with the Radiant A. For the Perpendicular CH being drawn, ACH and HCF will be the Angles of Incidence and Reflection, which being equal, their Complements ACB and FCE are so too: But the Angle BCG is equal to FCE, as being vertical to it; therefore in the Triangles ABC and GBC the Angles at C are equal, the Side BC is common, and the Angles at B are also equal to each other, as being right ones; therefore the Lines AB and BG, which respect the equal Angles at C, are also equal, and consequently the Point G, the Focus of the incident Rays AB, AC, is at the same Distance behind the Surface, that the Point A is before it. Q. E. D.

Case II. Of converging Rays.

This is the Converse of the former Case. For supposing FC and AB to be two converging incident Rays, CA and BA will be the reflected ones (the Angles of Incidence in the former Case

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To illustrate this, let AF, CD, EB, (Fig. 43.) represent three parallel Rays falling upon the concave Surface FB, whose Center is C. To the Points F and B draw the Lines CF, CB; these being drawn from the Center will be

Case being now the Angles of Reflection, and *vice versa*) having the Point A for their Focus; but this, from what was demonstrated above, is at an equal Distance from the reflecting Surface with the Point G, which in this Case is the imaginary Focus of the incident Rays, FC, and AB.

Observat. I. It is not here, as in the Refraction of Rays in passing through a plain Surface, where some of the refracted Rays proceed as from one Point, and some as from another (See *Observat. I. Chap. 3, in the Notes*) but they all proceed after Reflection as from one and the same Point, however obliquely they may fall upon the Surface; for what is here demonstrated of the Ray AC holds equally of any other, as AI, AK, &c.

The Case of parallel Rays incident on a plain Surface, is included in this Proposition; for in that Case we are to suppose the Radiant to be at an infinite Distance from the Surface, and then by the Proposition, the Focus of the reflected Rays will be so too; that is, the Rays will be parallel after Reflection, as they were before.

Proposition II.

Of the Reflection of parallel Rays from a spherical Surface.

When parallel Rays are incident upon a spherical Surface, the Focus of the reflected Rays will be the middle Point between the Center of Convexity and the Surface.

This Proposition admits of two Cases.

Case I. Of parallel Rays falling upon a convex Surface.

Dem. Let AB, DH, (Fig. 46) represent two parallel Rays incident on the convex Surface BH, the one perpendicularly, the other obliquely; and let C be the Center of Convexity; suppose HE to be the reflected Ray of the oblique Incident one DH proceeding as from F, a Point in the Line AB produced. Through the Point H draw the Line CI, which will be perpendicular to the Surface at that Point, and the Angles DHI and IHF, being the Angles of Incidence and Reflection, will be equal.

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be perpendicular to the Surface at those Points. The incident Ray CD also passing through the Center will be perpendicular to the Surface, and therefore will return after Reflection in the same Line; but the oblique Rays AF

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and

To the former of these, the Angle HCF is equal, *the Lines AC and DH being parallel*, and to the latter the Angle CHF *as being vertical*; wherefore the Triangle CFH is *Isoceles*; and consequently the Sides CF and FH are equal: But supposing BH to *vanish*, FH is equal to FB, and therefore upon this Supposition FC and FB are equal, that is, the *Focus* of the reflected Rays is the middle Point between the Center of Convexity and the Surface. Q. E. D.

Case II. Of parallel Rays falling upon a concave Surface.

Dem. Let AB, DH (Fig. 47) be two parallel Rays incident, the one perpendicularly, the other obliquely, on the concave Surface BH, whose Center of Concavity is C. Let BF and HF be the reflected Rays meeting each other in F; this will be the middle Point between B and C. For drawing through C the Perpendicular CH, the Angles DHC and FHC, *being the Angles of Incidence and Reflection*, will be equal, to the former of which the Angle HCF is equal, *as alternate*; and therefore the Triangle CFH is *Isoceles*. Wherefore CF and FH are equal: But if we suppose BH to *vanish*, FB and FH are also equal, and therefore CF is equal to FB; that is, the focal Distance of the reflected Rays is the middle Point between the Center and the Surface. Q. E. D.

Observat. II. It is here observable, that the farther the Line DH, either in Figure the 46 or 47, is taken from AB, the nearer the Point F falls to the Surface. For the farther the Point H recedes from B, the larger the Triangle CFH will become; and consequently since it is always an *Isoceles* one, and the Base CH, being the Radius, is every where of the same Length, the equal Legs CF and FH will lengthen; but CF cannot grow longer unless the Point F approach towards the Surface. And the farther H is removed from B, the faster F approaches to it.

This is the Reason, that whenever parallel Rays are considered, as reflected from a spherical Surface, the Distance of the oblique one from the perpendicular one is taken so small with respect to the focal Distance of that Surface, that without any physical Error it may be supposed to vanish.

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and EB will be reflected into the Lines FM and BM situated on the contrary side their respective Perpendiculars CF and CB. They will therefore proceed converging after Reflection towards some Point as M, in the Line CD; which Point, by what is demonstrated in the Note last referred to, will be in the Middle between C and D.

III.

From hence it follows, that if a Number of parallel Rays as AB, CD, EG, &c. fall upon a convex Surface, as expressed Figure the 48, and if BA, DK, the reflected Rays of the incident ones AB, CD, proceed as from the Point F, those of the incident ones CD, EG, viz. DK, GL, will proceed as from N, those of the incident ones EG, HI, as from O, &c. because the farther the incident ones CD, EG, &c. are from AB, the nearer to the Surface are the Points F, f, f, in the Line BF, from which they proceed after Reflection; so that properly the Foci of the reflected Rays BA, DK, GL, &c. are not in the Line AB produced, but in a curve Line passing through the Points F, N, O, &c.

The same is applicable to the Case of parallel Rays reflected from a concave Surface, as expressed by the pricked Lines on the other half of the Figure, where PQ, RS, TV, are the incident Rays; QFSf, Vsf, the reflected ones intersecting each other in the Points X, Y, and F; so that the Foci of those Rays are not in the Line FB, but in a Curve passing through those Points.

Had the Surface BH in Figure 46, or 47, been formed by the Revolution of a Parabola about its Axis having its Focus in the Point F, all the Rays reflected from the convex Surface would have proceeded as from the Point F, and those reflected from the concave would have fallen upon it, however distant their incident ones AB, DH, might have been from each other. For in the Parabola, all Lines drawn parallel to the Axis make Angles with the Tangents to the Points where they cut the Parabola (that is, with the Surface of the Parabola) equal to those which are made with the same Tangents by Lines drawn from thence to the Focus. (*De L'Hospital Sections Coniques. Liv. I. Prop. 5.*) Therefore, if the incident Rays describe those parallel Lines, the reflected ones will necessarily describe these other, and so will all proceed as from, or meet in, the same Point.

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III. Converging Rays falling on the like Surface are made to converge more.

For every thing remaining as above, let GF, HB, be the incident Rays. Now because these Rays have larger Angles of Incidence than the parallel ones AF and EB in the foregoing Case, their Angles of Reflection will also be larger than theirs; they will therefore converge after Reflection, suppose in the Lines FN, and BN, having their Point of Con-

Proposition III.

Of the Reflection of diverging and converging Rays from a spherical Surface.

When Rays fall upon any spherical Surface, if they diverge, the Distance of the Focus of the reflected Rays from the Surface is to the Distance of the Radiant Point from the same (or, if they converge, to that of the imaginary Focus of the incident Rays) as the Distance of the Focus of the reflected Rays from the Center is to the Distance of the Radiant Point (or imaginary Focus of the incident Rays) from the same.

This Proposition admits of ten Cases.

Case I. *Of diverging Rays falling upon a convex Surface.*

Dem. Let RB, RD (Fig. 49) represent two diverging Rays flowing from the Point R as from a *Radiant*, and falling the one perpendicularly, the other obliquely, on the convex Surface BD, whose Center is C. Let DE be the reflected Ray of the incident one RD, produce ED to F, and through R draw the Line RH parallel to FE till it meets CD produced in H. Then will the Angle RHD be equal to EDH the Angle of Reflection, *as being alternate to it*, and therefore equal also to RDH which is the Angle of Incidence; wherefore the Triangle DRH is *Isosceles*, and consequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are similar, (or to express it in *Euclid's* Way, the Sides of the Triangle RHC are cut proportionably. 2 Elem. 6.) and therefore FD is
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Concourse N farther from C than the Point M; that to which the parallel Rays AF and EB converged to in the foregoing Case: And their precise Degree of Convergency as determined in the Note, will be greater than that wherein they converged before Reflection.

IV. Diverging Rays, falling upon the like Surface, are after Reflection parallel, diverging, or converging. If they diverge from the *Focus* of parallel Rays, they then become parallel; if from

to RH, or its equal RD, as CF to CR; but BD *vanishing*, FD and RD differ not from FB and RB, wherefore FB is to RB also, as CF to CR; that is, the Distance of the *Focus* from the Surface is to the Distance of the *Radiant* Point from the same, as the Distance of the *Focus* from the Center is to the Distance of the *Radiant* from thence. Q. E. D.

Case II. Of converging Rays falling upon a concave Surface.

Dem. Let KD and CB be the converging incident Rays having their imaginary *Focus* in the Point R, which was the *Radiant* in the foregoing Case. Then as RD was in that Case reflected into DE, KD will in this be reflected into DF; for, since the Angles of Incidence in both Cases are equal, as they are by being vertical, the Angles of Reflection will be so too; so that F will be the *Focus* of the reflected Rays: But it was there demonstrated that FB is to RB as CF to CR, that is, the Distance of the *Focus* from the Surface is to the Distance (in this Case) of the imaginary *Focus* of the incident Rays, as the Distance of the *Focus* from the Center is to the Distance of the imaginary *Focus* of the incident Rays from the same. Q. E. D.

Case III. Of converging Rays falling upon a convex Surface, and tending to a Point between the Focus of parallel Rays and the Center.

Dem. Let BD (Fig. 50) represent a convex Surface whose Center is C, and *Focus* of parallel Rays is P; and let AB, KD, be two converging Rays incident upon it, and having their imaginary *Focus* at R, a Point between P and C. Now because KD tends

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from a Point nearer to the Surface than that, they will diverge, but in a less Degree than before Reflection; if from a Point between that and the Center, they will converge after Reflection, and that to some Point on the contrary Side the Center, but situated farther from it than the Point they diverged from: If the incident Rays diverge from a Point beyond the Center, the reflected ones will converge to one on the other Side of it, but nearer to it than the

tends to a Point between the *Focus* of parallel Rays and the Center, the reflected Ray D E will diverge from some Point on the other Side the Center, suppose F; as explained above in the Text under Proposition the VIIth. Through D draw the Perpendicular C D and produce it to H, then will K D H and H D E be the Angles of Incidence and Reflection, which being equal, their vertical ones R D C and C D F will be so too, and therefore the *Vertex* of the Triangle R D F is bisected by the Line D C: Wherefore (3 El. 6) F D and D R, or, B D vanishing, F B and B R are to each other as F C to C R; that is, the Distance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident ones, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

Case IV. Of diverging Rays falling upon a concave Surface and proceeding from a Point between the Focus of parallel Rays and the Center.

Dem. Let R B, R D, be the diverging Rays incident upon the concave Surface B D, having their *Radiant* in the Point R, the imaginary *Focus* of the incident Rays in the foregoing Case. Then as K D was in that Case reflected into D E, R D will now be reflected into D F. But it was there demonstrated that F B and R B are to each other as C F to C R; that is, the Distance of the *Focus* is to that of the *Radiant*, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

The Angles of Incidence and Reflection being equal, it is evident, that if in any Case the reflected Ray be made the incident one, the

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the Point they diverged from; and if they diverge from the Center, they will be reflected thither again.

1. Let them diverge in the Lines MF, MB, proceeding from M the *Focus* of parallel Rays; then as the parallel Rays AF and EB were reflected into the Lines FM and BM (Proposit. 2) these Rays will now on the contrary be reflected into them.

2. Let them diverge from N a Point nearer to the Surface than the *Focus* of parallel Rays, they will then be reflected into the diverging Lines

the incident will become the reflected one: And therefore the four following Cases may be considered respectively as the Converse of the four foregoing; for in each of them the incident Rays are supposed to coincide with the reflected ones in the other. Or they may be demonstrated independently of them as follows.

Case V. Of converging Rays falling upon a convex Surface, and tending to a Point nearer the Surface than the Focus of parallel Rays.

Dem. Let ED, RB (Fig. 49) be the converging Rays incident upon the convex Surface BD whose Center is C, and *Focus* of parallel Rays is at P; and let the imaginary *Focus* of the incident Rays be at F, a Point between P and B, and let DR be the reflected Ray. From C and R draw the Lines CH, RH, the one passing through D, the other parallel to FE: Then will the Angle RHD be equal to HDE, the Angle of Incidence, as alternate to it, and therefore equal to HDR, the Angle of Reflection; wherefore the Triangle HDR is *Isosceles*, and consequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are similar, and therefore RH, or RD, is to FD as CR to CF; but BD vanishing, RD and FD coincide with RB and FB, wherefore RB is to FB as CR to CF; that is, the Distance of the *Focus* from the Surface is to the Distance of the imaginary *Focus* of the incident Rays, as the Distance of the *Focus* from the Center is to the Distance of the imaginary *Focus* of the incident Rays from the same. Q. E. D.

Case

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Lines FG and BH which the incident Rays GF and HB described, that were shewn to be reflected into them in the foregoing Proposition; but the Degree wherein they diverge, as demonstrated in the Note, will be less than that wherein they diverged before Reflection.

3. Let them proceed diverging from X a Point between the *Focus* of parallel Rays and the Center, they then make less Angles of Incidence than the Rays MF and MB which became parallel by Reflection, they will consequently have less Angles of Reflection, and proceed therefore converging towards some Point as Y; which Point will always fall on the contrary Side the Center, because a re-
O reflected

Case VI. Of diverging Rays falling upon a concave Surface, and proceeding from a Point between the Focus of parallel Rays and the Surface.

Dem. Let FD and FB represent two diverging Rays flowing from the Point F as a *Radiant*, which was the imaginary *Focus* of the incident Rays in the foregoing Case. Then as ED was in that Case reflected into DR, FD will be reflected into DK, (for the Reason mentioned in Case the second) so that the reflected Ray will proceed as from the Point R: But it was demonstrated in the Case immediately before-going, that RB is to FB as CR to CF; that is, the Distance of the *Focus* from the Surface is to that of the *Radiant* from the same, as the Distance of the former from the Center is to that of the latter from the same. Q. E. D.

Case VII. Of converging Rays falling upon a convex Surface, and tending towards a Point beyond the Center.

Dem. Let AB, ED, (Fig. 50) be the incident Rays tending to F, a Point beyond the Center C, and let DK be the reflected Ray of the incident one ED. Then because the incident Ray
ED

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lected Ray always falls on the contrary Side the Perpendicular with respect to that on which its incident one falls; and, as demonstrated in the Note, it will, be farther distant from the Center than X.

4. If the incident ones diverge from Y, they will after Reflection converge to X, those which were the incident Rays in the former Case being the reflected ones in this.

And lastly, if the incident Rays proceed from the Center, they fall in with their respective Perpendiculars, and for that Reason are reflected thither again.

V. Parallel Rays reflected from a convex Surface are rendered diverging.

To shew this, let AB, GD, EF, (Fig. 44) be three parallel Rays falling upon the convex Sur-

ED tends to a Point beyond the Center, the reflected Ray DK will proceed as from one on the contrary Side, suppose R; as explained in the Text under Proposition the VIIth. Through D draw the Perpendicular CD and produce it to H. Then will EDH and HDK be the Angles of Incidence and Reflection, which being equal, their vertical ones CDF and CDR will be so too: Consequently the *Vertex* of the Triangle FDR is bisected by the Line CD: Wherefore (3 Elem. 6) RD is to DF, or BD vanishing, RB is to BF as RC to CF; that is, the Distance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident Rays, as the Distance of the former from the Center is to the Distance of the latter from the same. Q. E. D.

Case VIII. Of diverging Rays falling upon a concave Surface, and proceeding from a Point beyond the Center.

Dem. Let FB, FD, be the incident Rays having their *Radiant* in F, the imaginary *Focus* of the incident Rays in the foregoing Case. Then as ED was in that Case reflected into DK, FD will now be reflected into DR; so that R will be the *Focus* of the

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Surface BF whose Center of Convexity is C, and let one of them, *viz.* GD, be perpendicular to the Surface: Through B, D, and F, the Points of Reflection, draw the Lines CV, CG, and CT, which because they pass through the Center will be perpendicular to the Surface at those Points. The incident Ray GD being perpendicular to the Surface will return after Reflection in the same Line, but the oblique ones AB and EF in the Lines BK and FL situated on the contrary Side their respective Perpendiculars BV and FT. They will therefore diverge after Reflection as from some Point M in the Line GD produced; which Point, as demonstrated in the Note, will be in the middle between D and C.

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VI.

reflected Rays. But it was demonstrated in the foregoing Case, that RB is to FB as RC to CF; that is, the Distance of the *Focus* of the reflected Rays from the Surface is to the Distance of the *Radiant* from the same, as the Distance of the *Focus* of the reflected Rays from the Center is to the Distance of the *Radiant* from thence. Q. E. D.

The two remaining Cases may be considered, as the Converse of those under Proposition the second of this Note, because the incident Rays in these are the reflected ones in them; or they may be demonstrated in the same Manner with the foregoing, as follows.

Case IX. Converging Rays falling upon a convex Surface, and tending to the Focus of parallel Rays, become parallel after Reflection.

Dem. Let ED, RB, (Fig. 49.) represent two converging Rays incident on the convex Surface BD, and tending towards F, which we will now suppose to be the *Focus* of parallel Rays; and let DR be the reflected Ray and C the Center of Convexity of the reflecting Surface. Through C draw the Line CD, and produce it

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VI. Diverging Rays reflected from the like Surface are rendered more diverging.

For, every thing remaining as above, let GB, GF, be the incident Rays. These having larger Angles of Incidence than the parallel ones AB and EF in the preceeding Case, their Angles of Reflection will also be larger than theirs; they will therefore diverge after Reflection, suppose in the Lines BP and FQ, as from some Point N farther from C than the Point M; and the Degree wherein they will diverge, as determined in the Note, will exceed that wherein they diverged before Reflection.

VII.

it to H, drawing RH parallel to ED produced to F. Now it has been demonstrated (Case 5) where the incident Rays are supposed to tend to the Point F, that RB is to FB as RC to CF: But F in this Case being supposed to be the *Focus* of parallel Rays, it is the middle Point between C and B (by Proposition 2d) and therefore FB and FC are equal, and consequently the two other Terms in the Proportion, *viz.* RB and RC, must be so too; which can only be upon a Supposition that R is at an infinite Distance from B; that is, that the reflected Rays BR and DR be parallel. Q. E. D.

Case X. Diverging Rays falling upon a concave Surface, and proceeding from the Focus of parallel Rays, become parallel after Reflection.

Dem. Let RD, RB (Fig. 50) be two diverging Rays incident upon the concave Surface BD, as supposed in Case the fourth; where it was demonstrated that FB is to RB as CF to CR. But in the present Case RB and CR are equal, because R is supposed to be the *Focus* of parallel Rays; therefore FB and FC are so too. Which cannot be unless F be taken at an infinite Distance from B; that is, unless the reflected Rays BF and DF be parallel Q. E. D.

Observat.

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VII. Converging Rays reflected from the like Surface, are parallel, converging or diverging. If they tend towards the *Focus* of parallel Rays, they then become parallel; if to a Point nearer the Surface than that, they converge, but in a less Degree than before Reflection; if to a Point between that and the Center, they shall diverge after Reflection, as from some Point on the contrary Side the Center, but situated farther from it than the Point they converged to; if the incident Rays converge to a Point beyond the Center, the reflected ones will diverge as from one on the contrary Side of it, but nearer to it than the Point the incident ones converged to: And if the incident Rays converge towards the Center, the reflected ones will proceed as from thence.

O 3

1. Let

Observat. 3. *It is here observable, that in the Case of diverging Rays falling upon a convex Surface (see Fig. 49) the farther the Point D is taken from B, the nearer the Point F, the Focus of the reflected Rays, approaches to B, while the Radiant R remains the same. For it is evident from the Curvature of a Circle that the Point D (see Fig. 51) may be taken so far from B, that the reflected Ray DE shall proceed as from F, G, H, or even from B, or from any Point between B and R, and the farther it is taken from B, the faster the Point, from which it proceeds, approaches towards R: As will easily appear if we draw several incident Rays with their respective reflected ones, in such Manner that the Angles of Reflection may be all equal to their respective Angles of Incidence, as is done in the Figure. The like is applicable to any of the other Cases*

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1. Let them converge in the Lines KB and LF tending towards M the *Focus* of parallel Rays; than as the parallel Rays AB, EF were reflected into the Lines BK and FL (*Proposit. 5*) those Rays will now on the contrary be reflected into them.

2. Let them converge in the Lines PB, QF, tending towards N a Point nearer the Surface than the *Focus* of parallel Rays, they will then be reflected into the converging Lines BG and FG, in which the Rays GB, GF, proceeded that were shewn to be reflected into them

Cases of diverging or converging Rays incident upon a spherical Surface. This is the Reason that, when Rays are considered as reflected from a spherical Surface, the Distance of the oblique Rays from the perpendicular one is taken so small, that it may be supposed to vanish.

From hence it follows, that if a Number of diverging Rays are incident upon the convex Surface BD at the several Points B, D, D, &c, they shall not proceed after Reflection as from any one Point in the Line RB produced, but as from a curve Line passing through the several Points F, f, f, &c. The same is applicable in all the other Cases.

Had the Curvature BD (Fig. 49) been Hyperbolical having its Foci in R and F, then R being the Radiant (or the imaginary Focus of incident Rays) F would have been the Focus of the reflected ones, and vice versa, however distant the Points B and D might be taken from each other. In like Manner had the Curve BD (Fig. 50.) been Elliptical having its Foci in F and R, the one of these being made the Radiant (or imaginary Focus of incident Rays) the other would have been the Focus of the reflected ones, and vice versa. For both in the Hyperbola and Ellipsis, Lines drawn from each of their Foci through any Point make equal Angles with the Tangent to that Point; (De L'Hospital Sections Coniques, Liv. II. Prop. 8. & Liv. III. Prop. 11.) Therefore, if the incident Rays proceed to or from one of their Foci, the reflected ones

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them in the Proposition immediately foregoing: but the Degree wherein they will converge, as demonstrated in the Note, will be less than that wherein they converged before Reflection.

3. Let them converge in the Lines RB and SF proceeding towards X, a Point between the *Focus* of parallel Rays and the Center; their Angles of Incidence will then be less than those of the Rays KB and LF which became parallel after Reflection, their Angles

O 4

of

ones will all proceed, as from or to the other. So that in order that diverging or converging Rays may be accurately reflected to or from a Point, the reflecting Surface must be formed by the Revolution of an Hyperbola about its longer Axis, when the incident Rays are such that their Radiant, or imaginary Focus of incident Rays, shall fall on one Side the Surface, and the Focus of the reflected ones on the other: When they are both to fall on the same Side, it must be formed by the Revolution of an Ellipsis about its longer Axis. However upon Account of the great Facility with which spherical Surfaces are formed in Comparison of that with which Surfaces, formed by the Revolution of any of the Conic Sections about their Axes, are made, the latter are very rarely used. Add to this another Inconvenience, viz. that, the Foci of these Curves being Mathematical Points, it is but one Point of the Surface of an Object that can be placed in any of them at a Time, so that it is only in Theory that Surfaces formed by the Revolution of these Curves about their Axes render Reflection perfect.

Now because the focal Distance of Rays reflected from a spherical Surface cannot be found by the *Analogy* laid down in the third Proposition of this Note, without making use of the Quantity sought; I shall here give an Instance whereby the Method of doing it in all others will readily appear.

Prob. Let it be required to find the focal Distance of diverging Rays incident upon a convex Surface, whose *Radius* of Convexity

is

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of Reflection will therefore be less, on which Account they must necessarily diverge, suppose in the Lines BH and FI, from some Point as Y; which Point for the Reason given under Proposition the fourth will fall on the contrary Side the Center with respect to X, and, as demonstrated in the Note, will be farther from it than that.

4. If the incident Rays tend towards Y, the reflected ones will diverge as from X, those, which

is 5 Parts, and the Distance of the *Radiant* from the Surface is 20.

Sol. Call the focal Distance sought x , then will the Distance of the *Focus* from the Center be $5 - x$, and that of the *Radiant* from the same 25; therefore by Proposition the third, we have the following Proportion, *viz.* $x : 20 :: 5 - x : 25$, and, multiplying Extreams together and Means together, we have $25x = 100 - 20x$, which after due Reduction gives $x = \frac{100}{45}$.

If in any Case it should happen, that the Value of x should be a negative Quantity, the focal Point must then be taken on the contrary Side the Surface to that on which it was supposed that it would fall in stating the *Problem*.

If Letters instead of Figures had been made use of in the foregoing Solution a general *Theorem* might have been raised, to have determined the focal Distance of reflected Rays in all Cases whatever. See this done by *Dr. Brown* in his Supplement to *Gregory's Optics*, pag. 112. Edit. Second.

Because it was observed (Chap. III. in the Notes) that different incident Rays, though tending to or from one Point, would after *Refraction* proceed to or from different Points, a Method was there inserted of determining the *distinct* Point, which each separate Ray entering a spherical Surface converges to or diverges from after *Refraction*: The same has been observed here with regard to Rays reflected from a spherical Surface, (see *Observat.* 2 & 3) but

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which were the incident ones in one Case, being the reflected ones in the other.

And lastly, if the incident Rays converge towards the Center, they fall in with their respective Perpendiculars; on which Account they proceed after Reflection, as from thence.

but the Method of determining the *distinct* Point, to or from which any given incident Ray proceeds after *Reflection*, is much more simple. It is only necessary to draw the reflected Ray such, that the Angle of Reflection may be equal to the Angle of Incidence, which will determine the Point it proceeds to or from in any Case whatever.



CHAP.

C H A P. X.

*Of the Appearance of Bodies seen by
Light reflected from plain and spher-
ical Surfaces.*

IN the Beginning of the seventh Chapter, in which was explained the Appearance of Bodies seen through *refracting* Substances of various Forms, we laid down some Observations respecting the apparent Situation of Bodies seen by *refracted* Light: All which equally respect the apparent Situation of Bodies seen by *Reflection*, to them therefore we refer the Reader. But besides those, there is one peculiar to the Subject of this Chapter, viz. *That each Point in the Representation of an Object made by Reflection appears situated somewhere in an infinite right Line that passes through its correspondent Point in the Object, and is perpendicular to the reflecting Surface.*

The Truth of this appears sufficiently from the several Propositions laid down in the foregoing Chapter, in each of which Rays flowing from any *Radiant* are shewn to proceed after Reflection to or from some Point in a Line that passes through the said *Radiant*, and is perpendicular to the reflecting Surface. For Instance (Fig. 43) Rays flowing from Y
are

Of the Appearance of Bodies, &c. 119

are collected in X, a Point in the Perpendicular CD, that produced passes through Y; again (Fig. 44) Rays flowing from G proceed after Reflection, as from N, a Point in the Perpendicular CD, that produced passes through G; and so for the rest (a).

I. When an Object is seen by Reflection from a *plain* Surface, the Image of it appears at the *same Distance* behind the Surface that the Object is placed before it, of the *same Magnitude* therewith, and directly *opposite* to it.

To explain this, let AB (Fig. 52) represent an Object seen by Reflection from the plain Surface SV, and let the Rays AF, AG be so inclined to the Surface that they shall enter an Eye at H after Reflection; and let AE be

(a) This Observation, except where an Object is seen by Reflection from a plain Surface, relates only to those Cases where the Representation is made by means of such Rays, as fall upon the reflecting Surface with a very small Degree of Obliquity; because such, as fall at a considerable Distance from the Perpendicular, proceed not after Reflection as from any Point in that Perpendicular (see the second and third Observations in the Note to the foregoing Chapter) but as from other Points situated in a certain Curve, as there explained; upon which Account these Rays are neglected as forming a confused and deformed Representation. And therefore it is to be remembered, that however the Situation of the Eye, with respect to the Object and reflecting Surface, may be represented in the following Figures, it is to be supposed as situated in such Manner with respect to the Object, that Rays, flowing from thence and entering it after Reflection, may be such only as fall with a very small Degree of Obliquity upon the Surface; that is, the Eye must be supposed to be placed almost directly behind the Object, or between it and the reflecting Surface. The Reason why it is not always so placed, is only to avoid Confusion in the Figures.

perpen-

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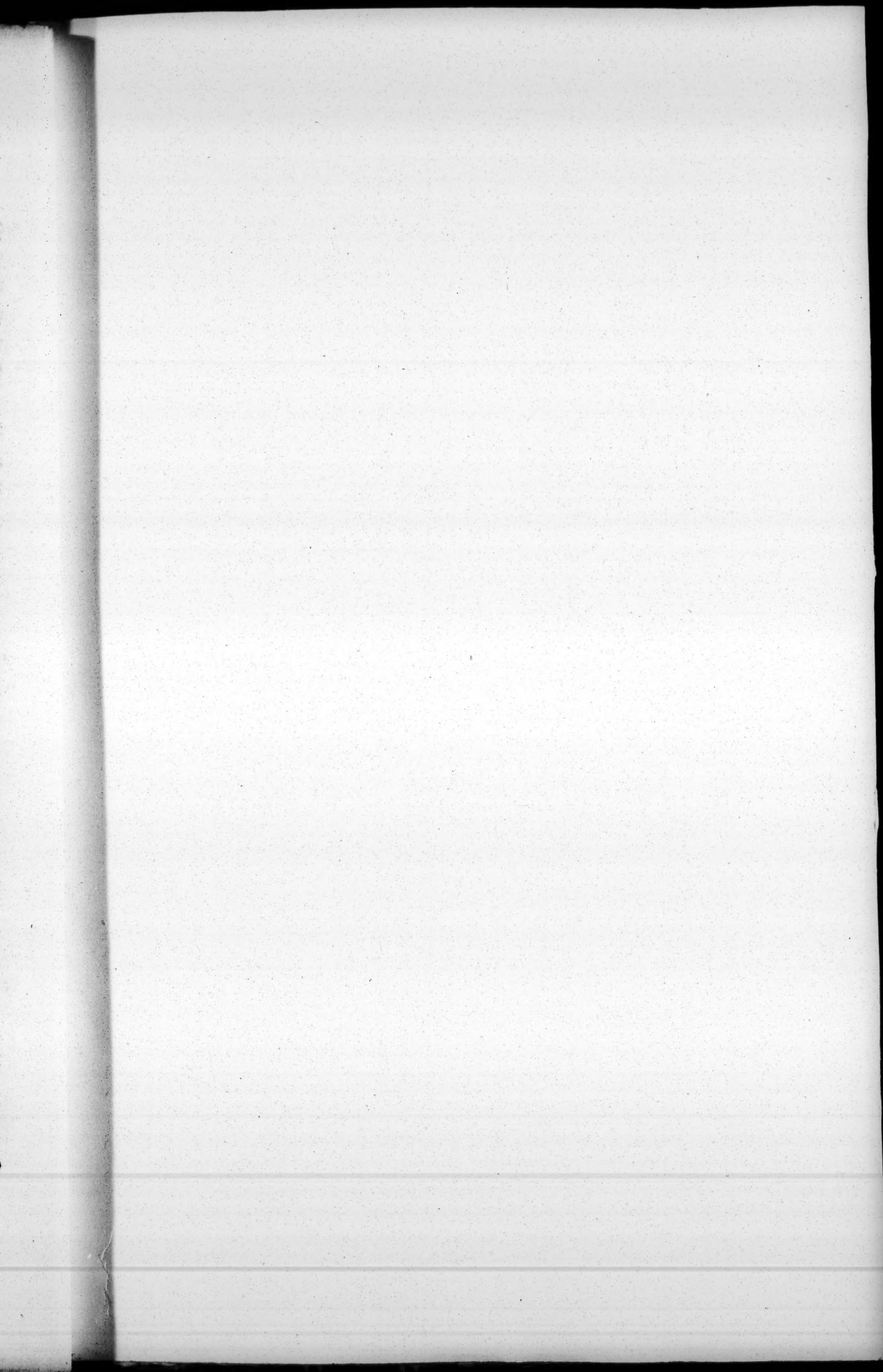
perpendicular to the Surface: Then by the Observation just laid down, the Point A will appear in some Part of the Line A E produced, suppose I; that is, the oblique Rays A F and A G will proceed after Reflection as from that Point, and further, because the reflected Rays F H, G K, will have the same Degree of Inclination to each other, that their incident ones have (as was shewn in Proposition the first of the foregoing Chapter) that Point must necessarily be at the same Distance from the Surface that the Point A is; the Representation therefore of the Point A, will be at the same Distance behind the Surface, that the Point itself is before it, and directly opposite to it: Consequently since the like may be shewn of the Point B, or any other, the whole Image I M will appear at the *same Distance* behind the Surface that the Object is before it, and directly *opposite* to it; and because the Lines A I, B M, which are perpendicular to the plain Surface, are for that Reason parallel to each other, it will also be of the *same Magnitude* therewith. As was to be shewn (b).

II. When an Object is seen by Reflection from a *convex* Surface, its Image appears *nearer* to the Surface, and *less* than the Object.

Let

(b) If the Object be placed before a common Looking-glass, and viewed obliquely, three, four, or more Images of it will appear behind the Glass.

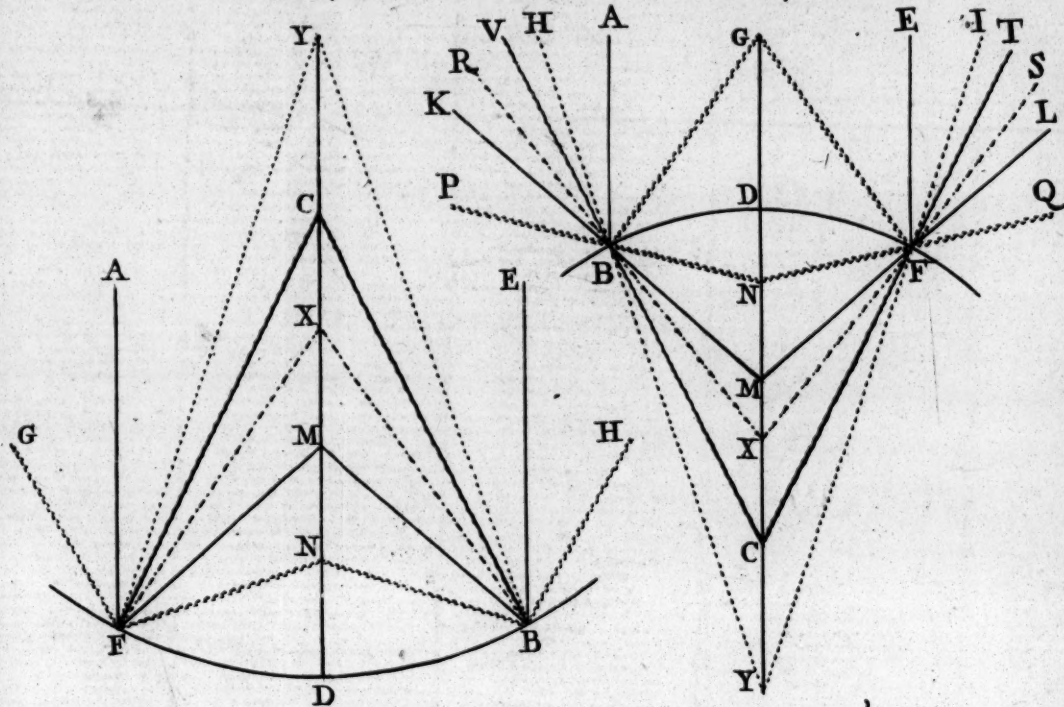
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Part III. Plate VII. p.120.

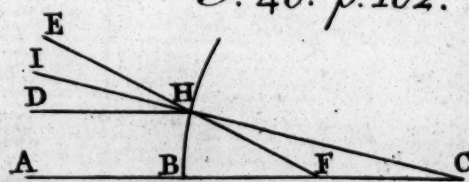
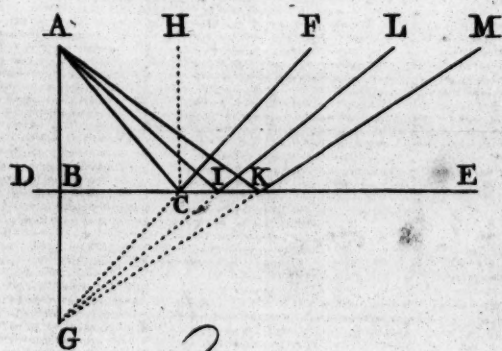
F.43. p.102.

F.44. p.110.



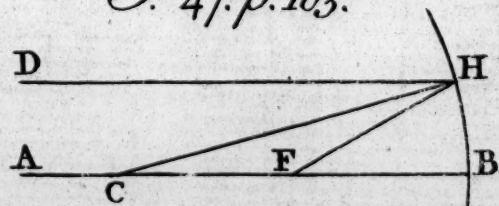
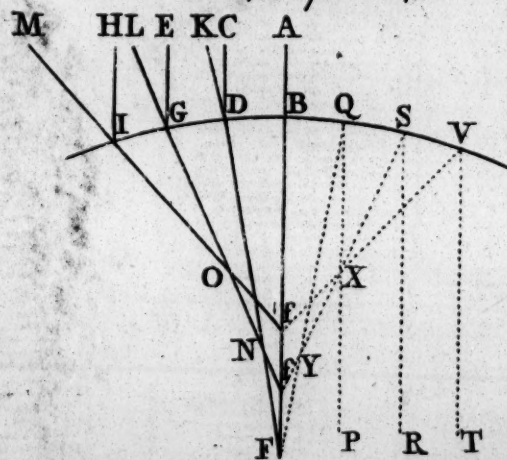
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F.48. p.104.

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Of the Appearance of Bodies, &c. 121

Let AB (Fig. 54) represent the Object, SV a reflecting Surface, whose Center of Convexity is C: And let the Rays AF, AG, be so inclined to the Surface, that after Reflection thereat, they shall enter the Eye at H: And let AE be perpendicular to the Surface: Then will the oblique Rays AF, AG, proceed after Reflection as from some Point in the Line AE produced, (by the Observation laid down at the Beginning of this Chapter) suppose from I; which Point, because the reflected Rays will diverge more

To explain this, let ABCD (Fig. 53) represent the Glass, and let EF be the *Axis* of a Pencil of Rays flowing from E, a Point in an Object situated there. The Rays of this Pencil will in Part be reflected at F, suppose into the Line FG, (see the Manner in which Light is reflected Chap. 18.) What remains will (after Refraction at F which we don't consider here) pass on to H; from whence (on Account of the Quicksilver which is spread over the second Surface of Glasses of this Kind to prevent any of the Rays from being transmitted there) they will be strongly reflected to K. where Part of them will emerge and enter an Eye at L: By this means one Representation of the said Point will be formed in the Line LK produced, suppose in M. Again, another Pencil whose *Axis* is EN, first reflected at N, then at O, and afterwards at P, will form a second Representation of the same Point at Q. And thirdly, another Pencil whose *Axis* is ER, after Reflection at the several Points R, S, H, T, V, successively, will exhibit a third Representation of the same Point at X; and so on *in infinitum*. The same being true of each Point in the Object, the whole will be represented in the like Manner; but the Representations will be faint, in Proportion to the Number of Reflections the Rays suffer and the Length of their Progress within the Glass. We may add to these another Representation of the same Object in the Line LO produced, made by such of the Rays as fall upon O, and are from thence reflected to the Eye at L.

This may be tried by placing a Candle before the Glass as at E, and viewing it obliquely, as from L.

than

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than the incident ones (Prop. the sixth of the foregoing Chapter) must be nearer to the Surface than the Point A. And since the same is true also of the Rays which flow from B, or any other Point, the Representation IM will be *nearer* to the Surface than the Object; and because it is terminated by the Perpendiculars AE and BF which incline to each other, as concurring at the Center, it will also appear *less*.

III. When an Object is seen by Reflection from a *concave* Surface, the Representation of it is *various*, both with regard to its Magnitude and Situation, according as the Distance of the Object from the reflecting Surface is *greater* or *less*.

I. When the Object is nearer to the Surface, than its *Focus* of parallel Rays, the Image falls on the *opposite* Side the Surface, is more *distant* from it, and *larger* than the Object.

Thus, let AB, (Fig. 55) be the Object, SV the reflecting Surface, F the *Focus* of parallel Rays, C its Center. Through A and B the Extremities of the Object draw the Lines CE, CR, which will be perpendicular to the Surface, and let the Rays AR, AG, be incident upon such Points of it that they shall be reflected into an Eye at H. Now because the *Radiant* Points A and B are nearer the Surface than F the *Focus* of parallel Rays, the reflected Rays will diverge (Chap. IX. Prop. 4) and will

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will therefore proceed as from some Points on the opposite Side the Surface; which Points, by the Observation laid down at the Beginning of this Chapter, will be in the Perpendiculars AE, BR, produced, suppose in I and M: But they will diverge in a less Degree than their incident ones (see the Proposition just referred to) and therefore the said Points will be farther from the Surface than the Points A and B. The Image therefore will be on the *opposite* Side the Surface with respect to the Object, it will be *more distant* than it, and consequently, being terminated by the Perpendiculars CI and CM, it will also be *larger*.

2. When the Object is placed in the *Focus* of parallel Rays, the reflected Rays enter the Eye parallel (Chap. IX. Prop. 4) in which Case the Image ought to appear at an *infinite* Distance behind the reflecting Surface; but the Representation of it, for the like Reasons that were given in the foregoing Case, being *large* and *distinct*, we judge it not much farther from the Surface than the Image (c).

3. When the Object is placed between the *Focus* of parallel Rays and the Center, the Image falls on the *opposite* Side the Center, is *larger* than the Object, and in an *inverted Position*.

Thus let AB (Fig. 56) represent the Object, SV the reflecting Surface, F its *Focus* of pa-

(c) See what has been said concerning the apparent Situation of Objects seen by parallel Rays, in Chapter VII.

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parallel Rays, and C its Center. Through A and B the Extremities of the Object draw the Lines CE and CN which will be perpendicular to the Surface; and let AR, AG, be a Pencil of Rays flowing from A. These Rays proceeding from a Point beyond the *Focus* of parallel Rays will after Reflection converge towards some Point on the opposite Side the Center (Chap. IX. Prop. 4, Case 3) which will fall upon the Perpendicular EC produced; but at a greater Distance from C than the Radiant A from which they diverged, (by the Proposition and Case just referred to.) For the same Reason, Rays flowing from B will converge to a Point in the Perpendicular NC produced, which shall be farther from C than the Point B; from whence it is evident, that the Image IM is *larger* than the Object AB, that it falls on the *contrary* Side the Center, and that their Positions are *inverted* with respect to each other.

4. If the Object be placed beyond the Center of Convexity the Image is then formed *between the Center and the Focus of parallel Rays*, is *less* than the Object, and its Position is *inverted*.

This Proposition is the Converse of the foregoing: For as in that Case Rays proceeding from A were reflected to I, and from B to M; so Rays flowing from I and M will be reflected to A and B; if therefore an
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Object be supposed to be situated beyond the Center in IM, the Image of it will be formed in AB, *between that and the Focus of parallel Rays*, will be *less* than the Object, and *inverted*,

5. If the Middle of the Object be placed in the Center of Convexity of the reflecting Surface, the Object and its Image will be *coincident*; but the Image will be *inverted* with respect to the Object.

That the Place of the Image and the Object should be the same in this Case needs little Explication; for the Middle of the Object being in the Center, Rays flowing from thence will fall perpendicularly upon the Surface, and therefore necessarily return thither again; so that the Middle of the Image will be *coincident* with the Middle of the Object. But that the Image should be inverted is perhaps not so clear. To explain this, let AB (Fig. 57) be the Object having its middle Point C in the Center of the reflecting Surface SV; through the Center and the Point R draw the Line CR which will be perpendicular to the reflecting Surface, join the Points AR and BR, and let AR represent a Ray flowing from A, this will be reflected into RB, for C being the middle Point between A and B the Angles ARC and CRB are equal; and a Ray from B will likewise be reflected to A; and therefore the Posi-

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tion

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sition of the Image will be *inverted* with respect to that of the Object (*d*).

6. If in any of the three last Cases, in each of which the Image is formed on the same Side the reflecting Surface with the Object, the Eye be situated farther from the Surface than the Place where the Image falls, the Rays of each Pencil, crossing each other in the several Points of the Image, will enter the Eye as from a real Object situated there; so that the Image will appear *pendulous* in the Air between the Eye and the reflecting Surface, and in the Position wherein it is formed, *viz. inverted* with respect to the Object, in the same Manner that an Image formed by *refracted* Light appears to an Eye placed beyond it; which was fully explained under the fourth Proposition of the seventh Chapter, and therefore needs not be repeated here.

But as to what relates to the Appearance of the Object when the Eye is placed nearer to the Surface than the Image, that was not there fully inquired into. That Point shall therefore now be more strictly examined under the following Case, which equally relates to *refracted* and *reflected* Light.

(*d*) In this Proposition it is to be supposed that the Object AB is so situated with respect to the reflecting Surface that the Angle ACR may be right; for otherwise the Angles ARC and BRC will not be equal, and Part of the Image will therefore fall upon the Object and part off.

7. If

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7. If the Eye be situated between the reflecting Surface and the Place of the Image, the Object is then seen *beyond* the Surface; and the farther the Eye recedes from the Surface towards the Place of the Image, the *more confused, larger, and nearer* the Object appears.

To explain this, let AB (Fig. 58) represent the Object, IM its Image, one of whose Points, M, is formed by the Concurrence of the reflected Rays DM, EM, &c. which before Reflection came from B; the other, I, by the Concurrence of DI, EI, &c. which came from A; and let *ab* be the Pupil of an Eye situated between the Surface DP and the Image. This Pupil will admit the Rays Ha, Kb, which, because they are tending towards I, are such as came from A, and therefore the Point A will appear diffused over the Space RS. In like Manner the Pupil will also receive into it the reflected Rays Ka and Lb, which, because they are tending towards M, by Supposition came from B; and therefore the Point B will be seen spread as it were over the Space TV, and the Object will seem to fill the Space RV; but the Representation of it will be *confused*, because the intermediate Points of the Object, being equally enlarged in Appearance, there will not be Room for them between the Points S and T, but they will coincide in part one with another; for Instance, the Appearance of that Point in the Object, whose Representation falls upon *c* in

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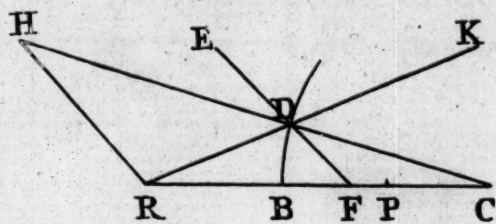
the Image, will fill the Space *mn*, and so of the rest. Now if the same Pupil be removed into the Situation *ef*, the reflected Rays *Ee* and *Gf* will then enter the Eye, and therefore one Extremity of the Object will appear to cover the Space *XY*; and because the Rays *Of* and *Le* will also enter it in their Progress towards *M*, the Point *B*, from whence they came, will appear to cover *ZV*; the Object therefore will appear *larger* and *more confused* than before. And when the Eye recedes quite to the Image, it sees but one single Point of the Object, and that appears diffused all over the reflecting Surface: For Instance, if the Eye recedes to the Point *M*, then Rays flowing from the Point *B* enter it upon whatever Part of the Surface they fall: And so for the rest. The Object also appears *nearer* to the Surface, the farther the Eye recedes from it towards the Place of the Image, probably because as the Appearance of the Object becomes more and more confused, its Place is not so easily distinguished from that of the reflecting Surface itself, till at last when it is quite confused (as it is when the Eye is arrived at *M*) they both appear as one, the Surface assuming the Colour of the Object (*e*).

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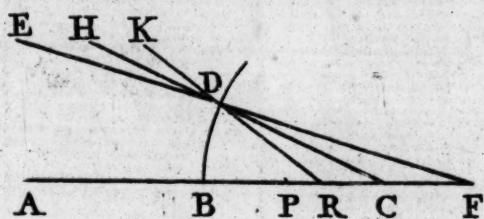
(*e*) As to the precise apparent Magnitude of an Object seen after this Manner, it is such that the Angle it appears under shall be equal to that which the Image of the same Object would appear under, were we to suppose it seen from the same Place: That is, the apparent Object (for such I must call it to distinguish

Part III. Plate VIII. p. 128.

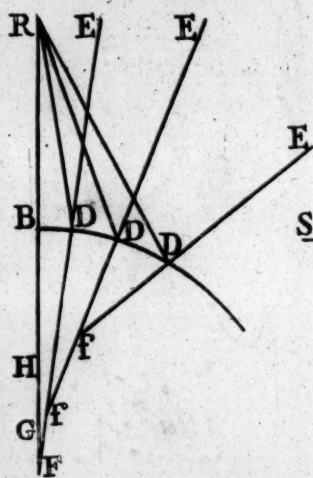
F. 49. p. 105.



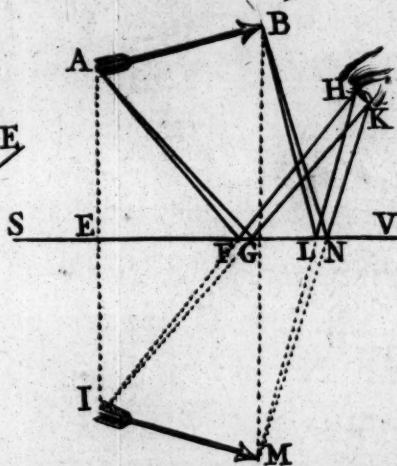
F. 50. p. 106.



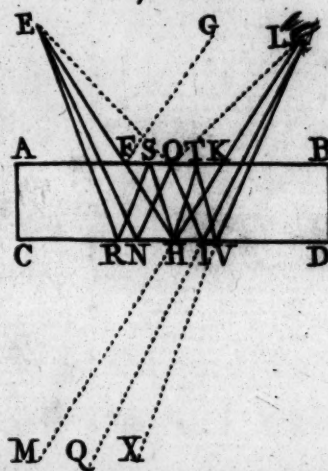
F. 51. p. 113.



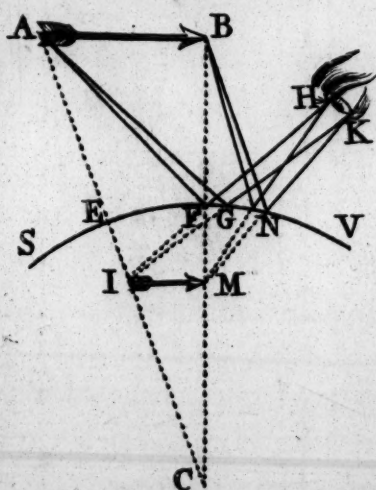
F. 52. p. 119.



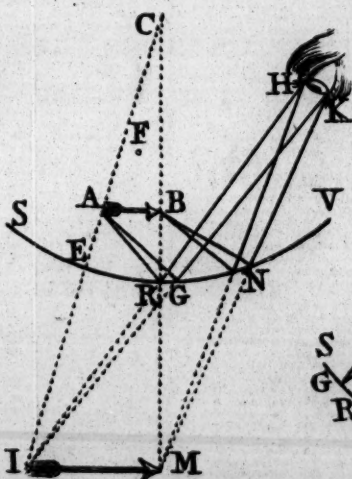
F. 53. p. 120.



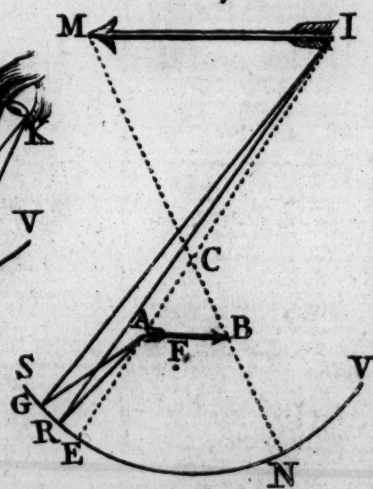
F. 54. p. 121.



F. 55. p. 122



F. 56. p. 123



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As each Point in the Representation of an Object made by Reflection is situated somewhere in a right Line that passes through its correspondent Point in the Object, and is perpendicular to the reflecting Surface, as was shewn in the Beginning of this Chapter; we may from hence deduce a most easy and expeditious Method of determining both the

distinguish it from the Image of the same Object) and the Image subtend equal Angles at the Eye.

Dem. Here we must suppose the Pupil of the Eye to be a Point only, because the Magnitude of that causes some small Alteration in the apparent Magnitude of the Object; as we shall see by and by. Let then the Point *a* represent the Pupil, then will the extreme Rays that can enter it be *Ha* and *Ka*, the Object therefore will appear under the Angle *HaK*, which is equal to its vertical one *MaI*, under which the Image *IM* would appear, were it to be seen from *a*. Again, if the eye be placed in *f*, the Object appears under the Angle *GfO* equal to *IfM* which the Image subtends at the same Place, and therefore the apparent Object and Image of it subtend equal Angles at the Eye. Q. E. D.

Now if we suppose the Pupil to have any sensible Magnitude, such suppose that its Diameter may be *ab*, then the Object seen by the Eye in that Situation will appear under the Angle *HxL*, which is larger than the Angle *HaK* under which it appeared before; because the Angle at *x* is nearer than the Angle at *a*, to the Line *IM*, which is a Subtense common to them both.

From this Proposition it follows, that, were the Eye close to the Surface at *K*, the real and apparent Object would be seen under equal Angles (for the real Object appears from that Place under the same Angle that the Image does, as will be shewn at the End of this Chapter) therefore when the Eye is nearer to the Image than that Point, the Image will subtend a larger Angle at it than the Object does; and consequently since the Image and apparent Object subtend equal Angles at the Eye, the apparent Object must necessarily be seen under a *larger* Angle than the Object itself, wherever the Eye be placed between the Surface and the Image.

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Magnitude and Situation of the Image in all Cases whatever. Thus,

Through the Extremities of the Object *AB*, and the Center *C*, (Fig. 59, 60, or 61) draw the Lines *AC*, *BC*, and produce them as the Case requires; these Lines will be perpendicular to the reflecting Surface, and therefore the Extremities of the Image will fall upon them. Through *F* the middle Point of the Object and the Center, draw the Line *FC* and produce it till it passes through the reflecting Surface, this will also be perpendicular to the Surface. Through *G*, the Point where this Line cuts the Surface, draw the Lines *AG* and *BG* and produce them this way or that, till they cross the former Perpendiculars; and where they cross, there *I* and *M* the Extremities of the Image will fall. For supposing *AG* to be a Ray proceeding from the Point *A* and falling upon *G*, it will be reflected to *B*; because *FA* is equal to *FB*, and *FG* is perpendicular to the reflecting Surface; and therefore the Representation of the Point *A* will be in *BG* produced as well as in *AC*, consequently it will fall on the Point *I* where they cross each other. Likewise the Ray *BG* will for the same Reason be reflected to *A*, and therefore the Representation of the Point *B* will be in *AG* produced as well as in some Part of *BC*, that is in *M* where

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where they cross. From whence the Proposition is clear.

If it happens that the Lines will not cross which way soever they are produced, as in (Fig. 62) then is the Object in the *Focus* of parallel Rays of that Surface, and has no Image formed in any Place whatever. For in this Case the Rays AH, AG, flowing from the Point A, become parallel after Reflection in the Lines HC, GB, and therefore do not flow as to or from any Point: In like Manner Rays flowing from B are reflected into the parallel Lines KB and GA; so that no Representation can be formed by such Reflection.

From hence we learn another Circumstance relating to the Magnitude of the Image made by Reflection; *viz.* that it subtends the same Angle at the *Vertex* of the reflecting Surface that the Object does. This appears by Inspection of the 59, 60, or 61st Figure, in each of which the Angle IGM, which the Image subtends at G the *Vertex* of the reflecting Surface, is equal to the Angle AGB, which the Object subtends at the same Place; for in the two first of those Figures they are vertical, in the third they are the same. And

Farther, the Angle ICM, which the Image subtends at the Center, is also equal to the Angle ACB which the Object subtends at the same Place; for in the two first Figures

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they are the same, in the last they are vertical to each other.

From whence it is evident, that the Object and its Image are to each other in Diameter, either as their respective Distances from the *Vertex* of the reflecting Surface, or as their Distances from the Center of the same.

IV. As Objects are multiplied by being seen through transparent *Media*, whose Surfaces are properly disposed, as was explained (Chap. VII. Prop. 4) so they may also by reflecting Surfaces. Thus,

1. If two reflecting Surfaces be disposed at right Angles, as the Surfaces AB, BC, (Fig. 63) an Object at D may be seen by an Eye at E, after one Reflection at F, in the Line EF produced; after two Reflections, the first at G, the second at H, in the Line EH produced; and also after one Reflection made at A, in the Line EA produced.

2. If the Surfaces be parallel, as AB, CD, (Fig. 64) and the Object be placed at E and the Eye at F, the Object will appear multiplied an *infinite* Number of Times: Thus, it may be seen in the Line FG produced, after one Reflection at G; in the Line FH produced, after two Reflections, the first at I, the second at H; and also in FP produced, after several successive Reflections of the Ray EL, at the Points L, M, N, O, and P: And so on *in infinitum*. But the greater the Number of Reflections

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lections are, the weaker the Representation will be.

There are reflecting Surfaces made, some cylindrically convex, others cylindrically concave; both which Kinds are designed to render the Image of an Object deformed, or the contrary, by augmenting or diminishing it in one Dimension and not in another; for the former Sort are convex one way, *viz.* round their *Axes*, and plain the other, *viz.* lengthwise; the other are concave round their *Axes*, and plain the other way: Upon which Account Surfaces of these Kinds necessarily have such Effects.

When a very deformed Picture, or rather no Picture at all, but a seemingly irregular and accidental Position of Colours is placed before one of these Surfaces, and seen by Reflection from thence, a beautiful and well proportioned Image shall appear therein. But to explain *Phænomena* of this Kind is not the Intent of this *Compendium*. Enough has been said to explain the Principles of *Catoptrics*. Pass we on now to the third and last Division of *Optics*, *viz.* the *Doctrine of Colours*.

C H A P. XI.

Of the different Refrangibility in the Rays of Light; of the Colours the distinct Species of them are disposed to excite; and of the Cause of that Variety of Colours which is observable in Bodies.

IN treating of the Refraction of Light in the Beginning of this Part, we supposed that all Light in passing out of one *Medium* into another of different Density, is equally refracted in the same or like Circumstances. This is the Notion the Philosophers before Sir *Isaac Newton*'s Time had of it; but that indefatigable and circumspect Author has discovered that it is not so, but that *there are different Species of Light; and that each Species is disposed both to suffer a different Degree of Refrangibility in passing out of one Medium into another, and to excite in us the Idea of a different Colour from the rest; and that Bodies appear of that Colour which arises from the Composition of the Colours the several Species they reflect are disposed to excite.*

There are Abundance of Experiments made by Sir *Isaac Newton* and others for the Confirmation

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firmation of this Doctrine; I shall only select the following ones, which will sufficiently illustrate the Proposition, and evince the Truth of it. And

First, There are different *Species* of Light, and each *Species* is disposed to suffer a different Degree of Refrangibility, and to excite the *Idea* of a different Colour.

To shew this, let a Room be darkened, and the Sun permitted to shine into it through a small Hole in the Window-shutter, and be made to fall upon a Glass *Prism* (by which is meant a Piece of Glass of a triangular Form, such as is represented in Figure 65) then will the Sun's Light in passing through this *Prism* suffer different Degrees of Refraction, and by that means be parted into different Rays, which Rays being received upon a clean white Paper will exhibit the following Colours, *viz. Red, Orange, Yellow, Green, Blue, Indigo, and a Violet Purple.* Thus, let AB (Fig. 66) represent the Window-shutter, C the Hole in it, DEF the Prism, ZY a Ray of Light coming from the Sun, which passes through the Hole and falls upon the Prism at Y, and if the *Prism* were removed would go on to X, but in entering its first Surface EF shall be refracted into the Course YW, falling upon the second in W, where in going out into the Air it shall be refracted again. Let the Light now, after it has passed the *Prism*, be received upon a
Sheet

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Sheet of white Paper GHIK held at a proper Distance, and it will exhibit upon the Paper a Picture or Image at LM of an oblong Figure, whose Ends are semicircular and Sides strait. And it shall be variegated with Colours after the following Manner. From the Extremity M to some Length suppose to the Line *no*, it shall be of an intense *Red*; from *no* to *pg* it shall be of an *Orange* Colour; from *pg* to *rs* it shall be *Yellow*; from thence to *tu* it shall be *Green*; from thence to *wx* *Blue*; from thence to *yz* *Indigo*; and from thence to the End *Violet*. And if the whole Image be divided lengthwise into 360 equal Parts, the *Red* shall take up 45 of them, the *Orange* 27, the *Yellow* 48, the *Green* 60, the *Blue* 60, the *Indigo* 40, and the *Violet* 80 (*f*).

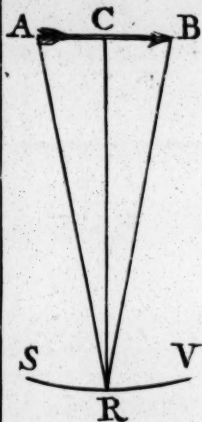
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(*f*) Sir *Isaac Newton* in his *Optics* has shewn how from the Refraction of the most refrangible and least refrangible Rays, to find the Refraction of all the intermediate ones. His Rule is this, if the Sine of Incidence be to the Sine of Refraction in the least refrangible Rays as AV to BC (Fig. 67) and to the Sine of Refraction in the most refrangible as AV to BD; and if CE be taken equal to CD, and then ED be so divided in F, G, H, I, K, L, that ED, EF, EG, EH, EI, EK, EL, EC, may be proportional to the eight Lengths of musical Chords, which shall sound the Notes in an Octave, ED being the Length of the *Key*, EF the Length of the *Tone* above that Key, EG the Length of the *lesser Third*, EH of the *Fourth*, EI of the *Fifth*, EK of the *greater Sixth*, EL of the *Seventh*, and EC of the *Octave* above that Key; that is, If the Lines ED, EF, EG, EH, EI, EK, EL, and EC bear the same Proportion to each other as the Numbers

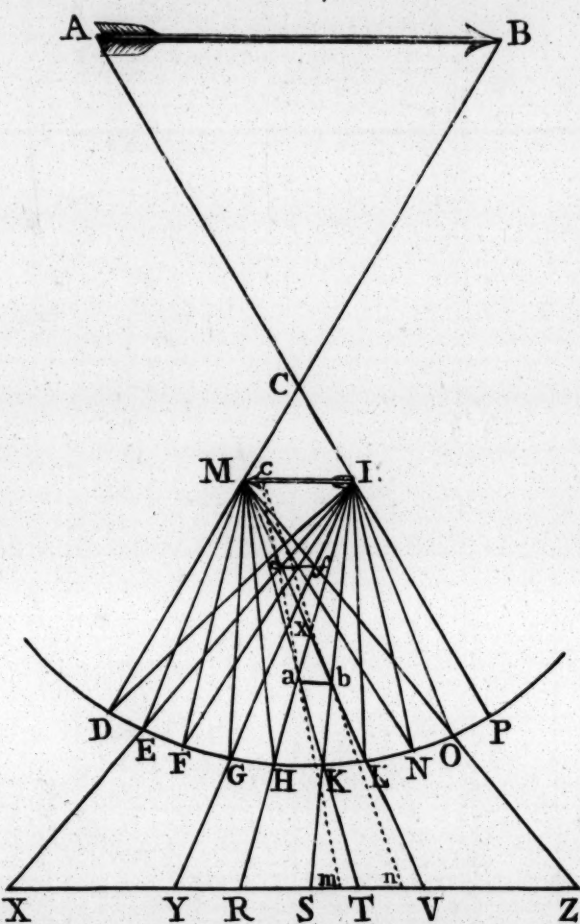
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Part III. Plate IX. p. 136.

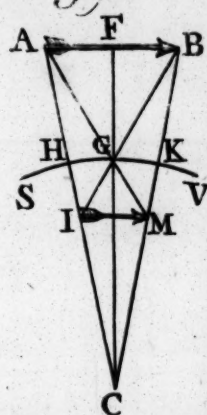
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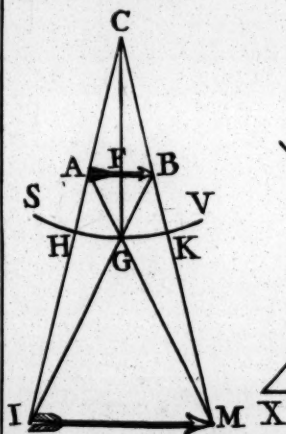
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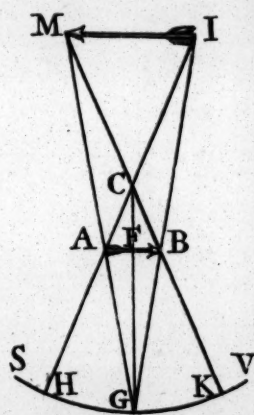
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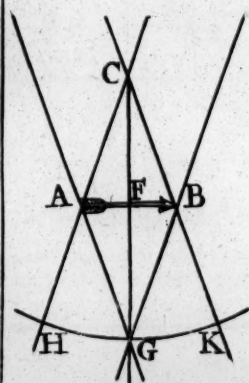
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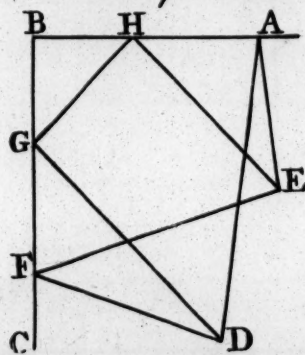
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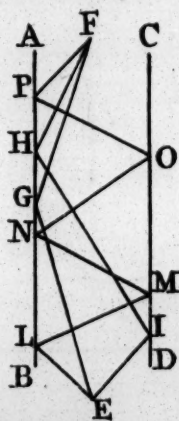
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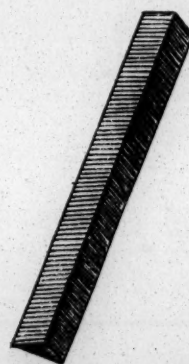
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To render this Proof complete, we must now shew that these Dispositions of the Rays of Light, to produce some one Colour, and some another, which manifest themselves after being refracted, are not wrought by any Action of the Prism upon them, but are originally inherent in those Rays; and that the Prism only affords each *Species* an Occasion of shewing its distinct Quality, by separating them one from the other, which before, while they were blended together in the unfracted Light of the Sun, lay concealed.

This will be proved by the following Experiment. Things remaining as in the foregoing one,

$1, \frac{8}{9}, \frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{9}{16}, \frac{1}{2}$, respectively, then shall BD and BF be the Limits of the Sines of Refraction of the *Violet* Rays; that is, the *violet* coloured Rays shall not all of them have precisely the same Sine of Refraction, but none of them shall have a greater Sine than BD nor a less than BF, though there be *Violet* coloured Rays which answer to any Sine of Refraction that can be taken between these two. In the same manner BF and BG are the Limits of the Sines of Refraction of the *Indigo*; BG and BH are the Limits belonging to the *Blue*; BH and BI, the Limits pertaining to the *Green*; BI and BK, the Limits for the *Yellow*; BK and BL, the Limits for the *Orange* coloured Rays; and lastly BL and BC, those of the Sines of Refraction belonging to the *Red*.

And particularly, when Light passes out of Glass into Air, if the Sine of its Angle of Incidence be 50, the Sine of the Angle of Refraction of the *Red* will be between 77 and $77\frac{1}{8}$, of the *Orange* coloured between $77\frac{1}{8}$ and $77\frac{1}{3}$, of the *Yellow* between $77\frac{1}{3}$ and $77\frac{2}{3}$, of the *Green* between $77\frac{2}{3}$ and $77\frac{1}{2}$, of the *Blue* between $77\frac{1}{2}$ and $77\frac{2}{3}$, of the *Indigo* between $77\frac{2}{3}$ and $77\frac{7}{9}$ and of the *Violet* coloured Rays between $77\frac{7}{9}$ and 78.

let

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let another Prism, as NO (Fig. 68) be placed either close to, or at some Distance from the first, in a perpendicular Situation with respect to the former, so that it may refract the Rays issuing from the first sideways. Now, if this Prism could separate the Light which falls upon it into coloured Rays, as the other did, it would divide the Image breadthwise into Colours, as before it was divided lengthwise; but no such Thing is observable: For the Image shall only be thrown out of the perpendicular Situation LM into the oblique one PQ; the upper Parts, which were more refracted in the former Case, being more refracted in this, and therefore made to recede farther sideways from their former Situation L, than the lower ones are from M. And farther, each Colour shall be uniform from Side to Side in the oblique Image, as well as in the perpendicular one.

If there be any Objection against the Sufficiency of this Proof, it must be, that the Rays, when they fall upon the second Prism, are not all in like Circumstances, with regard to their Inclination to its Surface; I shall therefore, to obviate that Objection, add one more Experiment which seems to be peculiarly adapted to that Purpose. It is as follows:

Two boards AB, CD, (Fig. 69) being erected in a darkened Room at a proper Distance, one of them AB, being near the Window-shutter EF, a Space being only left for the Prism

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Prism GHI to be placed between them; so that Part of the Rays, which enter the Hole M, may, after passing through the Prism, be transmitted through a smaller Hole K made in the Board AB, and passing on from thence go out at another Hole L, made in the Board CD, of the same Size as the Hole K, and small enough to transmit the Rays of one Colour only at a Time: Let another Prism PQR be placed behind the Board CD to receive the Rays passing through the Holes K and L, and after Refraction by that Prism, let the Rays fall upon the white Surface ST. Suppose, first, the *Violet* Light to pass through the Holes, and to be refracted by the Prism PQR to *s*, which, if that Prism were not there, would have passed on to W. If the Prism GHI be turned about slowly, so that the incident Ray ZY may fall more obliquely upon it, while the Boards and the other Prism remain fixed, in a little Time another Colour, suppose *Indigo*, which we may suppose before to have proceeded to *i*, will pass through the Holes K and L, and, if the Prism PQR were away, would proceed like the former Rays to the same Point W. Now the Refraction of this Prism will not carry these Rays to *s* as it did the other, but to some Place less distant from W, as to *t*. But it is manifest that the Holes K and L being in the same Situation in each Case, both Sorts of Rays enter the Prism PQR under the same
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Circumstances, for they are equally inclined to its Surface RP, and enter it at the same Point thereof; which shews that the one *Species* is more diverted out of its Course by Refraction than the other is, when the Circumstances of Incidence are the same in each. Farther, if the Prism GHI be turned about till the Rays which exhibit *Blue* pass through the Hole L, these will fall upon the Surface ST below *t*, as at *u*, and therefore are subject to a less Degree of Refraction than such as produce *Indigo*. And thus by proceeding it will be found that the *Green* is less refracted than the *Blue*, and so of the remaining Colours, according to the Order in which they are represented in an Image formed by a single Prism (*g*).

(*g*) There are Abundance of Experiments made by the Author of this Doctrine and others for the Confirmation of it, as was observed above. To give them all at full Length would tire the Reader. As that if a Body be painted one half Red, and the other Blue, and then viewed through a Prism; the apparent Place of the one half shall be different from that of the other: And if it be painted with a Mixture of these two Colours, it shall appear confused and deformed. Both which evidently shew that the Rays, which each of these Colours reflect, suffer different Degrees of Refraction in passing through the Prism. And if two Bodies be painted, the one Red and the other Blue, and the Rays which flow from them be made to pass through a convex *Lens*, the *Focus* made by the Concurrence of the Rays which flow from that which is painted with Blue, shall fall at a less Distance behind the *Lens*, than that which is made by those which come from the Red one. See the Experiment made by Dr. Desaguliers in a very accurate Manner (Philosoph. Transf. No. 426) in Opposition to Signor Ritzetti, who disputes the Conclusiveness of Sir Isaac Newton's Experiments.

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And also each *Species* of Rays is disposed to excite in us the *Idea* of a different Colour.

This is sufficiently clear from what has been already said, and is farther confirmed by what follows, *viz.* That whatever *Species* of Rays are thrown upon any Body, they make that Body appear of their own Colour. Thus *Minium* in *red* Light appears of its own Colour; but in *yellow* Light it appears yellow; and in *green* Light it appears green; in *blue*, blue; and in *violet-purple* coloured Light it appears of a purple Colour: In like manner Verdigrease will put on the Appearance of that Colour in which it is placed. But each of these Bodies appears most luminous and bright when enlightned with its own Colour, and dimmest in such as are most remote from that. 'Tis certain therefore each Ray is disposed to excite its own Colour, which is neither to be alter'd by Refraction nor Reflection,

Thus much in Confirmation of the first Part of the Proposition, *viz.* That there are different *Species* of Light, that each *Species* is disposed to suffer a different Degree of Refrangibility, and to excite in us the *Idea* of a different Colour. We proceed now to the second Part of the Proposition, *viz.*

2. That Bodies appear of that Colour, which results from a Composition of those Colours,

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which

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which the several *Species* they reflect are disposed to excite.

We have just now seen that each Ray, whatever be the Colour of the Body it is reflected from, is able to excite no other *Idea* than that of its own Colour, and that coloured Bodies reflect not all the different Sorts of Rays that fall upon them in equal Plenty; but some Sorts, *viz.* those of their own Colour, much more copiously than others. We will now proceed to shew, that the other Colours may be produced from a Mixture of those seven, which Rays of Light when separated by a Prism are disposed to exhibit. From whence it will be rational to conclude, that Bodies appear of that Colour which arises from the Mixture of those which they reflect.

1. All the *prismatic* Colours (*viz.* those which are made by the Prism) mixed together appear White a little inclining to Yellow, such as is that of the Sun's Light.

To shew this, let a convex *Lens* be placed between the Prism and the Paper which receives the Image, in order that the Rays separated by it may be collected into a *Focus*; and let the *Focus* fall upon the Paper, then will the Spot where it falls appear White. And that the Whiteness of this focal Point is owing to the Union of those Colours appears from hence, that if we remove the Paper from the
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focal Point, and suffer the Rays to cross each other in the *Focus*, and if when they have proceeded to some Distance beyond, they be then received upon the Paper, the same coloured Image will be exhibited, but inverted, because the Rays cross each other in the *Focus*; an evident Proof that the Whiteness of the Spot was owing to nothing but the Mixture of the Rays constituting the several Colours of the Image. But if the Rays of any particular Colour be intercepted before they are collected in the said Spot, it then appears not only of a different Colour from what it did before, but different from any of the *prismatic* Colours taken separately,

Or if the Circumference of a Wheel be painted with the *prismatic* Colours taken in the same Proportion with respect to each other in which they are exhibited in the Image made by the Prism, and the Wheel be turned swiftly about, the Circumference of that Wheel shall appear *white*: If they are taken in other Proportions, the Colour of the Wheel when turned about will vary accordingly. From whence this Part of the Proposition is also abundantly clear.

No Composition of these Colours will produce Black: That being no Colour, but the Defect, or Absence of all Colour whatever.

That *Species* of Light, which is disposed to suffer a greater Degree of Refraction, requires

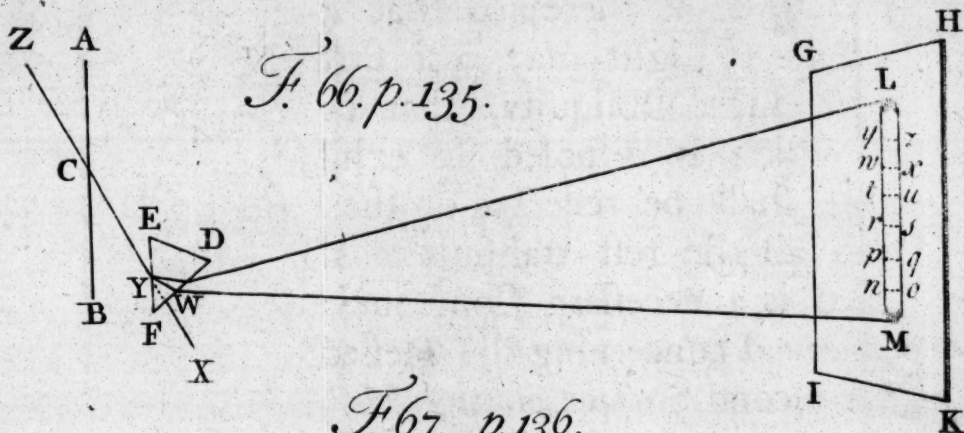
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proportionally less Obliquity at the second Surface of any *Medium* to occasion a *total Reflection* of it there; so that it is possible that a Ray of Light may pass through a *Medium* with such Obliquity, that only that Part of it which is disposed to exhibit a *violet* Colour shall be reflected at the second Surface, and all the rest transmitted there. This indeed is a necessary Consequence of what was observed concerning the Reflection of Light at the second Surface of any *Medium*, (Chap. 8) *viz.* that the Reflection of a Ray is total, when the Obliquity of the incident Ray is such, that the Angle of Refraction ought to be equal to, or to exceed a right one. I say this is a Consequence of that, because the Angle of Refraction of the *violet* coloured Light is larger than the Angle of Refraction of any other, though their Angles of Incidence be equal. And accordingly thus it happens, as appears by the following Experiment.

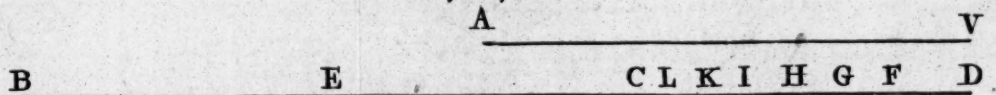
Let A B (Fig. 70) represent the Window-shutter of a darkened Room; C an Hole to let in a Ray of the Sun; DEF, GHI, two Prisms so applied together that the Sides EF and GI be contiguous, and the Sides DF and GH parallel: In this Situation Light will pass through them without any Separation into Colours; for the opposite Sides being parallel, if the Rays are refracted one Way where they

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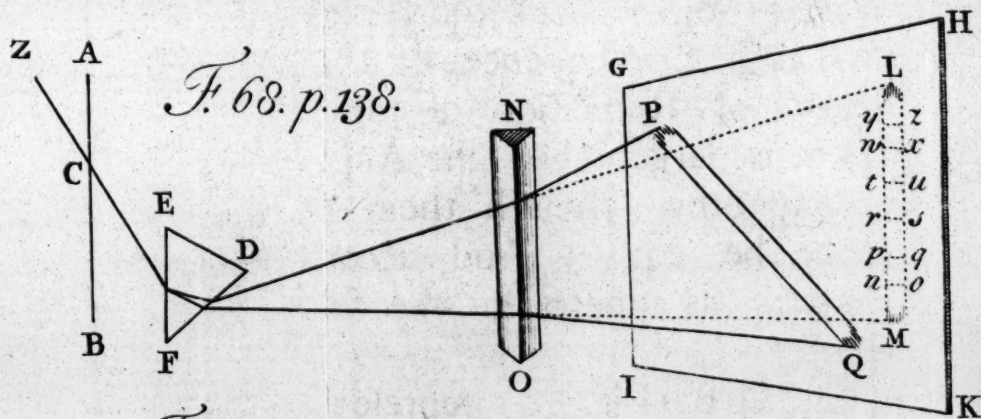
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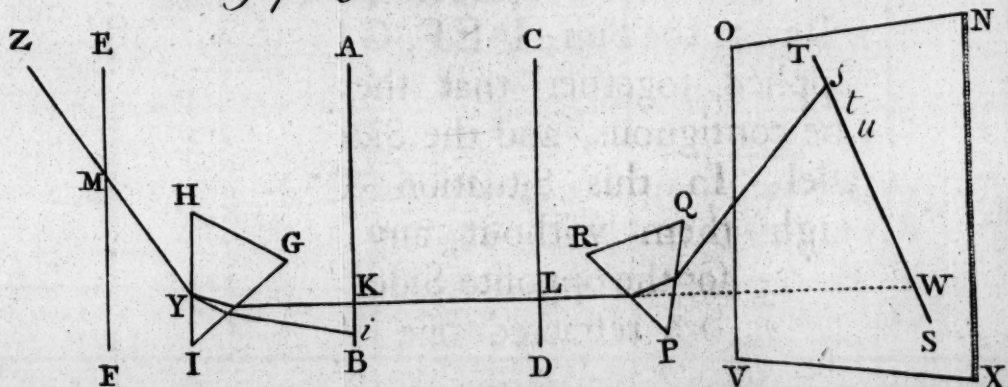
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go in, they will be as much refracted the contrary Way where they go out (See Page 51.) But if it be afterwards received by a third Prism KLM, it will be divided so as to form upon any white Body NOYU the usual Colours, *Violet* at *l*, *Indigo* at *m*, *Blue* at *n*, and *Red* at *r*. Now let it be supposed that the Surfaces EF and GI are not quite close together, but that the Rays, in passing from one to the other, pass through a *Medium* (*viz.* the Air) of different Density from that of the Prisms: And that the Ray ZC is not so much inclined to the second Surface of the first Prism as to cause a total Reflection of any one *Species* there; then will Part only of each *Species* be reflected and Part transmitted, agreeably to what was observed (Chapter 8) concerning the Manner of Reflection. Let now the reflected Rays be received by a fourth Prism TXV; these, after passing through it, will paint upon a white Surface RS the Colours of the Prism, *viz.* *Red* at *s*, *Orange* at *t*, *Yellow* at *v*, and *Violet* at *z*. Let now the Prisms DEF, GHI, be slowly turned about, keeping still the same Situation with respect to each other, until the Obliquity of the Rays ZC to the Surface EF be so far increased, that there shall begin to be a total Reflection of them there. In which Case it is observable, that first of all the *violet* Light will be *totally* reflected, and will therefore disappear

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at *l*, appearing instead thereof at *z*, and increasing the *Violet* Light which fell there before. And when the Rays *Z C* become more oblique by the Prisms being turned farther about, the *Indigo* shall be totally reflected, disappearing at *m*, but falling upon *y*, and making the *Violet* there more intense, And by turning the Prisms still farther about, all the remaining Colours will be successively removed from the Surface *PQ* to *RS*.



CHAP.

C H A P. XII.

Of the Qualifications in Bodies, which dispose them to reflect the Rays of different Colours.

WE are now to inquire what it is that gives Bodies this Power of reflecting, some one sort of Rays most copiously, and some another. And this is probably no other than the different Magnitude of the Particles whereof they are composed; as will appear from the following Observations.

If Water be prepared with Soap so as to render it sufficiently tenacious, and then blown up into a Bubble; it is observable, that as the Bubble grows thinner, and thinner (as it will do by reason of the Water's continually running down from the Top of it, till it breaks) different Colours will arise one after another at the Top of the Bubble, spreading themselves into Rings, and descending till they vanish at the Bottom in the same Order they arose at the Top. Thus, in an Experiment of this Kind, tried by Sir *Isaac Newton*, the Colours arose in this Order; first *Red*, then *Blue*; to which succeeded *Red* a second Time, and

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Blue immediately followed; after that *Red* a third Time, succeeded by *Blue*; to which followed a fourth *Red*, but succeeded by *Green*; after this a more numerous Order of Colours, first *Red*, then *Yellow*, next *Green* and after that *Blue*, and at last *Purple*; then again, *Red*, *Yellow*, *Green*, *Blue*, *Violet* followed each other; and the last Order of Colours that arose was *Red*, *Yellow*, *White*, *Blue*; to which succeeded a *dark* Spot that afforded scarce any Light, though it was observed to cause some very obscure Reflection, for the Image of the Sun or a Candle might be faintly discerned in it; and this last Spot spread itself more and more, till the Bubble broke.

Now 'tis apparent that the only Reason, why those different Colours succeeded each other at the Top of the Bubble in the abovementioned Manner, was because its Thickness in that Part continually varied, till it broke. It remained therefore to examine what was the Thickness of the Bubble at the Top, at the Time it exhibited each particular Colour. And this was effected by the following Contrivance, *viz.* by taking the Object-Glass of a long Telescope, such having but a very small Degree of Convexity, and placing it upon a flat Glass: These Glasses by Reason of the Convexity of the former would touch but in one Point, and the Distance between them, where they did not touch, would be exceedingly small, but larger the farther

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we consider it from the Point of Contact. Now Water being put between these Glasses, the same Colours appeared as in the Bubble, in the Form of Circles or Rings surrounding the Point appeared black, like the Top of the Bubble when it was thinnest. Next to this Spot lay a blue Circle, and next without that a white one, and so on in the same, but contrary Order to that in which the Colours arose on the Top of the Bubble.

Now the Distance between the Glasses, that is, the Thickness of the Body of Water between them, where it exhibited any one Colour of a particular Order, was equal to the Thickness of the Bubble at the Time the same Colour appeared upon it. For though the *Medium* the Light must pass through to come at the Water is in one case, Glass, and in the other, Air; that makes no Difference in the *Species* of the Colour reflected from the Water: For Pieces of *Muscovy* Glass, made thin enough to appear coloured, would have their Colours faded, but not the *Species* of them altered by being made wet with Water. But it was found that transparent Bodies of different Density would not, under the same Thicknesses, exhibit the same Colours: For if the forementioned Glasses were laid upon each other without any Water between them, the Air between them would then afford the same Colours as the Water,
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but more expanded, so that each Ring had a larger Diameter, though they bore all the same Proportion to each other; so that the Thickness of the Air proper to reflect each Colour was in the same Proportion larger, than the Thickness of the Water adapted to reflect the same.

Farther, all the Light which is not reflected by the thin Substances, whether of Air or Water contained between the Glasses, is transmitted through them; for when viewed from the other Side, they exhibit also coloured Rings as before, but in a contrary Order; for the middle Spot, which in the other View appears black for Want of reflected Light, now looks perfectly white; next without this Spot the Light appears tinged with a yellowish *Red*; where the *White* appeared before, it now seems *Black*; and so of the rest.

It is farther observable, that the forementioned thin Plates, whether of Air or Water, do not appear of the same Colour when viewed obliquely, as when seen direct: For if the Rings and Colours between a convex and plain Glass be viewed first in a direct Manner, and then under different Degrees of Obliquity, the Rings will be observed to dilate themselves as the Obliquity is increased. But a Plate of Air between the Glasses alters its Colour much sooner than the Water in the Bubble which is surrounded with Air. For in the Water
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when viewed obliquely the same Colour might be seen at more than twelve Times the Thickness it appeared at under a direct View; but when the Air was viewed under such an Obliquity that the Thickness of the Plate, where it was observed, was but half as much again as when it was viewed directly, a different Colour appeared.

Lastly, the same Colour reflected from a denser Substance reduced to a thin Plate, and surrounded by a rarer, will be more brisk, than the same Colour, when reflected from a thin Plate formed of the rarer Substance and surrounded by the denser, as was found by blowing Glass very thin, which exhibited in the open Air more vivid Colours, than the Air does between two Glasses.

As to the Thickness of the Plate of Air by which the several Colours were reflected, it was found by carefully measuring the Distances of the Rings from the Point where the Glasses touched, that the Distance between the Glasses where the first Order of Colours was reflected, was from $\frac{1}{178000}$ to $\frac{2}{178000}$ Part of an Inch; that where the second, was from $\frac{3}{178000}$ to $\frac{4}{178000}$; that were the third, from $\frac{5}{178000}$ to $\frac{6}{178000}$, and so on in a *Series* of the odd Numbers: And that the Distance of the Glasses, where the first Order of Colours that was transmitted passed through, was from 0 to $\frac{1}{178000}$ Part of

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an Inch; that where the second, was from $\frac{2}{178000}$ to $\frac{3}{178000}$; that where the third, from $\frac{3}{178000}$ to $\frac{5}{178000}$, and so on in a *Series* of the even Numbers. And the Thickness of a Plate of Water, where it reflected or transmitted the same Colours, was $\frac{3}{4}$ of the Thickness of the Plate of Air.

Now we learn from Experiments made with the Microscope, that the least Parts of almost all Bodies are transparent; or the same may be experienced in the following Manner: Take a very thin Plate of the opaquest Body, and the Room being darkened apply it to a small Hole in the Window-shutter, and it will sufficiently discover its Transparency. This Experiment cannot be so well performed with a white Body, because of the strong reflective Power in such; but even those, when dissolved in *Aqua Fortis* or other proper *Menstruum*, do also become transparent. Wherefore if we should suppose any Body reduced to a Thinness proper to produce any particular Colour and then broken into Fragments, in all Probability each Fragment would exhibit that Colour, and an Heap of such Fragments would constitute a Body of that Colour: So that the Cause, why some Bodies reflect one Sort of Rays most copiously, and some another, is probably no other than the different Magnitude of their constituent Particles

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cles (*b*). This will be farther confirmed by Particulars. The Colours in the same Part of a Peacock's Tail vary as the Tail changes its Posture, with respect to the Eye; just so the thin Plates of Air or Water appear of a different Colour in the same Plate when view'd directly, from what they do when seen obliquely, as was observed above. The Colours of Silks, Cloths, and other Substances, which Water or Oil can intimately penetrate, become faint and dull by being wet with such Fluids, and recover their Brightness when dry; just as we observed, that Plates of *Muscovy* Glass grew faint and dim by wetting. All which Particulars, and many more that might be produced, give abundant Proof of the present Point (*i*).

(*b*) This Sir *Isaac Newton* thinks a probable Ground for making Conjecture concerning the Magnitude of the constituent Particles of Bodies. The Green of Vegetables he takes to be of the third Order, as likewise the Blue of Syrup of Violets. The azure Colour of the Sky, he thinks is of the first Order, as also the most intense and luminous White; but if it is less strong, he then conjectures it to be a Mixture of the Colours of all Orders. Of the latter Sort he takes the Colour of Linen, Paper, and such like Substances to be; but white Metals to be of the former Sort. For producing *Black*, the Particles must be smaller than for exhibiting any of the Colours, *viz.* of a Size answering to the Thickness of the Bubble where it reflected little or no Light, and for that Reason appeared colourless.

(*i*) See Sir *Isaac Newton's* Optics, *passim*.

C H A P. XIII.

Of the Cause of Opacity and Transparency in Bodies.

WHEN two *Mediums* or transparent Substances of equal Density are contiguous, or as near to each other as the Glasses were, where the Light was wholly transmitted, in the Experiment made with the Object Glass mentioned in the foregoing Chapter, a Ray of Light will pass from one to the other without suffering either Reflection or Refraction; but if they differ in Density, the Light will undergo both; Part of it being reflected and Part refracted. Just so it is with a Ray of Light in passing through the different Particles of the same Body. For Instance, if when the Ray has passed through any one Particle of a Body, it finds another contiguous to it, it will enter that Particle without Interruption; but if at its Emerision out of that Particle, it enters a Pore sufficiently large, Part of it will be transmitted and Part reflected. Thus will the Light every time it enters a Pore, unless it be an exceeding small one, be in part reflected: So that nothing more seems necessary to render a Body opaque, than that the Particles, of which it is composed, touch but in few Points, and that the Pores

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Pores of it be very numerous and large; so that the Light, which enters it, may by numerous Refractions and Reflections be stifled and lost within it. On the contrary, if the Particles of a Body touch each other in many Points, and its Pores be few and small, or filled with a Substance of nearly an equal Density with the Particles of the Body, that Body will be transparent.

In Confirmation of this, we may observe, that opaque Bodies become transparent by filling their Pores with a Substance nearly of the same Density with that of their solid Parts: As when Paper is made wet with Water or Oil; when Linen Cloth is dipped in Water, oiled, or varnished, or the *Oculus Mundi* Stone steeped in Water. Besides, as filling the Pores of an opaque Body renders it transparent, so on the other hand evacuating the Pores of a Body that is transparent, or separating the Parts of it from one another, makes it opaque; as Salt or wet Paper by being dried, and Glass by being powdered, lose their Transparency, or Water beat up into Froth.

Besides which Instances, abundance more might be brought in Confirmation of what is here laid down; but these are sufficient.

But because it may be Matter of Surprise, that Bodies should be sufficiently porous to transmit Light in that plenty we observe they do,

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do, and yet be hard or solid; I shall conclude the Subject of this Chapter and of the Doctrine of Light and Colours, with shewing the Consistency of such a Supposition. In order to which, let us imagine a Body whose constituent Particles are of a such a Form that, when laid together, the Vacancies between them may be equal in Bigness to the Particles; how this may be done, and yet the Body be hard, is easy to comprehend. Now the solid Parts of a Body thus formed will be but half its Bulk; and if we suppose each constituent Particle of this Body to be formed of less Particles with Vacuities between them, equal to each Particle as before, the solid Parts of this Body will then be but a fourth Part of its Bulk; and if each of these lesser Particles again be formed in the same Manner, the solid Parts of the Body shall be but one Eighth of its Bulk: And thus if the Composition be continued according to the same Rule, the solid Parts of the Body may be made to bear as small a Proportion to its whole Magnitude as shall be desired, notwithstanding which, the Body, by means of the Contiguity of the Parts, shall be capable of being hard in any Degree. Thus, Matter being infinitely divisible, any the least Portion of it may be supposed to be wrought into a Body of any designed Dimensions how great soever, and yet the Pores of that Body none of them

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them greater than the smallest Magnitude proposed at Pleasure; and yet the Parts of the Body shall so touch, that the Body itself shall be hard or solid. Which shews that the whole Globe of the Earth, nay, all the known Bodies of the Universe, for any thing that appears to us to the contrary, may be composed of no greater Quantity of Matter than what might be reduced into any determinate Space how small soever.



DISSERTATION II.

Of the Cause of Reflection of Light.

THE Opinions of Philosophers relating to the Cause of this difficult *Phænomenon*, are principally four, which I shall here lay down and examine particularly; after which I shall give my own Thoughts, concerning it.

And,

1. It was the Opinion of Philosophers before Sir *Isaac Newton* discovered the contrary, that Light is reflected by impinging upon the solid Parts of Bodies; but that it is not so, is clear from the following Reasons.

And first, it is not reflected at the first Surface of a Body by impinging against it.

For it is evident, that in order to the due and regular Reflection of Light, that is, that the reflected Rays should not be dispersed and scattered one from another, there ought to be no Rasures or Unevenness in the reflecting Surface large enough to bear a sensible Proportion to the Magnitude of a Ray of Light; because if the Surface abounds with such, the reflected Rays will rather be scattered like a Parcel of Pebbles thrown upon a rough Pavement, than reflected with that Regularity with which Light is observed to be from a well

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well polished Surface. Now those Surfaces which to our Senses appear perfectly smooth and well polished, are far from being so; for to polish is no other than to grind off the larger Eminences and Protuberances of the Metal with the rough and sharp Particles of Sand, Emery, or Putty, which must of Necessity leave behind them an Infinity of Rasures and Scratches, which though inconsiderable with regard to the former Roughnesses, and too minute to be discerned by us, must nevertheless bear a large Proportion to, if not vastly exceed the Magnitude of the Particles of Light.

Secondly, It is not reflected at the second Surface, by impinging against any solid Particles.

That it is not reflected by impinging upon the solid Particles which constitute this second Surface, is sufficiently clear from the foregoing Argument; the second Surfaces of Bodies being as incapable of a perfect Polish as the first; and it is farther confirmed from hence, viz. that the Quantity of Light reflected differs according to the different Density of the *Medium* behind the Body: And that it is not reflected by impinging upon the Particles which constitute the Surface of the *Medium* behind it, is evident, because the strongest Reflection of all at the second Surface of a Body, is when there is a *Vacuum* behind it. This therefore wants no farther Proof. (See the Man-

ner in which Light is reflected in Chapter the 8th.)

II. It has been thought by some (*k*), that it is reflected at the first Surface of a Body, by a repulsive Force equally diffused over it, and at the second, by an attractive Force.

1. If there be a repulsive Force diffused over the Surface of Bodies, that repels Rays of Light at all Times, then, since by increasing the Obliquity of a Ray we diminish its perpendicular Force (which is that only, whereby it must make its Way through this repulsive Force) however weakly that Force may be supposed to act, Rays of Light may be made to fall with so great a Degree of Obliquity on the reflecting Surface, that there shall be a *total* Reflection of them there (*l*), and not one Particle of Light be able to make its Way through, which is contrary to Observation; the Reflection of Light at the first Surface of a transparent Body being never total in any Obliquity

(*k*) See *Muſchenbrock*, Element. Phific. Cap. 35.

(*l*) *D.m.* Let AB (Fig. 71) represent the reflecting Surface, ABCD the repellent Power diffused over it, EF a Ray of Light incident upon it at the Point F, and let the Line EF by its Length expreſs the Force with which the Ray moves. This Force is reſolvable into the Forces EG and EH, or, which is the ſame thing, HF and GF, which latter is the ſole Force by which the Ray endeavours to pierce through the repulsive Power. But this Force may be diminished *in infinitum* by augmenting the Obliquity of the Ray EF, and therefore it may be made leſs than that of the repelling Power, in which Caſe the Ray will neceſſarily be reflected: And ſince the

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Obliquity whatever. The *Hypothesis* (s) therefore in this Particular must be false.

2. As to the Reflection at the second Surface by the attractive Force of the Body, this may be considered in two Respects; first, when the Reflection is *total*; secondly, when it is *partial*.

And first, in Cases where the Reflection is *total*, the Cause of it is undoubtedly that same attractive Force by which Light would be refracted in passing out of the same Body. This is manifest from that Analogy which is observable between the Reflection of Light at this second Surface, and its Refraction there. For otherwise, what can be the Reason that the total Reflection should begin just when the Obliquity of the incident Ray, at its Ar-

the same is demonstrable of any Ray, (let it move with a greater or less Force than the Ray EF) the Obliquity of the Rays may be so great, that there shall be a total Reflection of them. Q. E. D.

(m) And yet from an Experiment made by Sir *Isaac Newton*, (See his *Optics*, Book III) it appears that there is a repulsive Force between Light and some Bodies. The Experiment is as follows. If over an Hole in a Window-shutter be fixed a thin Piece of Lead, or the like, in which there is an Hole about the fortieth Part of an Inch in Diameter; and if when the Sun shines through that Hole, an Hair be held in its Ray at some Distance from the Hole; the Progress of the Rays after they have passed by the Hair will be as expressed in the 72d Figure, where A is a Section of an Hair, BC an Hole in a Window-shutter; BL, UF, DE, &c. Rays passing through it, in the Middle of which is placed the Hair A; and let RP be a Paper held at some Distance behind the Hair. Things being thus disposed, the Ray BL shall fall upon O, UF upon Q, DE upon R, GH upon P, &c. as represented in the Figure.

rival at the second Surface, is such, that the refracted Angle ought to be a right one; or when the Ray, were it not to return in Reflection, ought to pass on parallel to the Surface, without going from it? For in this Case it is evident, that it ought to be returned by this very Power, and in such Manner that the Angle of Reflection shall be equal to the Angle of Incidence: Just as a Stone thrown obliquely from the Earth, after it is so far turned out of its Course by the Attraction of the Earth, as to begin to move horizontally, or parallel to the Surface of the Earth, is then by the same Power made to return in a Curve similar to that which it described in its departure from the Earth, and so falls with the same Degree of Obliquity that it was thrown with.

But secondly, as to the Reflection at the second Surface, when it is *partial*; an attractive Force *uniformly* spread over it, as the Maintainers of this *Hypothesis* conceive it to be, can never be the Cause thereof. Because, it is inconceivable that the same Force, acting in the same Circumstances in every Respect, can sometimes reflect the *Violet* coloured Rays and transmit the *Red*, and at other times reflect the *Red* and transmit the *Violet*.

This Argument concludes equally against a repulsive Force *uniformly* diffused over the first

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first Surface of a Body, and reflecting Light there; because some Bodies reflect the *Violet* and transmit the *Red*, others reflect the *Red* and transmit the *Violet* at their first Surface; which cannot possibly be upon this Supposition, the Rays of which-ever of these Colours we suppose to be the strongest.

III. Some, being apprehensive of the Insufficiency of a repulsive and attractive Force diffused over the Surfaces of Bodies and acting uniformly, have supposed that by the Action of Light upon the Surfaces of Bodies the Matter of these Forces is put into an *undulatory Motion*, and that where the Surface of it is subsiding, Light is transmitted, and in those Places where it is rising, Light is reflected (*n*). But this seems not to advance us one Jot farther; for in those Cases, suppose where *Red* is reflected and *Violet* transmitted, how comes it to pass that the *Red* impinges only on those Parts when the Waves are rising, and the *Violet* when they are subsiding?

IV. The next *Hypothesis*, that I shall take notice of, is that remarkable one of Sir *Isaac Newton's Fits of easy Reflection and Transmission*, which I shall now explain and examine.

That Author, as far as I can apprehend his Meaning in this Particular, is of Opinion, that

(*n*) See *Muschenbroek Element, Physic. Cap. 35.*

Light, in its Passage from the luminous Body, is disposed to be *alternately* reflected by and transmitted through any refracting Surface it may meet with; that these Dispositions (which he calls *Fits of easy Reflection and easy Transmission*) return successively at equal Intervals: And that they are communicated to it at its first Emission out of the luminous Body it proceeds from, probably by some very subtle and elastic Substance diffused through the Universe, and that in the following Manner. As Bodies falling into Water, or passing through the Air cause Undulations in each, so the Rays of Light may excite Vibrations in this elastic Substance. The Quickness of which Vibrations depending on the Elasticity of the *Medium* (as the Quickness of the Vibrations in the Air, which propagate Sound, depend solely on the Elasticity of the Air, and not upon the Quickness of those in the sounding Body) the Motion of the Particles of it may be quicker than that of the Rays; and therefore when a Ray at the Instant it impinges upon any Surface, is in that Part of a Vibration of this elastic Substance which conspires with its Motion, it may be easily transmitted, and when it is in that Part of a Vibration which is contrary to its Motion, it may be reflected. He farther supposes, that when Light falls upon the first Surface of a Body, none is reflected there, but

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but all that happens to it there is, that every Ray that is not in a Fit of easy Transmission is there put into one, so that when they come at the *other* Side (for this elastic Substance easily pervading the Pores of Bodies, is capable of the same Vibrations within the Body as without it) the Rays of one Colour shall be in a Fit of easy Transmission, and those of another in a Fit of easy Reflection, according to the Thickness of the Body, the Intervals of the Fits being different in Rays of a different Kind. This very well accounts for the different Colours of the Bubble and thin Plate of Air and Water (mentioned in Chap. XII.) as is obvious enough; and likewise for the Reflection of Light at the second Surface of a thicker Body; for the Light reflected from thence is also observed to be coloured, and to form Rings according to the different Thickness of the Body, when not intermix'd and confounded with other Light, as will appear from the following Experiment. If a Piece of Glass be ground concave on one Side and convex on the other, both its Concavity and Convexity having one common Center, and if a Ray of Light be made to pass through a small Hole in a Piece of Paper held in that common Center, and be permitted to fall on the Glass; besides those Rays which are regularly reflected back to the Hole again, there will be others reflected

to the Paper, and form coloured Rings surrounding the Hole, not unlike those occasioned by the Reflection of Light from thin Plates. The same will happen if the Rays be reflected from a metalline *Speculum*, but the Light will not be *coloured*; which shews that the Colours arise from that Light which is reflected from the Back-side, and that in the following Manner: Beside that Light which is regularly reflected from the farther Surface of the Glass, there is some reflected irregularly, which passing from the back Surface under different Obliquities, does as it were pass through Glasses of different Thicknesses, and therefore is in Part reflected back again when it comes to the first Surface, and is in part transmitted through it, the transmitted Light, when received upon the white Paper, exhibiting the Rings of Colours abovementioned (o).

As to the Light which is supposed to be reflected at the first Surface, his Opinion seems to be, that it is not *there* reflected, as I observed above, but that it really enters the Surface, and is reflected from the Back-side of the first *Series* of Particles that lie therein; so that according as these Particles are larger or smaller, the Rays of Light which at their Entrance into them

(o) This Experiment succeeds better, when the Back part of the Glass is Quick-silvered over.

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(for they are transparent, whether the Body they compose be so or not, See Page 154) are thereby put into Fits of easy Transmission; at their Emerſion at the other Side are ſome in a Fit of easy Transmission, others in a Fit of easy Reflection, according as the Interval of their Fits are large or ſmall. So that the Particles of a Body may be of ſuch a Size that they ſhall reflect the *Red* and transmit the *Violet*; or that they may reflect the *Violet* and transmit the *Red*; or, in general, that the ſtrongeſt and moſt forcible Rays may be transmitted, while the weaker are reflected; or the weaker may be transmitted, while the ſtronger are reflected.

Thus I have endeavoured to clear up the Account Sir ISAAC NEWTON has left us of his own Sentiments concerning this Matter. But after all, I cannot ſay that I think his Solution the true one. It is too much clogged with Suppoſitions; neither is it conſonant to that Simplicity, Uniformity, and Regularity, with which Nature is every where obſerved to act. The Time will come, when the Principles of *Attraction* and *Repulſion* will be found alone ſufficient to account for this perplexing *Phænomenon*. Would any one that has a *Genius* for a Work of this Kind, and *Opportunity* to make the neceſſary Experiments, aſſume thoſe Sir *Iſaac Newton* has left, and

add others, as his Judgment should direct him; he would soon be able to give us as easy and simple a Solution of the *Reflection of Light*, as we already have of any other *Phænomenon* whatever (*p*).

(*p*). Perhaps it may be of Service to one that shall undertake this, to acquaint him of a Fact relating to this Matter, which every Philosopher is not apprised of, *viz.* That a Piece of Iron, when heated, assumes all the Colours of the Rainbow before it becomes red hot.



DISSERTATION III.

Of Microscopes and Telescopes.

THAT the Telescope is of modern Invention is most certain; neither does it appear that Microscopes or Optic Glasses of any Kind were known to the Ancients, though there are two Passages brought to shew that such Glasses were of Use among them. The one is quoted by *Pancirollus* from *Plautus*, *Cedo Vitrum, necesse est Conspicillo uti*; the other is taken from *Pliny*, *C. Julius Medicus, dum inungit Specillum, &c.* The former of these Quotations is a Fiction, no such Passage being to be found in the Writings of *Plautus* (q); and the Word *Specillum* in *Pliny* is not to be understood of an Optic-glass of any Kind, but of a Probe or other Instrument made use of by the Surgeons of that Time (r).

It is contended, that *Alexander de Spina*, a Native of *Pisa*, was the first that made the Use of Glasses known to the World; but our Countryman, Friar *Bacon*, who died one and twenty Years before him (s), was, in

(q) Vid. Lettere Memorabili del Abbate Michele Giustiani. Parte terza. Let. 16.

(r) See *Molineux's* Dioptrics, Part II. Ch. 6.

(s) In the Year 1292.

all Probability, acquainted with them first, for he wrote a Book of *Perspective*, in which, he plainly shews that he did not only understand the Nature of convex and concave Glasses, but the Use of them when combined in Telescopes (*t*); though he no where in that Treatise discovers the Manner in which they are to be put together.

The Telescope with the concave Eye-glass was first invented by a Mechanic of *Middleburg* in *Zeland*, called *Z. Johannides*, about the Year 1590, though *J. Lipperboy*, another *Dutchman*, is Candidate for the same Discovery (*u*). From whence, this Sort of Telescope is called *Tubus Batavus* (*x*).

Franciscus Fontana, a *Neapolitan*, contends, that he was the first Contriver of the Telescope composed of two convex Glasses, which is now the common astronomical Telescope (*y*); and *Rbeita* pretends to be the first that rendered that Telescope fit for terrestrial Uses, by adding two Eye-glasses to it (*z*). This Kind of Telescope is called *dioptrical*.

(*t*) See his *Perspective*, Part III. and his *Epistola ad Parisiensem*, Cap. 5.

(*u*) Vid. *Borellus* de vero Telescopii Inventore.

(*x*) This is by some called *Galileo's* Telescope, as invented by him; but *Galileo* acknowledges, that it was upon hearing that the *Dutchman* had contrived one, that he effected his.

(*y*) See his *Observationes cœlestium terrestriumq; Rerum*.

(*z*) *Ocul. Enoch & Eliæ*. Lib. IV. *Hist. Acad. Reg. Lib. V.* Sect. 1. Cap. 7.

The *Catadioptrical* or reflecting Telescope was invented by Sir *Isaac Newton*; of which we shall give a particular Description when we have explained the former Sort, and shewn the Defects of them.

Microscopes are of two Kinds, Simple and Compound. The first Sort consists of one Glass; the other of two or more.

The Simple Microscope is no other than a convex *Lens*, through which, as we have shewn (Chap. VII. Prop. 3) Objects appear magnified.

An Object seen through this Microscope appears magnified nearly in that Proportion which the Distance, at which an Object would be seen distinctly with the naked Eye, bears to the focal Distance of the Microscope.

Thus, let AB (Fig. 73) represent the Microscope, CD an Object placed at the focal Distance of parallel Rays, or something nearer, that the Rays of the same Pencil may be parallel to each other, or rather diverging in a small Degree, when they enter the Eye (this Circumstance being requisite to distinct Vision :) And let the Microscope be so small, that all the Rays that pass through it from the Object may enter the Pupil of the Eye EF at the same time, when placed close to it

as

as in the Figure (for, unless it be so small, it will scarce magnify sufficiently to obtain the Name of a Microscope.) Things being thus disposed, the Angle under which this Object appears will be GIH , or CID ; but this is nearly the same it would have appeared under, had there been no Microscope interposed (*a*). Notwithstanding which, the Object is properly enough said to appear magnified by this Microscope, because, without that, it could not have been seen distinctly at so small a Distance from the Eye, but must have been situated eight or ten Inches from it; and therefore, since Objects appear under a larger Angle the nearer they are to the naked Eye, this Object appears larger, or is magnified by means of the Microscope, in Proportion as it is seen distinctly at a less Distance with it than without it. Objects therefore are magnified hereby nearly in that Proportion, which the Distance of distinct Vision bears to the focal Distance of the Microscope (*b*).

(*a*) For had there been no Microscope interposed, the Angle CID would have been in the Middle of the Pupil, (see the Note in Page 61) and therefore something less, as being farther from the Object; but this is an *accidental* Circumstance depending on the Thickness of the *Lens*, and its Distance from the Center of the Pupil, and therefore not considered in the Theory.

(*b*) An Object will also appear distinct, though it be situated at a very small Distance from the Eye, by being viewed through a small Hole in a piece of Paper, the Reason of which was explained in Note Page 64. But then this Hole must be made so very small, that, unless the Object be strongly illuminated, it will appear very obscurely through it.

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Differt. III. *and* Telescopes. 173

The Form of a Compound Microscope is expressed in the 74th Figure, where AB represents a small convex *Lens*, whose focal Distance is such, that Rays flowing from the Point C may be collected in D; and EF is a larger *Lens* whose *Focus* of parallel Rays coincides with the Point D; and FG represents an Eye so situated that Rays proceeding from an Object at KL may enter the Pupil of it, after having passed through both Glasses. Things being disposed in this Manner, the Object KL will appear *magnified* and also *distinct*.

For first, let RCS represent a Pencil of Rays flowing from the Point C, these will meet their *Axis* again in the Point D by Supposition, and crossing there will enter the *Lens* EF diverging from its *Focus* of parallel Rays, and will therefore enter the Pupil of the Eye in Directions parallel to each other, and concur upon the *Retina* at Q; the Object will therefore appear *distinct*.

Secondly, a Pencil of Rays flowing from another Point of the Object, as L, will meet their *Axis* in M, and diverging from thence will, after being refracted by the *Lens* EF, become parallel with respect to each other; but with respect to the former they will converge, because with regard to them, they diverged before they passed through the *Lens* EF from I, a Point more distant than its

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Focus of parallel Rays They will consequently cross them at some Distance from it, suppose at H, where the Pupil of an Eye being placed to receive them, the Point L will be represented at O. And for the like Reason, the Point K being represented at P, the Object will be seen under the Angle PHO or EHF, which, as will be demonstrated in Note (e), is much larger than that under which it would have appeared to the naked Eye (c).

That Glass AB, which is situated next the Object, is called the *Object-glass*; that which is placed next the Eye, the *Eye-glass* (d).

The Proportion of magnifying, in a Microscope of this Kind, is nearly in a *Ratio* compounded of the Proportion which the Distance of the Image from the Object-glass bears to its Distance from the Eye-glass; and of that which the Distance of the Ob-

(c) It may be remarked here, that when we view an Object through an Instrument of this Kind, we are then in reality looking at the Image of that Object through a single Microscope. Thus, it is MN the Image of the Object KL, formed by the Concurrence of the Rays of each Pencil in their respective *Foci*, which we see through the *Lens* or single Microscope EF; so that the Addition of the Glass AB is only that we may have an Image of the Object to look at, larger than the Object itself.

(d) In some Microscopes there is a third Glass placed between the Object-glass and the Image, and is called a *Middle-glass*. This is placed there only to bring the Rays to a *Focus* the sooner, in order that the Image may fall nearer the Object-glass than it otherwise would do.

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ject from the Eye bears to its Distance from the Object-glass (*e*).

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(*e*) *Lemma*. Let EF (in Fig. 75) be a *Lens*, whose *Focus* of parallel Rays is D; and let Rays flowing from the Point I be collected in H; I say, that if X be the Middle of the *Lens*, IX will be to XH, as ID to DX.

Demonstration of the Lemma. On D the *Focus* of parallel Rays erect the Perpendicular DM; and let IE be an Incident, and EH a refracted Ray: Then whereas Rays flowing from D, and passing through the *Lens*, would after Refraction be parallel to each other and to DX, a Line drawn from thence through the Middle of the *Lens*; Rays flowing from M and passing through the *Lens*, will also be refracted into Lines parallel to themselves, and consequently to MX that which passes through the Middle of the *Lens*: consequently EH is parallel to MX; the Triangles therefore IMX and IEH are similar; and therefore IM is to ME as IX to XH: But IM is to ME also as ID to DX, the Triangles IMD and IEX being also similar; IX is therefore to XH as ID to DX. Q. E. D.

Demonstration of the Proposition. To avoid the Confusion which might arise from the Multiplicity of Lines in the 74th Figure, let only CXH and LEH, viz. the *Axes* of the Rays which proceed from the Points C and L, be represented as in Fig. 75, and draw the Line LH, then will CHL be the Angle under which half the Object would be seen by the naked Eye at H; but EHX is the Angle under which the same Half appears when viewed through the Microscope. Now this Angle is to the former in a *Ratio* compounded of the Angle EHX to EIX, and of the same EIX, or (which is equal to it because vertical) CIL to CHL; because the *Ratio* that any two Quantities bear to each other, is compounded of the *Ratio* which the first bears to any other, and of the *Ratio* which that other bears to the Second. But the first of these *Ratio*'s, viz. EHX to EIX, is as IX to XH, or which, as demonstrated in the *Lemma*, is the same thing as ID to DX; that is, as the Distance of the Image from the Object-glass to its Distance from the Eye-glass: And the other *Ratio*, viz. CIL to CHL, is as CH to CI, that is, as the Distance of the Object from the Eye to its Distance from the Object-glass. Therefore, &c. Q. E. D.

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After what has been said concerning the Structure of the Compound Microscope, and the Manner in which the Rays pass through it to the Eye, the Nature of the common *astronomical* Telescope will easily be understood, for it differs from the Microscope only in that the Object is placed at so great a Distance from it, that the Rays of the same Pencil, flowing from thence, may be considered as falling parallel to one another upon the Object-glass; and therefore the Image made by that Glass is looked upon as coincident with its *Focus* of parallel Rays.

The 76th Figure will render this very plain, in which ABC is the Object emitting the several Pencils of Rays ADF, BDF, &c. but supposed to be at so great a Distance from the Object-glass DF, that the Rays of the same Pencil may be considered as parallel to each other, they are therefore supposed to be

But it is proper to compare the Angle EHX with that under which the Object would appear to the naked Eye at a Distance proper for distinct Vision; because when a Person views an Object by the Help of a Microscope, he is often obliged to place his Eye at a Distance from the Object, very different from that at which he would choose to place it, were he to look at it with the naked Eye; and then, instead of the Distance of the Object from the Eye in the foregoing Proposition, we must substitute the Distance of distinct Vision; in which Case it will stand thus: The Proportion of magnifying is nearly in a *Ratio* compounded of the Proportion which the Distance of the Image from the Object-glass bears to its Distance from the Eye-glass, and of that which the Distance of distinct Vision bears to the Distance of the Object from the Object-glass.

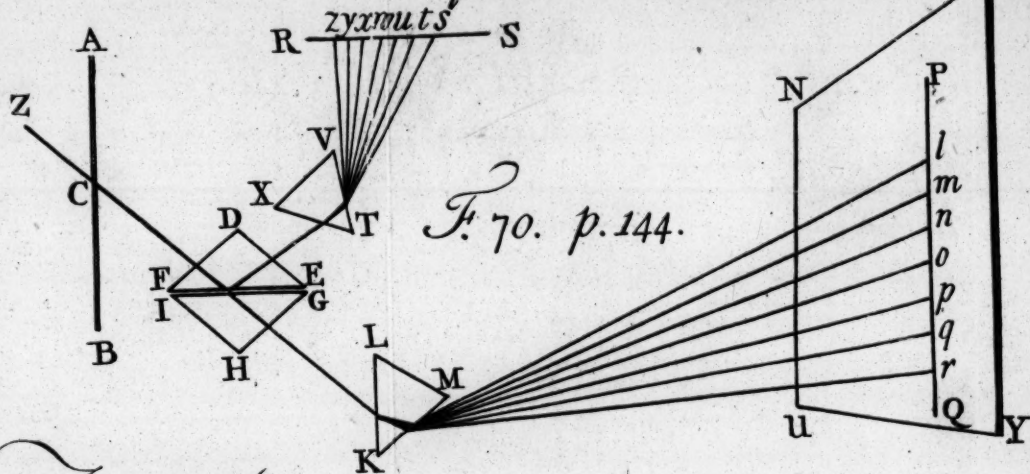
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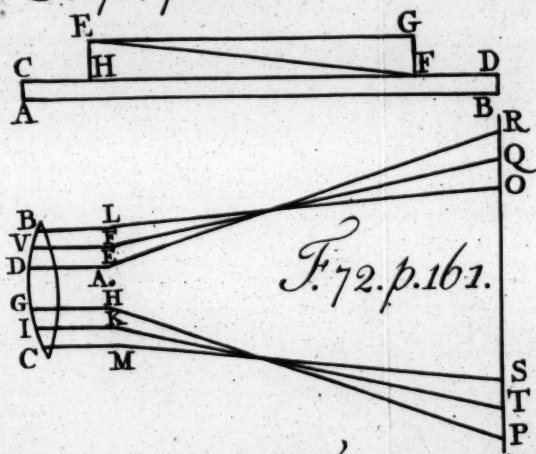
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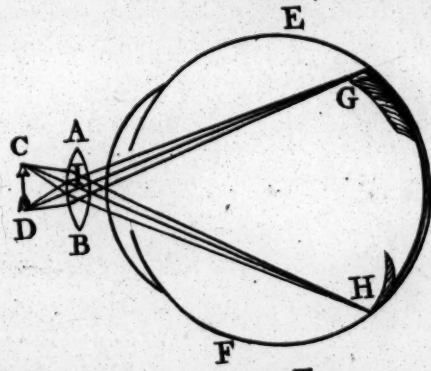
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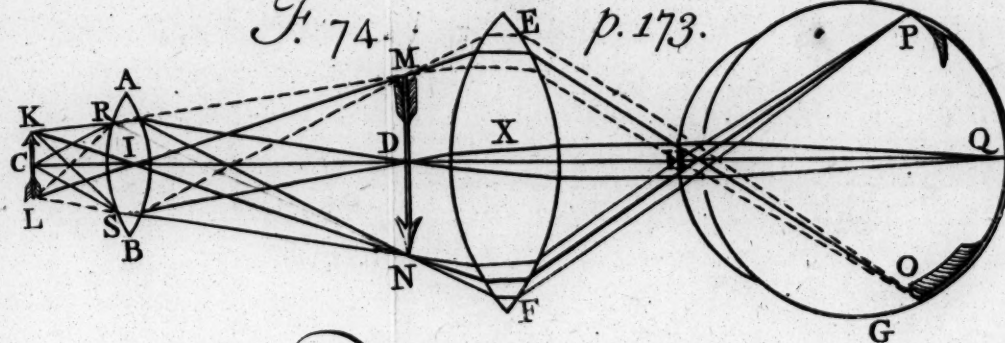
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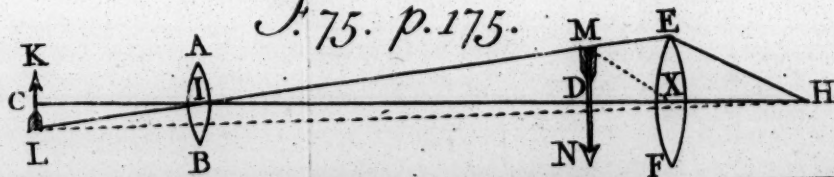
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F. 74. p. 173.



F. 75. p. 175.



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collected into their respective *Foci* at the Points G, H, I, situated at the focal Distance of the Object-glass DF. Here they form an Image, and crossing each other proceed diverging to the Eye-glass KM; which being placed at its own focal Distance from the Points G, H, I, the Rays of each Pencil, after passing through that Glass, will become parallel among themselves, but the Pencils themselves will converge considerably with respect to one another, even so as to cross at P, very little farther from the Glass KM than its *Focus*; because, when they entered the Glass, their *Axes* were almost parallel, as coming through the Object-glass at the Point E, to whose Distance the Breadth of the Eye-glass KM in a long Telescope bears very small Proportion. So that the Place of the Eye will be nearly at the focal Distance of the Eye-glass, and the Rays of each respective Pencil being parallel among themselves, and their *Axes* crossing each other in a larger Angle than they would do, if the Object were to be seen by the naked Eye, as we shall demonstrate in the Notes, Vision will be *distinct*, and the Object will appear *magnified*.

The Power of magnifying in this Telescope is as the focal Length of the Object-glass to the focal Length of the Eye-glass (*f*).

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(*f*) *Dem.* In order to prove this, we may consider the Angle AEC as that under which the Object would be seen by the
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It is evident from the Figure, that the visible *Area*, or Space which can be seen at one View when we look through this Telescope, depends on the Breadth of the Eye-glass, and not of the Object-glass; for if the Eye-glass be too small to receive the Rays GM, IK, the Extremities of the Object could not have been seen at all: A larger Breadth of the Object-glass conduces only to the rendering each Point of the Image more luminous by receiving a larger Pencil of Rays from each Point of the Object.

It is in this Telescope as was remarked of the compound Microscope in the Notes, (Page 174), that what we see, when we look through it, is not the Object itself, but only an Image of it at GI: Now that Image being inverted with respect to the Object, as it is, because the *Axes* of the Pencils that flow from the Object cross each other at E, Objects seen through a Telescope of this Kind necessarily appear *inverted*.

naked Eye; for in considering the Distance of the Object, the Length of the Telescope EP may be omitted, as bearing no Proportion to it. Now the Angle, under which the Object is seen by means of the Telescope, is KPM, which is to the other AEC, or its Equal KEM, as the Distance EL to LP, or which is the same Thing, by *Lemma* to the foregoing Note, as EH to HL. The Angle therefore, under which an Object appears to an Eye assisted by a Telescope of this Kind, is to that under which it would be seen without it, as the focal Length of the Object-glass to the focal Length of the Eye-glass.

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This is a Circumstance not at all regarded by Astronomers; but for viewing Objects upon the Earth, it is convenient that the Telescope should represent them in their natural Posture; to which Use the Telescope with three Eye-glasses, as represented Fig. 77, is peculiarly adapted, and the Progress of the Rays through it from the Object to the Eye is as follows:

AB is the Object sending out the several Pencils ACD, BCD, &c. which passing thro' the Object-glass CD, are collected into their respective *Foci* in EF, where they form an inverted Image. from hence they proceed to the first Eye-glass HI, whose *Focus* being at G, the Rays of each Pencil are rendered parallel among themselves, and their *Axes*, which were nearly parallel before, are made to converge and cross each other at K: The second Eye-glass LM, being so placed that its *Focus* shall fall upon K, renders the *Axis* of the Pencils which diverge from thence parallel, and causes the Rays of each which were parallel among themselves to meet again at its *Focus* NO on the other Side, where they form a second Image inverted with respect to the former, but erect with respect to the Object. Now this Image, being seen by the Eye at XY through the Eye-glass QR, affords a direct Representation of the Object, and under the same Angle that the first Image

EF would have appeared, had the Eye been placed at K, supposing the Eye-glasses to be of equal Convexity; and therefore the Object is seen equally magnified in this, as in the former Telescope, that is, as the focal Distance of the Object-glass to that of any one of the Eye-glasses, and appears *erect*.

If a Telescope exceeds 20 Feet, it is of no Use in viewing Objects upon the Surface of the Earth; for if it magnifies above 90 or 100 times, as those of that Length usually do, the Vapours, which continually float near the Earth in great Plenty, will be so magnified as to render Vision obscure.

The Telescope with the concave Eye-glass is constructed as follows:

AB (Fig. 78) is an Object sending forth the Pencils of Rays ADE, BDE, &c. which, after passing through the Object-glass, DE, tends towards FG (where we will suppose the *Focus* of it to be) in order to form an inverted Image there as before; but in their Way to it are made to pass through the concave Glass HI so placed that its *Focus* may fall upon S, and consequently the Rays of the several Pencils which were converging towards those respective focal Points F, S, G, will be rendered parallel among themselves; but the *Axes* of those Pencils crossing each other at K, and diverging from thence, will be rendered more diverging, suppose in the

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the Directions LM, NO. Now these Rays entering the Pupil of an Eye, will form a large and distinct Image PQ upon the *Retina*, which will be *inverted* with respect to the Object, because the *Axes* of the Pencils cross in K; and the Angle the Object will appear under will be equal to that which the Lines ML, ON, produced back through the Eye-glass, form at X.

'Tis evident, that the less the Pupil of the Eye is, the less is the visible *Area* seen through a Telescope of this Kind; for a less Pupil would exclude such Pencils as proceed from the Extremities of the Object AB, as is evident from the Figure. This is an Inconvenience that renders this Telescope unfit for many Uses, and is only to be remedy'd by the Telescope with the convex Eye-glasses, where the Rays which form the extreme Parts of the Image are brought together in order to enter the Pupil of the Eye, as explained above.

It is apparent also, that the nearer the Eye is placed to the Eye-glass of this Telescope, the larger is the *Area* seen through it; for, being placed close to the Glass, as in the Figure, it admits Rays that come from A and B, the Extremities of the Object, which it could not if it was placed farther off.

The Degree of magnifying in this Telescope is in the same Proportion with that in the

the other, *viz.* as the focal Distance of the Object-glass is to the focal Distance of the Eye-glass.

For there is no other Difference but this, *viz.* that as the extreme Pencils in that Telescope were made to converge and form the Angle KPM (Fig. 76) or QPR (Fig. 77) these are now made to diverge and form the Angle MXO (Fig. 78) which Angles, if the concave Glass in one has an equal refractive Power with the convex one in the other, will be equal, and therefore each Kind will exhibit the Object magnified in the same Degree.

There is a Defect in all these Kinds of Telescopes, not to be remedied by any Means whatever, which was thought only to arise from hence, *viz.* that *spherical* Glasses do not collect Rays to one and the same Point, as was observed (Chapter III. in the Notes) but it was happily discover'd by Sir *Isaac Newton*, that the Imperfection of this Sort of Telescope, so far as it arises from the spherical Form of the Glasses, bears almost no Proportion to that which is owing to the different Refrangibility of Light. This Diversity in the Refraction of Rays is about a twenty-eighth Part of the Whole, so that the Object-glass of a Telescope cannot collect the Rays which flow from any one Point in the Object into a less Room than the circular Space
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whose Diameter is about the fifty-sixth Part of the Breadth of the Glass (*g*). Therefore, since each Point of the Object will be represented in so large a Space, and the Centers of those Spaces will be contiguous, because the Points in the Object the Rays flow from are so, it is evident that the Image of an Object made by such a Glass must be a most confused Representation, though it does not appear so when viewed through an Eye-glass that magnifies in a moderate Degree; consequently the Degree of magnifying in the Eye-glass must not be too great with respect to that of the Object-glass, lest the Confusion become sensible.

Notwithstanding this Imperfection, a dioptrical Telescope may be made to magnify in any given Degree, provided it be of sufficient Length; for the greater the focal Distance of the Object-glass is, the less may

(*g*) To shew this, let *AB*, Fig. 79, represent a convex *Lens*, and let *CD* be a Pencil of Rays flowing from the Point *D*, and let *H* be the Point at which the least refrangible Rays are collected to a *Focus*, and *I*, that where the most refrangible concur; then, if *IH* be the twenty-eighth Part of *EH*, *IK* will be a proportionable Part of *EC* (the Triangles *HIK* and *HEC* being similar:) Consequently *LK* will be the twenty-eighth Part of *FC*. But *MN* will be the least Space into which the Rays will be collected, as appears by their Progress represented in the Figure. Now *MN* is but about half of *KL*, and therefore it is but about the fifty-sixth Part of *CF*, so that the Diameter of the Space, into which the Rays are collected, will be about the fifty-sixth Part of the Breadth of that Part of the Glass through which the Rays pass. Which was to be shewn.

be

be the *Proportion* which the focal Distance of the Eye-glass may bear to that of the Object-glass, without rendering the Image obscure. Thus, an Object-glass, whose focal Distance is about four Feet, will admit of an Eye-glass whose focal Distance shall be little more than one Inch, and consequently will magnify almost forty-eight times; but an Object-glass of forty Foot *Focus* will admit of an Eye-glass of only four Inch *Focus*, and will therefore magnify 120 times; and an Object-glass of an hundred Foot *Focus* will admit of an Eye-glass of little more than six Inch *Focus*, and will therefore magnify almost 200 times (*b*).

But

(*b*) The Reason of this Disproportion in their several Degrees of magnifying is to be explained in the following Manner: Since the Diameters of the Spaces, into which Rays flowing from the several Points of an Object are collected, are as the Breadth of the Object-glass, it is evident that the Degree of Confusedness in the Image is as the Breadth of that Glass (for the Degree of Confusedness will only be as the Diameters or Breadths of those Spaces, and not as the Spaces themselves.) Now the focal Length of the Eye-glass, that is, its Power of magnifying, must be as that Degree; for, if it exceeds it, it will render the Confusedness sensible; and therefore it must be as the Breadth or Diameter of the Object-glass. The Diameter of the Object-glass, which is as the square Root of its Aperture or Magnitude, must be as the square Root of the Power of magnifying in the Telescope, for unless the Aperture itself be as the Power of magnifying, the Image will want Light; the square Root of the Power of magnifying will be as the square Root of the focal Distance of the Object-glass; and therefore the focal Distance of the Eye glass must be only as the square Root of that of the Object-glass. So that in making Use of an Object-glass of a longer *Focus*, suppose than one that

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But Telescopes of such prodigious Length being very inconvenient and unfit for Practice, the *catadioptrical* or reflecting Telescope, as it is commonly called, invented by Sir *Isaac Newton*, is infinitely preferable to them; for one of these, six Feet in Length, shall magnify as much as one of the other of an hundred. The Form of the Tube, and the Progress of the Rays through it, are as described in Figure the 80th, where *ABCD* is the Tube, *BC* a concave reflecting Metal, *EF* a plain reflecting Metal fixed to the Tube by Means of the Stem *HI*. *MN* represents a distant Object emitting Pencils of Rays from each Point, two only of which are here represented, and those cut off before they reach the Metal, to prevent Confusion in the Figure. Now it is evident from what has been explained above (Chap. X. Proposit. 3. Case 4.) that these Rays, were they not intercepted in their Way, would return after Reflection at the concave Surface *BC*, and form an inverted Image at *OP*, supposing

is given, you are not obliged to apply an Eye-glass of a proportionably longer *Focus* than what would suit the given Object-glass, but such a one only whose focal Distance shall be to the focal Distance of that which will suit the given Object-glass, as the square Root of the focal Length of the Object-glass, you make Use of, is to the square Root of the focal Length of the given one. And this is the Reason that longer Telescopes are capable of magnifying in a greater Degree than shorter ones, without rendering the Object confused or coloured.

that

that to be the Place of the *Focus* of reflected Rays. But in this Case the reflected Rays are intercepted in their Return to that Place by the plain Metal, and are thereby thrown side-ways, and, instead of forming the Image O P, are made to form the Image Q R; which, because the Rays have as yet suffered no Refraction, is not liable to the Imperfection which arises from the different Refrangibility of the Rays of Light, nor to any other except what may arise from an imperfect Polish, or the Want of the Form of one of the *conic* Sections in the Reflector BC; and therefore may be viewed by an Eye at T with a very small *Lens* or Eye-glass KL, without appearing either coloured or confused.

It being inconvenient to find the Object with a Telescope of this Form, a small dioptrical Telescope with two Hairs or Wires run through the Tube in the common *Focus* of the two Glasses, and crossing each other at right Angles, is generally fixed upon it in such a Manner that the *Axis* of one Telescope shall be parallel to that of the other, so that when the Object appears in one at the Intersection of the Hairs, the other may be duly posited for viewing the same Object through its Side.

But this Method of finding an Object is very incommodious for viewing terrestrial Objects,

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Objects, and therefore the same Kind of Telescope has been contrived and effected in the following Manner :

ABCDEFGH (Fig. 81) is the Tube, BG the concave reflecting Metal, with an Hole in it at IK. LM is another reflecting Concave fixed to the Tube by means of the Stem NO, the common *Focus* of the two Metals being at P. Things being thus disposed, let QR represent an Object emitting several Pencils of Rays, two of which are represented in the Figure. These, after Reflection, will form the two Extremities of the inverted Image ST (as explained Chap. X. Prop. 3. Case 4.) where the several Rays of the same Pencil cross each other, and, being afterwards reflected by the concave Surface LM, become parallel among themselves, but the Pencils themselves are made to converge, and, crossing each other at V, pass through the *Lens* CF, which having its focal Distance about V, makes the Pencils parallel, and at the same Time renders the Rays of each Pencil converging, so as to form an erect Image WX, which is seen by the Eye at Y through another *Lens* at DE.

This Kind of Telescope is called the *Gregorian*, as being attempted by J. Gregory, though in vain. (See his *Optica promota*, Proposit. 59.) It is now grown common, and is excellently well adapted for the viewing
terrestrial

terrestrial Objects, because a Tube of this Kind, of two or three Feet in Length, will magnify sufficiently for that Purpose.

If the Reader would see a particular History of the Invention of the several Sorts of reflecting Telescopes, he may consult the *Appendix* to the last Edition of *Gregory's Optics*, where he will find a full Account of it, together with the Letters that passed between the Inventors themselves upon that Occasion. And for such Authors as have explained the Nature of Microscopes and Telescopes in general, consult Mr. *Johnson's Quæstiones Philosoph.* Q. 34 & 35.



DISSERTATION · IV.

Of the Rainbow.

BEfore the different Refrangibility of Light was discovered, all Attempts to account for the Appearance of the Rainbow proved ineffectual; for it is no other than the Diversity of Refrangibility to which that *Phænomenon* is to be ascribed: As will appear from the following Explication of it; in which, because it is a *Phænomenon* not easily apprehended by Beginners, I hope to be excused, if I am more than ordinarily particular. To begin then:

The Rainbow is never seen, but when the Sun shines upon Drops of Rain falling on that Side of the Spectator which is opposite to the Sun.

To illustrate this, let A (Fig. 82) represent the Eye of a Spectator, and let B, C, D, E, be a Series of Drops of Rain falling from a Cloud, on which let the Sun be supposed to shine from the Parts about S, S, &c. then will there be exhibited the Appearance of a Rainbow in the Cloud; and it will be formed as follows. Let SB, SC, SD, &c. represent the Sun's Rays, which (because of the Sun's great Distance) we will suppose parallel

parallel; and let the Ray SC fall upon the Drop C at the Point C: Then will so much of it as enters the Drop be refracted towards the Perpendicular, and proceed on, suppose to F, where Part of it will be transmitted, and Part reflected, suppose to G (*i*): Of that which is reflected to G, some will be there reflected and some transmitted; that which is transmitted will, on account of the Diversity of Refrangibility to which Light is subject (*k*) be separated by Refraction, and made to exhibit the several *prismatic* Colours, viz. *Red, Orange, Yellow, &c.* And if the *Red* Light proceeds from the Drop in the Line GR, the *Orange*, suffering a greater Degree of Refraction, will proceed in one situated above this, suppose in GO, and the *Yellow* in GY, &c. and the *Violet* in GV; therefore, to an Eye placed any where in the Line GR, the Drop C will exhibit a *Red* Colour, that is, the Cloud will appear *Red* in that Place. To an Eye placed any where in the Line GO, the same Drop would exhibit the *Idea* of *Orange* Colour, and so on through all the Colours of the *Prism*.

Now, let us consider the Passage of a Ray of Light through another Drop at a certain Distance below this, viz. the Drop D, on

(*i*) See the Manner in which Light is reflected, Chap. VIII.

(*k*) See Chap. XI.

which

Differ

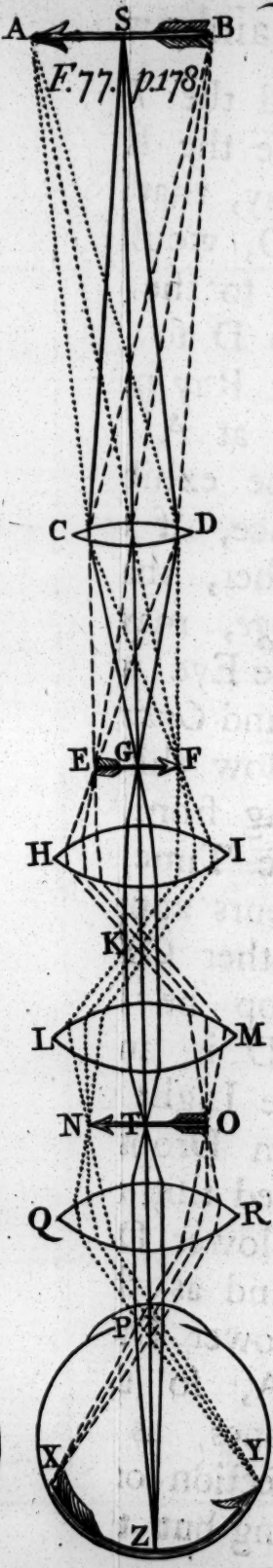
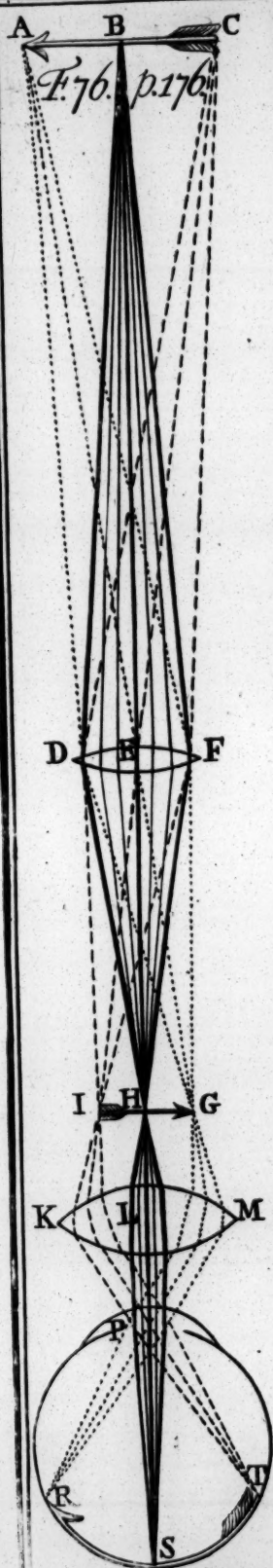
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which let *SD* be the incident Ray. This Ray, after having been refracted and reflected as the former was in the other Drop, will emerge separated into the Rays *HR*, *HO*, *HY*, &c. of which, if *HR* exhibits the *Red*, *HO* will paint the *Orange*, *HY* the *Yellow*, &c. and *HV* the *Violet* Colour; and the Ray incident upon this Drop being parallel to that which was incident upon the former, the Rays exhibiting the several Colours separated by this Drop, will be respectively parallel to the Rays exhibiting the correspondent Colours separated by means of the other Drop; that is to say, the Ray *HR*, which exhibits *Red* in this Drop, will be parallel to the Ray *GR*, which exhibits the same Colour in the other Drop; and so of the other corresponding Colours. Consequently the Rays *HO*, *HY*, &c. which exhibit *Orange*, *Yellow*, &c. in this Drop, will all converge towards *GR*, which exhibits *Red* in the other: And therefore each of these would cross that, if produced far enough. Let then the Ray *HV* in the Figure before us, which exhibits *Violet* in this Drop, be produced till it crosses that which exhibits *Red* in the other produced also, suppose at the Point *A* where the Eye of the Spectator is placed. To this Eye therefore, upon this Supposition, will be represented in the Cloud at the same Time two of the *prismatic* Colours, *viz.* *Red* and *Violet*, the

Red above at G, and the *Violet* below at H. But if we suppose the Eye placed farther back, where a Ray, that exhibits another Colour in the Drop D, would cross the Ray GR, or which comes to the same Thing, if we suppose the Drop D so much nearer to the Drop C, that that Ray may enter the Eye along with the other at A; then would the Colour of that Ray be exhibited along with the *Red*. For Instance, if the Drop D be placed so much higher, that the Ray HO which exhibits *Orange*, may cross the Ray GR at A, then to the Eye will be exhibited the Colours of *Red* and *Orange*; and if there be a third Drop below this, so placed that the *Yellow* proceeding from it shall enter the Eye also at the same Time, then will three of the *prismatic* Colours appear to that Eye, and so on for the other Colours, till the Situation of the Drop be supposed as low as where the Drop D is, and then the *Violet* and most refrangible Light is transmitted to the Eye; but from Drops below this, no Colour is transmitted thither, all the Rays which issue from a lower Drop, as E, passing below the Eye. And as those Rays, which pass through the lower Drops, are too low for the Eye at A, so those which come from the higher ones, as B, are too high, as appears by Inspection of the Figure; so that there is nothing but total Darknes both
above

Part III. Plate XII. p. 192.



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above and below. Now since the Colours of the Rainbow are the same with those of the *Prism*, it is evident, from what has been said, that between the Points G and H, all the Colours of that beautiful *Phænomenon* will appear, provided a sufficient Number of Drops be supplied from the Cloud, to prevent any *Hiatus* or Deficiency in the Light reflected by them.

But we have hitherto *tacitly* supposed, that the Rays SB, SC, SD, &c. were all incident with the same Degree of Obliquity upon the Surface of each Drop (that is, that they entered at the same Distance from that Point in the Surface of each Drop which is nearest the Sun) and that that Obliquity was certain and determinate: For Rays which enter the Drops with other Obliquities conduce nothing towards exhibiting the Colours of the Rainbow, and are therefore to be looked upon as *ineffectual* and *insignificant*. The Truth of which we shall now proceed to shew. After this we shall be enabled to explain the remaining Particulars relating to the *Bow*.

Let then SA, SB, SC, &c. (Fig. 83) represent the Sun's Rays falling upon the Drop XY, the first perpendicularly to the Surface of it, the other with different Degrees of Obliquity, according to their different Distances from the first; and let the Point at which the two first, *viz.* SA and SB, would meet by means of that Refraction which the oblique one SB

suffers in passing through the first Surface of the Drop, be H. Now it was remarked (Chap. III. Observat. 2. in the Notes) that when parallel Rays fall upon a convex Surface, the nearer any one of the oblique ones is to that which enters the Surface perpendicularly, the greater shall be the Distance at which it will meet the perpendicular one after Refraction at that Surface; that is, in the present Case, that the oblique incident Ray SB shall after Refraction at B, (supposing it to pass through the second Surface of the Drop without Refraction) meet the perpendicular Ray SA produced at a greater Distance than SC shall; and SC shall meet it at a greater than SD shall; SD at a greater than SE, &c. H then being supposed to be the Point where SB meets SA, let I be that where SC, K that where SD, L that where SE, M that where SF would meet it, &c. From whence we may observe,

That the farther we take the Rays from SA, the *nearer* the Points at which the refracted Rays fall upon the second Surface of the Drop approach to X, till we come to the Ray SD; after which, the farther we take them from SA, the farther the Points they fall upon are from X. For Instance, the Ray SB falls upon N; SC upon O; SD upon P; but SE does not fall beyond P, but upon O, and SF upon N, &c. So that upon every Point below P there are two Rays incident; and the one of
them

them is such as enters the Drop on one Side of the Ray SD, and the other on the other; and the farther from it the one is on the one Side, the farther the other is on the other; and also the farther they are from it, the farther the Point they meet at is from P. Thus SC and SE meet at O, the Rays SB and SF at N, &c. The Use of which Observation we shall see by and by. But let it be remembered, that I would be understood here and in what follows of some one Species of Rays only.

Now it is remarkable, that when two Rays fall upon a Drop, and at their Entrance are so refracted, as to meet in a Point at the other Surface, and are from thence reflected to some other Part of the Surface, and there pass out of it; they shall after such Emergency have the same Inclination towards each other, that they had before they entered the Drop. To explain this, let AB, CD, (Fig. 84) represent two Rays incident upon the Drop BEF, and let them, after Refraction at B and D, meet at the Point E, from whence being reflected, let them pass out at F and G, and be refracted into the Lines FH and GI; then whatever Inclination the incident Rays AB, CD, have to each other, the emerging Rays FH and GI will have the same; because the Angles of Incidence and Reflection at E being equal, the Rays EF and EG will have the same Inclination to each other, and to the Surface at F and G, that the Rays EB and

ED have to each other, and to the Surface at B and D. For if we conceive all these Rays to flow from the Point E as a Radiant, and BA, DC, to be the refracted ones of the incident ones EB, ED, as FH and GI are of EF and EG, it is evident, that under these Circumstances, the Rays BA and CA will have the same Inclination to each other, that FG and HI have; but the Degree of Refraction is the same, whether EB and ED, or AB and CD, be the incident Rays; because the refractive Power of the Drop is the same, whether the Rays pass one Way, or the other. The Proposition therefore is true.

From hence it follows, that the parallel Rays, SB, SF, (Fig. 83) which after Refraction meet at the same Point N, will if they are from thence reflected, suppose in the Lines NQ, NR, become parallel to each other, after their Emergency, suppose in the Lines QT, RV: But their intermediate ones SC, SD, and SE, which fall upon quite different Points at the second Surface of the Drop, and are from thence reflected, will not do so, but will go out in Directions oblique to one another and to them; and will therefore pass on, not only a different Way from them, but from one another: So that the Rays QT and RV will be left to themselves, being deprived of their *intermediate* ones, by which Means they are rendered, as to all Intents and Purposes of Vision, entirely useless and insignificant.

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Again, the Rays SC and SE, which meet at the Point O, will also be parallel among themselves after their Emergency from the Drop, but their *intermediate* ones will pass off another Way, though not so obliquely, with respect to them and to one another, as those in the foregoing Case; because the several Points they fall upon at the second Surface of the Drop, being situated between O and P, are nearer to each other, than the Points the *intermediate* Rays fell upon, in the former Case, were to N.

But such as are incident very near SD on each Side of it, will with their intermediate ones all fall upon, or at an insensible Distance from the Point P; so that these, after their Emergency, will all pass on parallel, or very nearly so, to each other; and therefore when they enter the Eye of a Spectator, though he be at a considerable Distance, will affect him sensibly enough to excite the *Idea* of their own Colour (for as was observed above, I speak now only of Rays of one Colour) which the other Rays considered in the forementioned Cases, for Want of their *intermediate* ones, were too weak to do, however near the Situation of the Eye might be. These therefore are the only Rays that exhibit the Colours of the *Bow*, and are hence called in Contradistinction to such as enter at other Points of the Drops, that is, with other Obliquities, *effectual* or *significant*.
It

It was proper therefore, in the Explication above, to suppose none to enter the Drop, but these. As to the Degree of Obliquity with which Rays must fall upon the Drops to become effectual, the Method of finding that will be shewn in the next Note.

Since then the effectual Rays enter each Drop with the same Degree of Obliquity (I still mean such Drops as exhibit the same Colour) the emerging Rays must necessarily make the same Angle with the incident ones in every Drop. The Magnitude of which shall be determined in the Note below (*a*). That is, the Angle which the Ray SC (Fig. 82) makes with the emerging Ray GR which exhibits *Red*; will be the same in all the Drops that exhibit that Colour, and so of the rest. Let then the
Line

(*a*) We are here to determine the Angle, which an incident *efficacious* Ray of any Colour makes with the emerging Ray of the same Colour. In order to this, let AB, CD, (Fig. 84) be two Rays incident upon the Drop BG, and let them be refracted to E, and after Reflection there, and second Refraction at F and G, let them emerge in the Lines FH and GI, making with the incident ones the Angle AKI, which is the Angle to be determined.

Let us call the *Ratio*, which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, I to R. Then from the Center I to the Lines BK, BE, DE, draw the Lines LM, LN, and LO, respectively perpendicular, and with the *Radius* LO describe the Arch OP, and draw the Line LB, and produce it to Q. Then will ABQ, or its Equal LBM, be the Angle of Incidence of the Ray AB, and LN its Sine: LBN will be the Angle of Refraction, and LM its Sine: Likewise LR will be the Sine of the Angle of incidence of the other Ray CD, and
LO

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Line AI be supposed to be drawn from the Sun through the Eye of the Spectator. This Line, because of the Sun's immense Distance, will be parallel to the Sun's Rays SB, SC, SD; and therefore the Angle GAI which this Line makes with any one of the emerging Rays, for Instance GR, will be the same that the incident ones makes with it. Let us then imagine the Line AI fixed, and the Line AG to revolve round it, always making the same Angle with it; then will the Line AG describe the Surface of a Cone whose *Apex* will be at A, and its *Axis* AI, and the Surface of this Cone will in all Parts of it make the same Angle with the Sun's Rays, because they are parallel to one

LO the Sine of its Angle of Refraction. We shall therefore have for the first Step of the following Process

this Proportion, *viz.*

And for the second

But by Construction

Therefore from the 2d and 3d Steps

Now if we subtract the two first Members of the fourth Step from the two first of the first Step respectively, by which Means the Proportion between the Terms will not be destroyed, we shall have

But by the Figure

And ————

Therefore from the fifth, sixth, and seventh Steps, we have

$$1 \quad LM : LN :: I : R$$

$$2 \quad LR : LO :: I : R$$

$$3 \quad LO = LP$$

$$4 \quad LR : LP :: I : R$$

$$5 \quad LM - LR : LN - LP :: I : R$$

$$6 \quad LM - LR = MR$$

$$7 \quad LN - LP = NP$$

$$8 \quad MR : NP :: I : R$$

Parallel

one another, and to the Axis of the Cone; therefore Drops of Rain, whatever Part of this Surface they pass through in falling, will, in the Instant of Time that they pass through it, send a *red* Ray towards the Eye of the Spectator; for it is not necessary that the Drops should be all at the same Distance from the Eye. In like manner Drops of Water, passing through the Surface of a lesser Cone made by the Revolution of the Line HV about the Line AI, will exhibit *Violet*; and so for the intermediate Colours. So that the Rainbow, were we to see it entire, would be a compleat Circle having its Center in the Line AI, and consequently directly opposite to the Sun with respect to the Spectator's Eye.

Parallel to BE draw the Line DX, and on the Lines DR and DX let fall the Perpendiculars BT and BX, from the Point B. Then, because we suppose the Rays AB and CD *efficacious* ones, and therefore infinitely nearer one another, the little Triangles BTD, BXD, and NOP may be considered as right-lined ones, and the latter, *viz.* NOP may be also looked upon as right-angled at P. Upon this Supposition, the Triangles BTD and BLM will be similar, for they are right-angled at T and M; and the Angles DBT and MBL want each the same Angle TBL to make either of them right, they are therefore also equal. For the like Reason the Triangles DBX and BNL are similar, being right angled at X and N, and wanting equally the Angle XBL to make their Angles at B right ones. But to the Triangle BDX the Triangle NPO is also similar, for they are right-angled at P and X, they have their Sides NP and BX parallel, as being each perpendicular to the same Line BE; and their Sides NO and BD are also parallel, because the Points N and O, where the Perpendiculars LN and LO fall, must be in the Middle of the Lines BE and DE. Farther, since the Lines BE and DE are coincident at E, and the Points N and O are in the Middle of each, BD is double of NO; and consequently,

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Eye. The Reason that it does not appear such, is because the Sun when the Rainbow is seen, is *in* or *above* the Horizon, and therefore the Center of the Bow being opposite to it, is *in* or *below* it, on the other Side the Spectator.

Since the Angle made by the Line SC with GR, or which is the same thing, GR with AI is 42 Degrees and two Minutes, as determined in the Note, it's evident that when the Sun is in the Horizon, the highest Point of the *Bow* is 42 Degrees and two Minutes above the Horizon,

quently, the Triangles NOP and BDX being similar, BX is double of NP. From all which we have the following Steps,

<i>viz.</i>	9	BL : BM :: BD : BT
And	10	BL : BN :: BD : BX
By comparing these two together	11	BM : BN :: BT : BX
But by Construction	12	BT = MR
And, as was just now shewn,	13	BX = 2NP
Consequently from the three last Steps	14	BM : BN :: MR : 2NP
But by the eighth Step	15	I : R :: MR : NP
Therefore from the last	16	I : 2R :: MR : 2NP
Consequently from the fourteenth and sixteenth Step	17	BM : BN :: I : 2R
But from the first Step	18	LM : 2LN :: I : 2R
Therefore from the two last	19	BM : BN :: LM : 2LN
And by changing the Places of the mean Terms in the last Step	20	BM : LM :: BN : 2LN
And by squaring each Term	21	BMq : LMq :: BNq : 4LNq
From whence by comparing the Antecedents and Consequents with the Antecedents we have	22	BMq + LMq : BMq :: BNq + 4LNq : BNq

But

rizon, for then the Line AI is parallel to it; and as the Sun rises the Height of the *Bow* diminishes, and with it the Portion that is visible, till it is 42 Degrees and two Minutes high; after which the *Bow* appears no more, because then the Point I is above 42 Degrees below it.

The *Phænomenon*, we have been explaining, constitutes what is called the *primary* or *interior Bow*; there is also another exterior to this, whose Colours are much more dilute and faint, which

But, because the Triangle BML is right angled at M

And for the like Reason BNq + LNq is equal to BLq, therefore

Therefore from the three last Steps

But BLN being a right-angled Triangle

Therefore from the two last Steps

Now because BML is a right-angled Triangle

Therefore from the two last Steps

And subtracting the two first Terms, viz. BLq and BMq out of the two last Terms respectively, we have

But by the first Step

Therefore substituting I and R in the Room of LM and LN in the 30th Step, we have

$$23 \quad BMq + LMq = BLq$$

$$24 \quad BNq + 4 LNq = BLq + 3 LNq$$

$$25 \quad BLq : BMq :: BLq + 3 LNq : BNq$$

$$26 \quad BNq = BLq - LNq$$

$$27 \quad BLq : BMq :: BLq + 3 LNq : BLq - LNq$$

$$28 \quad BLq = BMq + LMq$$

$$29 \quad BLq : BMq :: BLq + 3 LNq : BMq + LMq - LNq$$

$$30 \quad BLq : BMq :: 3 LNq : LMq - LNq$$

$$31 \quad LM : LN :: I : R$$

$$32 \quad BLq : BMq :: 3 Rq : Iq - Rq$$

The

Differt. IV. *Of the Rainbow.* 203

which for that Reason is called the *secondary* Bow. The Progress of the Rays of Light through the Drops of Water, in forming this, is as follows.

Let A (*Fig. 85*) represent the Eye of a Spectator, SB and SC two of the Sun's Rays entering the Drops, as expressed in the Figure, and after being twice reflected in each Drop, *viz.* at D and D, let them pass out, the one at E, the other at F, by which Means they will be separated into their homogenous Colours, the *Violet* and most refrangible Light being

The Proportion therefore which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, when Rays of any particular Colour pass out of Air into Water, being known, the Proportion, which the *Radius* BL bears to BM, will be thereby determined; and therefore the Angle BLM, of which BM is the Sine, will also be known, and therefore also the Angle LBM, which is equal to ABQ, the Degree of Obliquity wherewith the *efficacious* Rays enter the Drop. But the Line BM being known, the Line BN may be also had by the 17th Step, and therefore also the Angle BLN of which it is the Sine, and therefore the Angle LBN too, or its Equal LEN; and therefore also the Complement of this last to two right ones, *viz.* KEB. If now we subtract the Angle LBN out of LBM, we gain the Angle EBK, and consequently the third Angle in the Triangle FBK may be from hence known, *viz.* the Angle BKE. Now if we double this, we have the Angle AKI, which was the Angle sought.

If a Computation be made after this Manner with the *Ratio* of 108 to 81, or (which is the same thing) that of 4 to 3, for the *Red* Rays, the Angle AKI (that is, the Angle GAI in *Fig. 82*) will be found 42 Degrees and 2 Minutes; and if we use the Proportion of 109 to 81, which is the Proportion of Refraction in the *Violet* coloured Light, the Angle AKI (or HAI in *Fig. 82*) will be 40 Degrees 17 Minutes. And the Difference between these two Angles (that is, the Angle GAH in *Fig. 82*) will be the Breadth of the Bow.

conveyed

conveyed from the uppermost Drop to the Eye at A in the Line EV; while the *Red*, and least refrangible, is carried from the lower one in the Line FR, and the intermediate Colours from the intermediate Drops: So that in this Bow the Colours will be in an inverted Order, with respect to those of the other, the *Red* being the innermost in this, which was the outermost in that. The Colours in this will be more dilute than in that, because the Rays in this suffer two Reflections, in that but one; and the Angles, which the incident Rays in this make with the emerging ones, are larger than the like Angles in the other, as shall be determined in the next Note, *viz.* 50 Degrees 57 Minutes for the *Red*, and 54 Degrees 7 Minutes for the *Violet*; this Bow therefore is exterior to, and encompasses the former.

As to the Means by which Rays of Light become *efficacious* in the Formation of this Bow, it is exhibited in Figure 86, where AB, CD represent two parallel Rays incident with such Obliquity upon the Drop, that they shall cross each other before they reach the other Side; which that it is possible appears from what was said, with regard to the Progress of the Rays through the Drop XY in Figure 83. Let them then cross in the Point E such, that after Reflection at F and G they become parallel, suppose in the Lines FH and GI, then from the Nature of the Circle it is plain, that after
Reflection

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Reflection at H and I they will cross again, suppose at K, and after Refraction at V and W, will become parallel as at first. And such of these as also enter so very near one another, that their intermediate ones may suffer the like Refractions and Reflections with themselves, will be the *efficacious* ones, and exhibit the *Idea* of their own Colour, at a considerable Distance from the Drop. What the Obliquity is with which these must enter the Drops, and the Angle the emerging ones of each particular Colour will make with their incident ones, shall be determined in the Note below (*b*).

U

Now

(*b*) The Progress of the *efficacious* Rays through the Drop BH (Fig. 86) being as explained above, it is evident, because the Angles of Incidence are every where equal to the Angles of Reflection, that the Lines BG, GI and IW are all equal, and therefore the Arches BG, GI, and IW are so too; and likewise, that the Lines DF, FH, and HV are equal, and therefore also the Arches DF, FH, and HV: It is also apparent, that the Arch FG is equal to HI, therefore FG is half the Difference between the Arches FH and GI, and consequently it is half the Difference also between the Arches FD and GB which are respectively equal to these. Now, the whole Difference between these Arches is what remains when FG is taken from BD, therefore the Remainder when FG is taken from BD is double of FG, consequently FG itself is but one third Part of BD; for, if when one Quantity is taken from another, the Remainder be double to the Quantity taken away, it is plain that other must contain the Quantity taken away three Times.

Now the Rays AB and CD being supposed infinitely near one another, the curvilinear Spaces BED, and FEG may be consider'd as similar Triangles, and therefore EG is equal to a third Part of EB, consequently N (the Point where the Perpendicular LN falls upon BG) being the middle Point of the Line BG, EN is equal to EG, and therefore also a third Part of EB.

If now as in the 84th Figure, the Triangles BTD and BXD be formed, as also the Triangle NOP, NO will be a third Part of ED

Now if we imagine the Lines EA and FA (Fig. 85) to revolve about the Line AI which passes through the Eye of the Spectator and the Center of the Sun as before, and always to make the same Angles with it at A, they will describe the Surface of two Cones, in the larger of which will be situated the Drops that exhibit *Violet*, and in the lesser those which exhibit *Red*. So that this Bow also, were it to appear intire, would be a compleat Circle, and

BD, and NP a third Part of BX. Therefore resuming the former Process at the 11th Step, we may proceed as follows, *Viz.*

By Construction	11	BM : BN :: BT : BX
And by what was just observ'd	12	BT = MR
Therefore from the three last Steps	13	BX = 3 NP
But by the eighth Step	14	BM : BN :: MR : 3 NP
Therefore from the last	15	I : R :: MR : NP
Consequently from the 14th and 16th Steps	16	I : 3R :: MR : 3NP
But by the first Step	17	BM : BN :: I : 3R
Therefore from the last	18	LM : LN :: I : R
Therefore from the 17th and 19th	19	LM : 3LN :: I : 3 R
Or by changing the Places of the mean Terms in the last Step	20	BM : BN :: LM : 3 LN
And squaring the Terms	21	BM : LM :: BN : 3 LN
Therefore by comparing the Antecedents and Consequents with the Antecedents, we have	22	BMq : LMq :: BNq : 9 LNq
But by the Figure	23	BMp + LMq : BNq :: BNq + 9 LNq : BNq
And	24	BMq + LMq = BLq
	25	BNq + 9 LNq = BLq + 8 LNq

Therefore

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and the several Cones, through whose Surfaces

Therefore from the three last	26	$BLq : BMq :: BLq + 8 LNq$ $: BNq$
But by the Figure	27	$BNq = BLq - LNq$
Therefore from the two last	28	$BLq : BMq :: BLq + 8 LNq$ $BLq - LNq$
But by the Figure	29	$BLq = BMq + LMq$
Therefore from the two last	30	$BLq : BMq :: BLq + 8$ $LNq : BMq + LMq -$ LNq
And substracting the two first Terms out of the two last Terms, we have	31	$BLq : BM :: 8 LNq : LMq -$ LNq
But by the first Step	32	$LM : LN : I : R$
Therefore	33	$BLq : BMq :: 8Rq : Iq - Rq$

Now the Proportion of I to R being known, the Proportion which the *Radius* BL bears to BM is had by the last Step. But to avoid the Confusion which a Multiplicity of Lines may occasion, let the 86th Figure be transferred to the 87th with as many Lines as shall be necessary, in which let AB be the incident Ray, BG the refracted one as before. Then, because the Proportion between BL and BM is known, the Angle LBM may be had, which is equal to ABQ, the Measure of the Obliquity with which the efficacious Rays enter the Drop; and therefore also its Complement to two right ones SBL. And the Line BM being known, the Line BN may be had, because by the 17th Step BM is to BN as I to 3 R, and therefore also the Angle LBN, or its Equal LGB, and consequently BLG the remaining Angle of the Triangle BGL; but to this is equal the Angle GLH or HLV, and if these three be added together, and their Sum taken from four right ones, it will give the remaining Angle about the Center, *viz.* VLB, which being halved, gives the Angle SLB; but the Method of determining the Angle SBL was shewn before, and therefore LSB the remaining Angle of the Triangle LBS, may be had, which Angle doubled gives the Angle VSB or its Equal ASY, which is the Angle sought.

If a Computation be made after this Manner with the *Ratio* of 108 to 81 for the *Red* Rays, this Angle will be found to be 50 Degrees 57 Minutes; if with 109 to 81, for the *Violet*, it will be 54 Degrees 7 Minutes; and the Difference, *viz.* 3 Degrees 10 Minutes, will be the Breadth of the *Bow*.

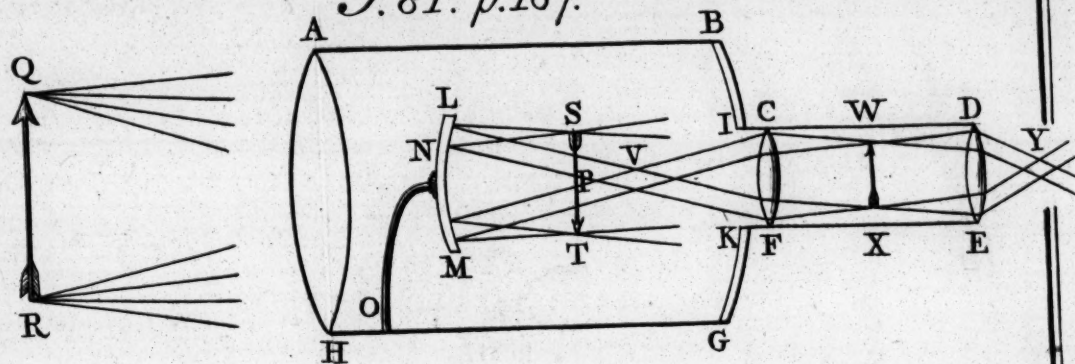
the Drops as they form the Colours of it pass, having one common *Axis* AI with those in whose Surfaces the Drops forming the Colours of the other *Bow* were placed, this will be exterior to, and concentric with it, and will therefore surround it, as observed above.

As Rays of Light when they enter a Drop of Water never all pass out, but as oft as they impinge upon the Surface are in Part reflected and in Part transmitted. It is evident that some Rays will pass out of each Drop after having suffered three Reflections, some after four, &c. these also will constitute *Rainbows*; but because the greatest Part of the Rays will be lost in suffering so many Reflections, that *Rainbow*, which is made by three Reflections, is scarce ever seen, much less such as are made by more, &c.

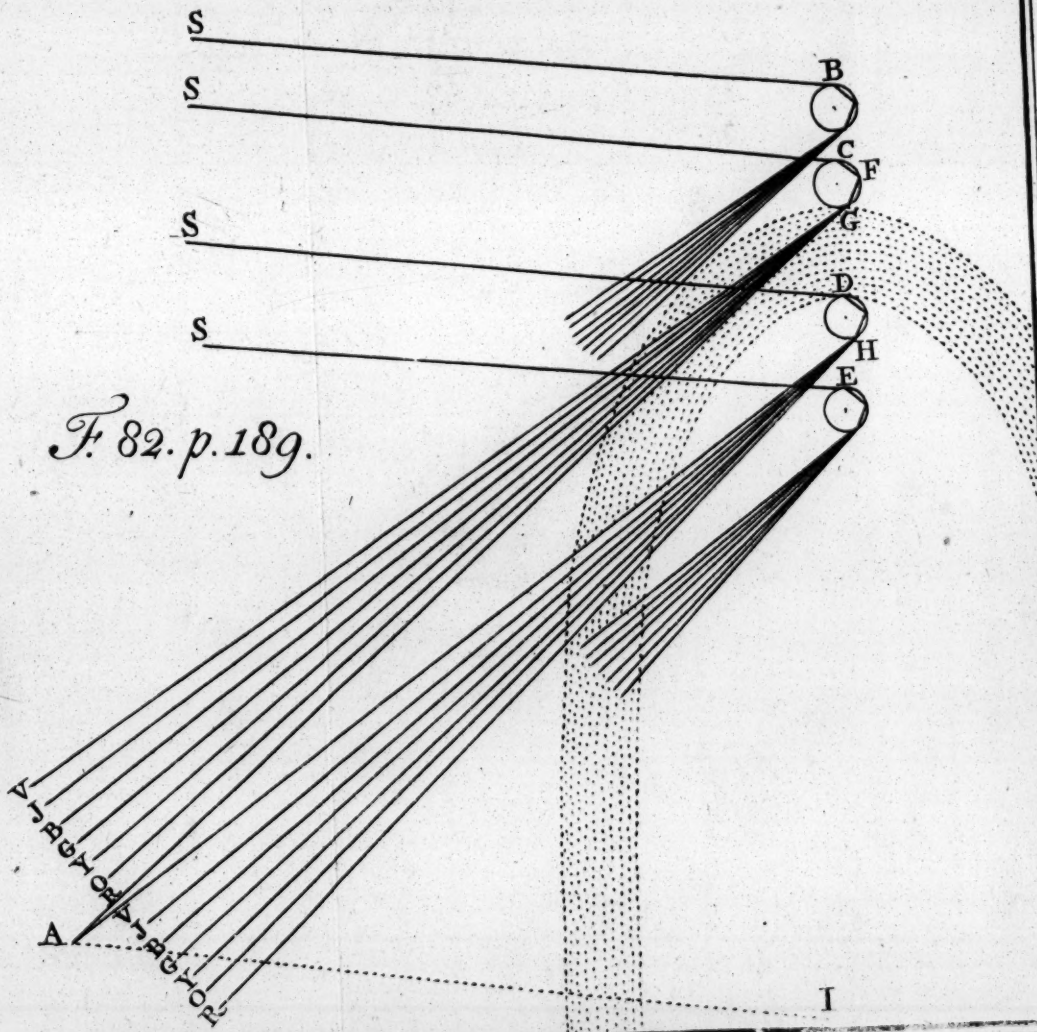
It is evident, that since the Line AI, viz. the common *Axis* of the Cones, on whose Surfaces the Colours of the *Bow* are formed, passes through the Eye of the Spectator, no two Persons can see the same *Bow* at the same Time, or rather, that the Rainbows, seen by two Persons at the same Time, are formed in different Drops of Rain and in different Parts of the Heaven.

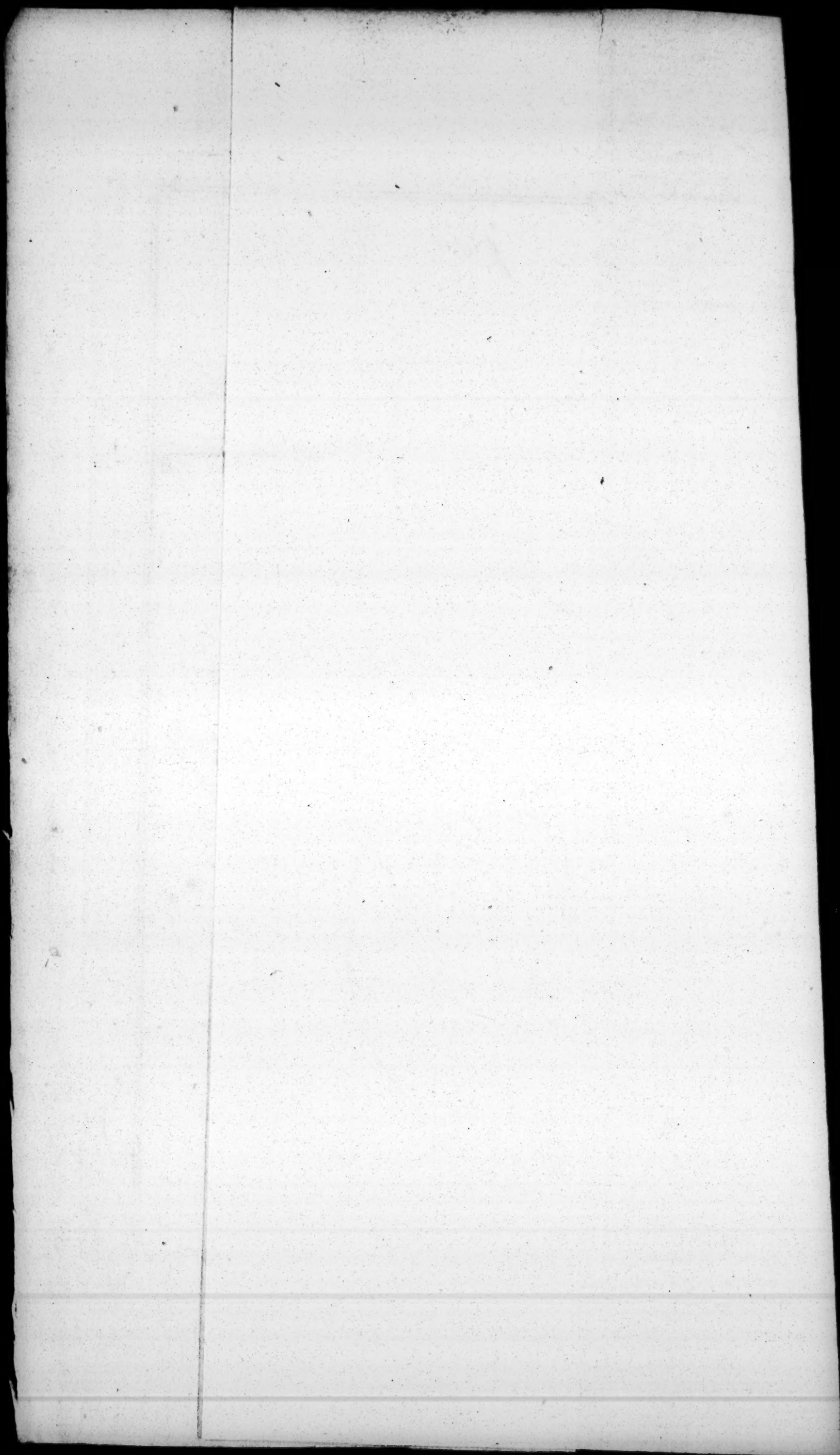
Accordingly, if a Person observes the Dew as it hangs upon the Grass when the Sun shines, he shall see the Colours of the *Bow* in the Drops of Dew; but as he walks along, the Colours shall remove from Drop to Drop. Some-

F. 81. p. 187.



F. 82. p. 189.





part III. Plate XIV. p. 208.

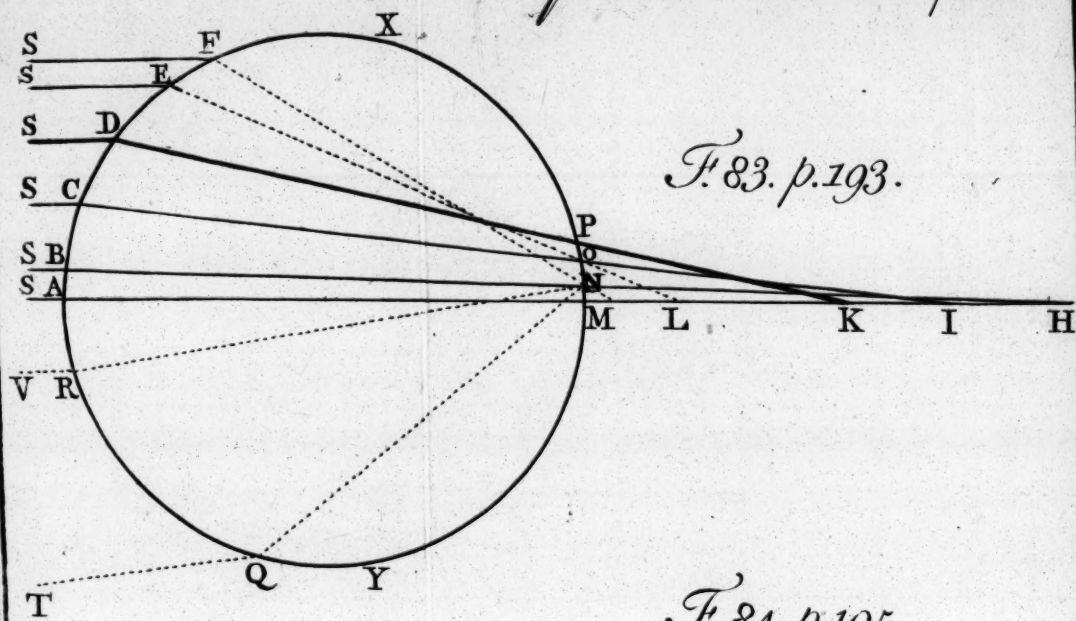
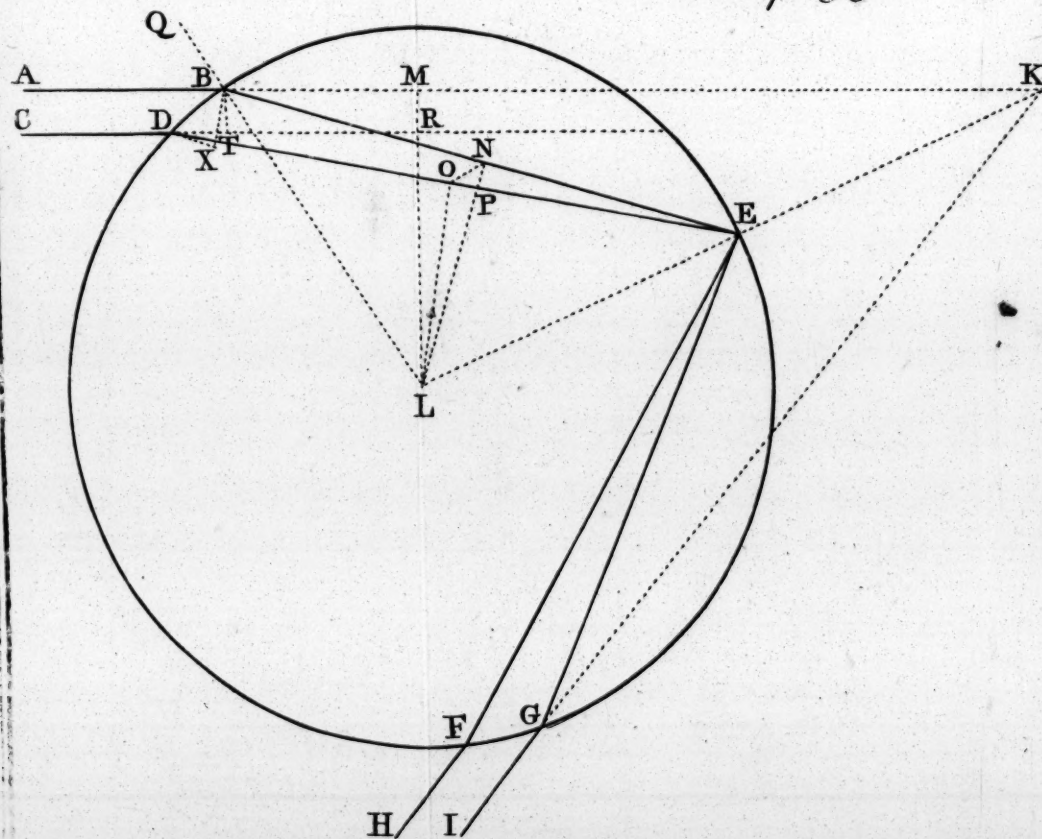


Fig. 84. p. 195.



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Sometimes the lower Part of the *Bow* shall appear upon the Ground, and the upper Part of it not at all, and then it looks like a Rainbow lying along the Ground with the Extremities of its Legs turned upward into the Air: This is when the Sky is clear towards the Sun, but foggy on the opposite Parts, and only to a small Height from the Ground.

The Moon sometimes occasions the Appearance of a *Rainbow* after the same Manner that the Sun does, but the Colours are much more faint and dilute.

And lastly, if Water be continually thrown up into the Air opposite to the Sun, as from a Fountain, and there breaks into small Drops, the Appearance of the Rainbow will be exhibited in them.

See more on this Subject in *Antonius de Dominis de Radiis Visus & Lucis*; and the Authors referred to by Mr. *Johnson*, in his *Philosoph. Quæst.* Chap. VII. Q. 45 & 46.

Of the Obscura Camera and the Magic Lanthorn.

THE *Obscura Camera* is of two Sorts; the one is no other than a convex *Lens* fixed in a Hole in a Window-shutter, which *Lens*, when no other Light is permitted to enter the Room except what passes through it, will represent all the external Objects that are visible through that Hole upon a white Paper held at the focal Distance of it, painted in their proper Colours. To illustrate this, let AB (Fig. 88) represent a Window-shutter, CD a convex *Lens* fixed in an Hole therein, and let EF be an external Object; then will this Object emit Rays of Light of its own Colour from each Part, which passing through the *Lens*, as the Figure represents them, will be collected into Points at GH, and being there received upon a white Paper or other Surface, will represent the Object painted in its proper Colours, which Colours will be the strongest of all when the Sun shines upon that Side of the Object that is next the Glass. But the Representation will be *inverted*, because the Pencils of Rays that flow from the Object cross in the Middle of the Glass.

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and the Magic Lanthorn. 211

The other Sort of *Obscura Camera* is that which is called the *Portable one*, and is of Use in drawing Pictures, taking Landskips, &c. It is contrived after the following Manner, AIKB (Fig. 88) is a Box, in an Hole in whose Side the *Lens* CD is fixed (or rather at the Extremity of a short Tube fixed in that Hole) and in the Situation LM is fixed a Piece of Looking-glass making an Angle with the Bottom of the Box of 45 Degrees; this, receiving the Rays in their Passage to GH, throws them upwards, and causes the Representation to be made in NO, which is there received upon the under Side of some thin Substance PQ, that is in a small Degree transparent, (as thin Paper or Glass about half polished) and so upon opening the Box appears ready to be drawn or copied out. But that the Colours may appear strong, the Light, when the Box is opened, must be kept from falling upon the Paper or Glass, as much as may be.

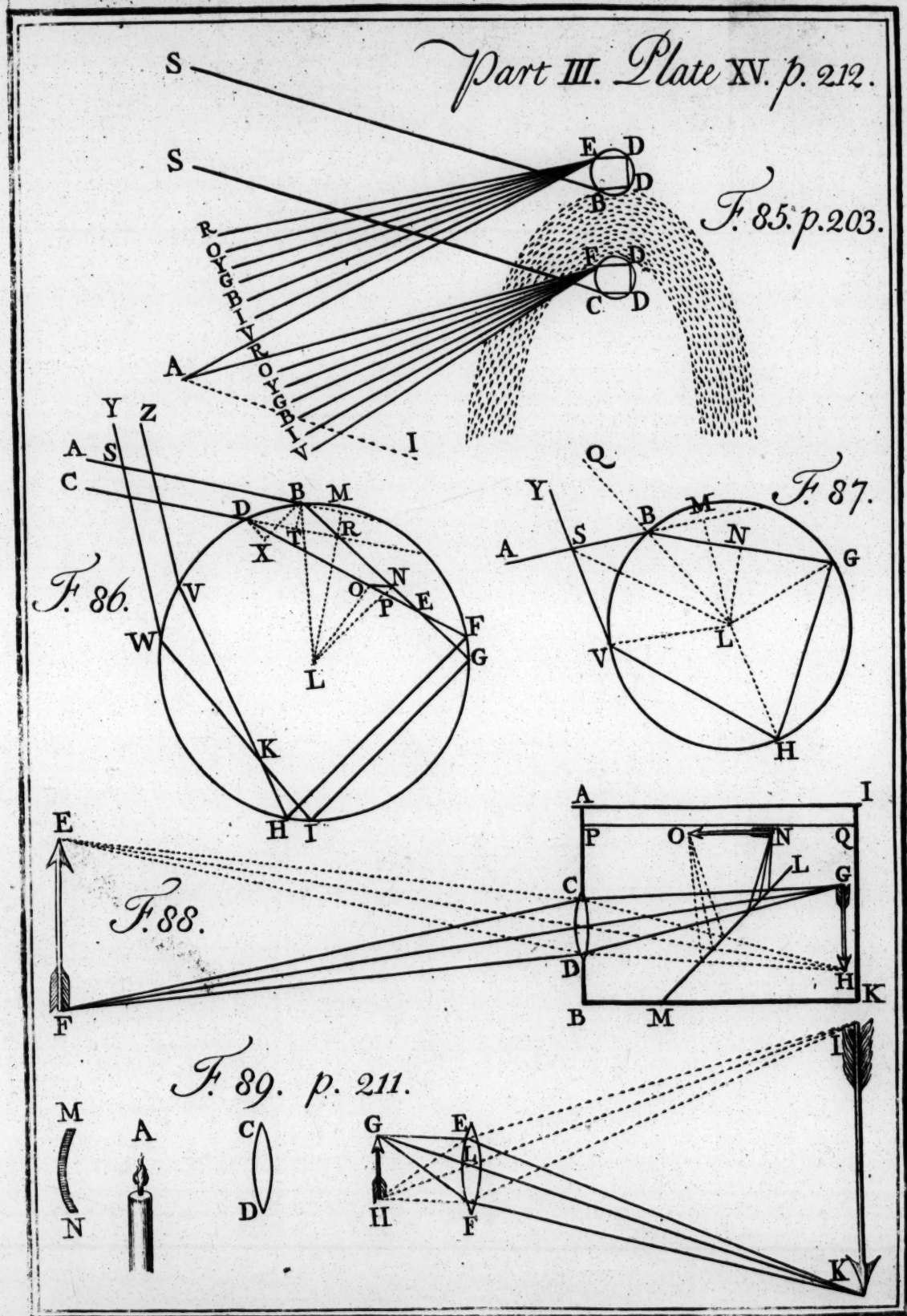
The *Magic Lanthorn* is an Instrument invented by *Kircher*, in order to represent Objects much larger and more luminous than they are. It is no other than a dark Lanthorn, in the Side of which there is fixed a short Tube, and in the Tube two convex *Lenses*, and between them a transparent painted Image of the Object we would represent. The Passage of the Rays through the *Lenses* and
the

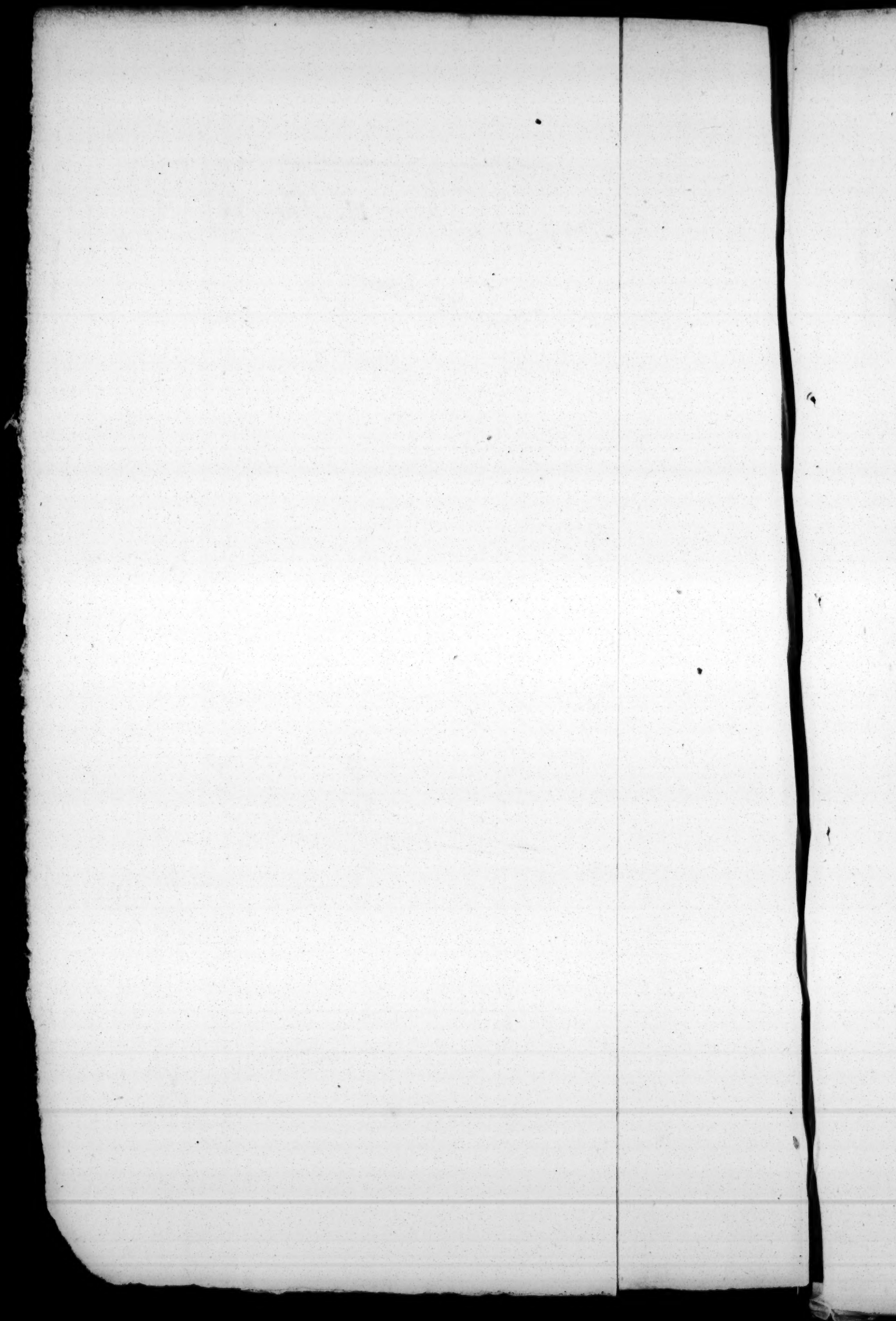
212 *Of the Obscura Camera, &c.*

the Image is thus. Let A (Fig. 89) represent a burning Lamp placed as in a common Lanthorn, and let CD, EF, be the two *Lenses* placed in the Tube abovementioned, and the Picture at GH. And let the Situation of the *Lens* CD be such, that the Light which falls upon it from the Lamp may be all thrown upon the Picture GH, by which means it will be strongly illuminated, and being transparent, will throw out Rays in Plenty the other Way: Which Rays, in passing through the other *Lens* EF, let us suppose to be collected into their respective *Foci* on an opposite Wall at IK, and to form an Image there. Which Image will be larger than the Picture in Proportion as the Distance IL is greater than LH; because the Angles ILK and GLH are equal; and the Room being dark every where else, it will appear very bright, if the Picture be strongly illuminated by the Lamp. And besides the abovementioned *Apparatus*, there is sometimes a concave Reflector placed within the Lanthorn behind the Lamp, as at MN, to give a stronger Illumination to the painted Image at GH.

The End of Part III.

Part III. Plate XV. p. 212.





A
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OF
Natural Philosophy.

With NOTES
Containing the
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AND
Some Occasional REMARKS.

PART IV.
ASTRONOMY.
Containing the *real* and *apparent* Motions of
the *Heavenly BODIES*.
With an Attempt to account for the *Phænomena*
of the *COMETS TAILS*,
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A
COMPENDIOUS SYSTEM
OF
Natural Philosophy.

PART IV.
ASTRONOMY.

The INTRODUCTION.

THE Science of *Astronomy* may be distinguished into three Parts; the first relating to the Motion of the Heavenly Bodies, as they *really* are in themselves; the second relating to the Phænomena of Appearances these Motions occasion to a Spectator on the Surface of the Earth; and the third to the physical Cause of those Motions.

The two first of these, *viz.* the real Motions of the Heavenly Bodies, and the Manner they seem to us to move in, are frequently the reverse of each other; that Body to us appearing to move *Westerly*, which in reality at the same Time moves *Easterly*; and that seeming to tend towards the *North*, which is standing

A ing

4 *The* INTRODUCTION.

ing still, or hastening towards the *South*; and the contrary. And because the apparent Motions of these Bodies depend upon their real ones, the Order I shall make use of, shall be first to lay down their real Motions; and then to show how the apparent ones arise therefrom: After which I shall proceed to consider the physical Cause thereof. But since the generality of People are apt to form their Judgment by their Senses only, and are loth to imagine Things different from what they appear to be, to prepare such Readers for a more easy Reception of what follows, it may not be amiss to premise the following Particulars by way of Introduction to this Part.

I. Let us imagine only one great Body in the Universe, placed in the midst of infinite Space, like a Ball in the Air; and then enquire whether of the two following Suppositions has the greater Probability on its Side; *viz.* That this Body will continue in its Place; or that it will not.

In pursuance of this Inquiry, perhaps it may be argued thus: Whereas no Bodies, on which we can make trial, are observed to continue in their Places without a Support, neither will this Body remain in the Place it is put, but will immediately descend, as it were, towards the lower Regions of Space.

This Argument is founded upon the Observation

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vation of Bodies falling to the Ground for want of Support, which is not a parallel Case. For to say the least that may be, we cannot be sure but that the Earth itself may be the Cause why Bodies descend towards it; as the Loadstone is the Cause why Steel approaches it when placed within a certain Distance from it. And if so, then for any thing that appears from hence to the contrary, this Body, being the only one in the Universe, may remain in its Place without a Support. In short, the Argument is just as if one should reason thus: It has never yet been observed, but that Steel when placed at a small Distance below a Loadstone will ascend upwards; therefore by Parity of Reason, if all the Bodies in the Universe were transformed into a Mass of Steel, and placed alone in the midst of infinite Space, that Mass would ascend. The Fallacy of which way of arguing is sufficiently apparent.

There is a much greater Probability of Truth in the opposite Way, and it is more rational to argue thus: By all the Experiments and Observations made upon Bodies, it does not appear, that any ever moves out of its Place, unless impelled by some other; or when it moves towards another, as in the Case of Bodies falling towards the Earth, or of Steel tending towards the Loadstone. Now the Loadstone is certainly some way or other the Cause why the Steel tends towards it, and therefore

6 *The* INTRODUCTION.

(by Parity of Reason) in all Probability the Earth is some way or other the Cause why Bodies are disposed to move towards it: And if so, then supposing all the Bodies in the Universe to be collected into one, it would most likely continue where it was put, without requiring any thing to support it, as having nothing which might impel it any way, or towards which it might move. This, I say, being the most natural and easy Deduction we can make in this Case, seems to have the greater Probability on its Side. And farther,

II. Since there are no Bodies, we can make Trial upon, but what are disposed to move towards some other Body, as Bodies towards the Earth, or Steel toward the Loadstone, it is most probable that the several Bodies, whereof the Universe consists, would all move towards one another, if not prevented.

III. Let us suppose all the Bodies in the Universe to be united in one, and that Body to be put into Motion. Now, whereas upon that Supposition there is nothing which may stop or destroy its Motion, it does not appear that it would ever stand still, or lose the least Part of it. For, by all the Experiments we can make upon the Bodies about us, there are none which, when put into Motion, can be stopped without some Force applied to them for that Purpose. They will *seem* indeed to stop of themselves, which is always owing
to

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to some Impediment. Accordingly it is found, that the less Resistance or Rubs Bodies meet with in their Way, the longer they continue their Motion (*a*). Whence we may infer, that this Body, once put into Motion, would ever continue in that State (*b*).

IV. Let us now suppose that all the Bodies in the Universe are comprised in two, the one of which let us imagine to be immensely greater than the other, and placed at some Distance from it. These Bodies, if not obstructed, will (agreeably to the Conclusion made in Section II.) approach towards each other; but because a little one requires proportionably less Force to put it in Motion than a larger one, the less will be the only one in which the Motion will be sensible, because we have supposed the other to be immensely greater, which will therefore be scarcely moved out of its Place by the less; just as it is with a Loadstone and a Piece of Steel hung up by two Strings near one another; if the Steel be much less than the Loadstone, it shall move almost all the Way towards the Loadstone, while that shall scarce stir out of its Place; but if the Steel much exceeds the Loadstone in Bulk and Weight, the Loadstone shall then move to the Steel, and not the Steel to the Loadstone, as in the other Case. This being granted,

(*a*) See Part I. Chap. IV.

(*b*) See more in Confirmation of this, Part I. Chap. IV.

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Let us conceive the less Body, while it is at some Distance from the larger one, to have a swift Motion given it in a Direction parallel to the Surface of the larger Body. This Motion will in some Measure prevent the Body from approaching to that other; and the greater the Velocity is that is thus given to it, the longer will the Time of its Approach be protracted thereby, and the greater Space it will move over before it comes to the Body. The Velocity therefore which is thus given to it, may be so adjusted to the Degree of Tendency it has towards the other Body, that it shall move quite round it without ever touching it, returning to the same Place where it was when the Motion was at first given it. In this Case, if it meets with no Opposition, it will move round it in the same Path over and over again, not unlike to a Stone whirl'd round the Hand in a Sling (*c*). And as a smaller Body may continually move round a larger in this Manner, so may several less ones move round the same larger one at the same Time, provided the Distances of the less ones from each other be so great, that their mutual Tendencies towards each other do not disturb their Motions (*d*).

(*c*) See this explained more at large, and illustrated, Part I. Chap. VIII.

(*d*) For the same Reason that the several smaller ones tend towards the larger, they will also tend one towards another, but with less Force in Proportion to their Smallness.

This

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This is the Case of the Planets and Comets with respect to the Sun, which is a Body immensely greater than any of them, placed as it were in the midst of an infinite Space, and round which they perform their respective revolutions at different Distances from it (*e*).

As to what we call the *Sky*, in which the Heavenly Bodies seem as it were fixed, it is no real Substance, but mere empty Space. The Reason why it appears to be a Substance, is as follows. Was there no Atmosphere surrounding the Earth (*f*), whose Particles might reflect other Rays of Light to our Eyes, than those which come directly from the Sun, all Parts of the Heavens, even in spite of Sunshine, would be quite dark; and the Stars

(*e*) This Doctrine being allowed, we may see the Possibility of the Earth being habitable quite round: For, if Bodies are not disposed to move any otherwise than as they tend towards some other, there can be no more Danger of our *Antipodes* falling from the Earth, than there is of our rising up into the Air. And as to their walking with their Heads downwards with respect to us, there is neither Inconvenience, nor seeming Oddness to them in that; for first, their Feet are turned towards the Surface of the Earth on which they walk, as well as ours are; and secondly, since both the Heavens and the Earth, which are the only Things they can judge of their Situation from, have the same Position with respect to their Bodies, when they stand or walk, that they have to us in the like Case, their Position cannot seem otherwise to them than ours does to us; for a Man cannot tell which way his Feet are turned, any otherwise than as his Judgment is directed by the Things about him; just as one cannot distinguish North from South in a strange Place, without a Compass, or seeing either the Sun or Stars.

(*f*) See the Nature and Constitution of the Atmosphere explained, Part II. Chap. III.

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would be visible at Noon-day (*g*). But since the Atmosphere of the Earth abounds with Particles capable of reflecting Light every way, some of it will fall upon our Eyes, to whatever Part of the Heavens they are directed. This Light, as all other, gives us the *Idea* of some Colour. Having an Impression of Colour upon our Minds, our Imagination presents us with a Substance for it to inhere in: Just as, when a Man views an Object in a Glass, the Light regularly reflected gives him an Idea of the Colours of that Object duly ranged; but it is his Imagination that forms the Image he thinks he sees.

Thus much by way of Introduction; which must not be understood as containing all the Reasons for these Things; but only as tending to shew the Possibility, and in some Degree the Probability, of them.

That which will be delivered afterwards, when we treat of the Physical Causes of the Motions of the Heavenly Bodies, will set this Doctrine in a clearer Light, and abundantly evince the Reasonableness of it. Proceed we in the mean time to consider the Order these Bodies move in, and their respective Distances from the Sun, which makes the Subject of the first Chapter.

(*g*) See Chap. XIII.

C H A P.

S E C T. I.

Of the Heavenly Bodies, and their real Motions.

C H A P. I.

THE Sun is a Body of prodigious Magnitude, situated in the midst of an unbounded Space, and the Fountain of Light and Heat to a certain Number of Planets and Comets, which continually move round it (*a*): All these, taken together, make up what is called the *Solar System*.

The Planets are in Number sixteen; six of which are called *Primary* ones; the other ten are distinguished by the Names of *Secondary* ones, *Moons*, or *Satellites*.

The *Primary* Planets are *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter* and *Saturn*. They revolve about the Sun at different Distances from it, moving from West to East (*b*), in Orbits (*c*)

(*a*) The Sun itself is not absolutely at Rest, but is subject to a very small Degree of Motion, which shall be consider'd when we treat of the Physical Causes of the Motions of the Heavenly Bodies.

(*b*) When any of the Heavenly Bodies are said by Astronomical Writers to move from West to East, or from East to West, it is always to be understood by way of the South. So that by the latter Expression is meant the same Way that the Sun appears to move in its daily Course, by the former is intended the contrary.

(*c*) By the Orbit of a Planet is meant the Way or Path it describes in moving round the Sun.

nearly

nearly circular, having the Center of the Sun directly within each of them, and nearly in the Middle; and they are not coincident one with another, but every one has about one Half a little above, and the other Half a little below each of the rest. Or, to express it in the common Way, the Planes (*d*) of their Orbits pass through the Center of the Sun, and are so situated, that they make but small Angles with each other.

Figure the first represents the *Solar System*, wherein the Point S is designed to denote the Center of the Sun; the Circle A B the Orbit or Path, which the nearest Planet *Mercury* describes in moving round it; CD, that in which *Venus* moves; FG, the Orbit of the Earth; HK, that of *Mars*; LN, that of *Jupiter*; and OP, that of *Saturn* (*e*). These Bodies, as they move round the Sun at very different

(*d*) The Plane of an Orbit is the Space included therein. Thus, if the Curve Line ADBE, in Fig. II. represents an Orbit, the Surface of the Paper within that Curve represents the Plane of that Orbit. Or, in other Words, the Plane of the Orbit of a Planet is a flat broad Space, out of which the Planet never strays.

(*e*) That the Sun is at Rest, and that the Planets move round it as described above, is an Opinion received of old by *Philolaus*, *Aristarchus* of *Samos*, and the whole Sect of the *Pythagoreans*. The *Egyptians* were early Observers of the Heavens, and from them probably this Notion was received in *Greece*; tho' the Notion of the Earth standing still in the Center, and the whole Heavens revolving round it, was generally received and defended, till the Time in which *Copernicus* flourished, viz. about the Year 1500, who restored the ancient Astronomy, and shewed in so clear a Manner how the Appearances of the Heavens might
be

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ferent Distances, the true Proportion of which is nearly represented in the Scheme, so they perform their respective Periods in very different Times; *viz.* *Mercury* in about three Months; *Venus* in about seven and a Half; the Earth in a Year; *Mars* in about two Years; *Jupiter* in twelve; and *Saturn* in not much less than thirty. And as they differ in their Periods, so they do surprisingly in their Mag-

be accounted for by it, that it became generally received, and from him was called the *Copernican System*.

There are two other *Hypotheses* of Note, *viz.* the *Ptolemaic* and the *Tychonic*. The *Ptolemaic* is so called from *Ptolemæus*, a Mathematician of *Pelufium* in *Egypt*, a great Defender of this *Hypothesis*. He supposes the fixed Stars to be stuck in the Firmament as they appear to the Eye; the Earth in the Center, round which he supposes the Moon to revolve; and at a greater Distance, *Mercury*; at a greater still, *Venus*; at a greater than this, the Sun; at a still greater, *Mars*; then *Jupiter*; then *Saturn*; and beyond all, the Firmament with the fixed Stars; all in the same Time they appear to move round us. This *Hypothesis* was excellently well adapted to amuse the Vulgar, the Motions of the Sun and Planets being represented by it much after the same Manner as they appear to Sense. But since by the Help of Telescopes it has been discover'd, that *Venus* puts on different *Phases*, like the Moon, the Reason of which will be shewn in Chap VI. this *Hypothesis* is entirely laid aside, as inconsistent therewith.

The *Tychonic Hypothesis* has its Name from *Tycho Brahe*, a *Danish* Nobleman. This Philosopher supposes that the Earth stands still, and that as the Sun moves round the Earth, the Planets move round that, just as the Secondaries move round their Primaries in the *Solar System* laid down above. There is another *Hypothesis* called the *Semitychonic*, and differs from the former only in this; that whereas that makes the Earth stand still, and the Sun with the Planets to move round it every Day; this gives the Earth a diurnal Motion round its *Axis*, and only supposes the Sun, with the Planets revolving about it, to move round the Earth once a Year.

nitudes. For supposing the Circumference of the Sun to be represented by a Circle of ten Inches in Diameter, the several Circles Q, V, W, X, Y, Z, would nearly express those of the Planets, *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn* respectively (*f*).

These Planets have no Light of their own; for when viewed through a Telescope, the Side only next the Sun is observed to be enlightened. They are also of the Form of a Globe; for one Edge of the shining Part sometimes appears hollow, sometimes strait, and sometimes convex, according as the Planet is situated with regard to the Spectator and the Sun; which it could not do, unless they were of that Form (*g*).

Farther, the Planets do not only move about the Sun, as we have observed, but turn also about their own *Axes* (*b*) at the same Time, and

(*f*) Their Magnitudes, Distances, Times of Revolution, with their Rotation about their *Axes*, are more accurately expressed in the following Table.

	Diameters in English Miles	Dist. from the Sun in English Miles.	Periodical Times.				Rotation about their <i>Axes</i> .			
			Days	H.	M.	S.	D.	H.	M.	S.
The Sun	763,460						25	6	0	0
Mercury	4,240	32,000,000	87	23	15	53	not discovered			
Venus	7,906	59,000,000	224	16	49	24	00	23	16	0
The Earth	7,970	81,000,000	365	6	9	14	00	23	56	4
Mars	4,444	123,000,000	686	23	27	30	00	24	40	0
Jupiter	81,155	424,000,000	4,332	12	20	25	00	9	55	54
Saturn	67,870	777,000,000	10,759	6	36	26	not known.			

(*g*) See the Reason why one Edge of the shining Part of a round Body appears sometimes strait, sometimes hollow, sometimes convex, largely explained Chap. VI.

(*b*) By the turning of a Planet about its *Axis* is meant, its turning about any two Points in its Surface, that are opposite to one another;

and the same Way they do about the Sun, viz. from West to East. As the former of these Motions is generally compared to the revolving of a Stone in a Sling about the Hand, this latter may be compared to the spinning of a Top, or the turning of a Wheel upon its Axle-tree. The Sun itself is not exempt from a Motion of this latter Kind.

This Motion of the Sun and some of the Planets about their *Axes* is discoverable by certain Spots, or distinguishable Parts on their Surfaces; which appearing first on one Extremity of their Disks (*i*), do by Degrees come forwards towards the Middle, and so pass on till they reach the opposite Edge thereof, where they disappear; and after they have lain hid about the same Space of Time that they continued visible, they appear again as at first. By the Motion of these Spots passing in a right Line over the Disk of a Planet, or rising upwards one Half of the Way, and descending the

another; and an imaginary Line, supposed to be drawn through the Planet from one of those Points to the other, is, while the Planet moves about those two Points, called its *Axis*. Thus if the Reader holds a Ball between his Finger and Thumb, and turns it round, a Line supposed to pass through it from Finger to Thumb, will in Astronomical Language be the *Axis* of that Ball. And if he takes it up by two other Points, and turns it, then will a Line passing through those other Points become its *Axis*.

(*i*) This is a Term used by Astronomers for the Face of the Sun, Moon, or other Heavenly Body, when it is considered as a flat round Surface, as it appears to be: And the Breadth of it is conceived to be divided into twelve equal Parts, which they call *Digits*.

other Half, the Situation of the Planet's *Axis*, about which it turns with respect to us, is evidently discovered. These Spots are very visible on the Surface of the Sun, *Venus*, *Mars*, and *Jupiter*; but by reason of the Nearness of *Mercury* to the Sun, and the great Distance of *Saturn* from us, no Observations have been made that could discover any Spots in them, so that it is uncertain whether these Bodies revolve about their *Axes* or not. As to the rest, the Times in which they thus revolve are expressed above in Note (*f*) in page 14. and the Inclination of their *Axes* in the Note below (*k*).

In *Jupiter*, besides his Spots, there are several broad Spaces running parallel to each other, called his *Belts*. These, as also his Spots, are observed to undergo several Changes, neither keeping the same Magnitude nor Distance from one another. And some of his Spots have appeared only for a Time.

As to the Spots of the Sun, they are subject to much greater Variety of Changes. New ones appear, and old ones vanish, others succeeding in their Room; several smaller ones run together, and form one larger; and larger

(*k*) The Inclination of the Axis of the Sun to the Plane in which the Earth moves, is seven Degrees, and a Half; that of *Venus* to the same Plane fifteen Degrees, according to an Observation of Signior *Blanchini*'s at *Rome* in the Year 1726; the Earth's 23 Degrees and 29 Minutes; and *Mars* and *Jupiter*'s are nearly at right Angles with their own Orbits.

ones

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ones are sometimes divided into smaller (*l*). However, several have remained on the same Part of its Body long enough to determine the Time of its Motion about its Axis. See Note (*f*) in page 14.

In the Beginning of this Chapter it was observed, that the Planets move round the Sun in Orbits nearly circular, and in Planes crossing each other in such Manner, as to make very small Angles with each other. As to the Form of their Orbits, they are *Ellipses* (*m*), having one of their *Foci* (*n*) in the Center of the Sun. They differ very little from Circles, or, to speak properly, their *Excentricities* (*o*) bear

(*l*) We learn from History, that the Sun has wanted its usual Brightness, shining with a dim and obscure Light for a Year together. This was probably owing to its being in a great Measure covered with Spots; for now there are sometimes Spots seen upon it, that are larger than the Surface of the whole Earth.

(*m*) An *Ellipse* is a Figure like an Oval; but strictly speaking, a Geometrical Curve, made by the tranverse Section of a Cone or a Cylinder. But the Nature of it may be understood from the following Description. Stick up two Pins at F and G (Fig. 2.) over which put a Thread, both Ends being tied together; then with a third Pin at P, within the Thread, keeping it upon the Stretch, describe the Curve ADBE, which is an *Ellipse*. And if we make Use of the same Thread, but increase or diminish the Distance between the Pins, the Figure described will still be an *Ellipse*, though of a different Kind. And when the Pins are brought close together, the Figure described degenerates into a Circle.

(*n*) The Points F and G, where the Pins were fixed, are called the *Foci* of the *Ellipse*, and the middle Point between them the Center.

(*o*) The Distance between the Center and either of the *Foci* is the *Excentricity*.

a very

a very small Proportion to their longer *Axes* (*p*). And as to the Situation in which their Planes lie, it is in all nearly the same with that of the Earth's Orbit. See the Excentricities of their Orbits, and their Inclinations to that of the Earth's, in the Notes (*q*) and (*r*).

The Primary Planets are again distinguished into two Kinds, *viz.* *Inferior* and *Superior*; those which are nearer the Sun than the Earth is, as *Mercury* and *Venus*, are called the *Inferior* Planets; the others the *Superior* ones.

The Secondaries are constant Attendants of the respective Primary ones to which they belong, revolving round them, while they themselves move round the Sun. Of the Primary Planets there are only the three largest, as far as the Observations of Astronomers have been able to discover, that have their Secondaries, *viz.* the *Earth*, *Jupiter*, and *Saturn*.

(*p*) The Line AB, which passes through the *Foci*, and is terminated at each End in the Ellipse, is called the *longer Axis*. The Line DE, which passes through the Center of the Ellipse, in a Direction perpendicular to the former, and is terminated by the Sides of the Ellipse, is the *shorter Axis*.

(*q*) The Excentricities of the several Orbits of the Planets are as follow: Supposing the Distance of the Earth from the Sun 1000 equal Parts, the Excentricity of *Mercury's* Orbit is about 80 such; of that of *Venus* 5; of that of the Earth 17; of *Mars* 141; of *Jupiter's* 250; and the Excentricity of *Saturn's* Orbit 247.

(*r*) Their Inclinations are as follows: *viz.* The Orbit of the Planet *Mercury* is inclined to that of the Earth 6 Degrees, 54 Minutes; that of *Venus* is inclined to it 3 Degrees, 24 Minutes; that of *Mars*, 1 Degree, 51 Minutes; that of *Jupiter*, but 1 Degree, 19 Minutes; and that of *Saturn*, 2 Degrees and 33 Minutes.

The

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The *Earth* is attended but by one, which is the Moon. This Secondary revolves round the Earth from West to East in 27 Days, 7 Hours, and 43 Minutes, at the Distance of about 60 Semidiameters and an half of the Earth from its Center (*s*); and performs its Rotation about its Axis, which is nearly perpendicular to the Plane of the Earth's Motion, in the same Time and the same Way that it revolves about the Earth.

The Planet *Jupiter* has four Moons, which revolve about it at different Distances and in different Times. The innermost performs its Revolution in about one Day and eighteen Hours, nearly at the Distance of six Semidiameters of *Jupiter* from its Center; the second revolves about it in about three Days and an half, at the Distance of about nine such Semidiameters; the third in about seven Days, at the Distance of 14 Semidiameters; the fourth and outermost Satellite requires almost 17 Days to perform its Course in, and is distant from its Center 25 Semidiameters nearly (*t*).

B

Saturn

(*s*) The Excentricity of the Moon's Orbit is about $\frac{3}{3}$ of the Semidiameter of the Earth; and its Inclination to that of the Earth is 5 Degrees and 18 Minutes. The Reason that the Moon appears so large, and to outline all the other Planets, both Primary and Secondary, is because of the Smallness of its Distance from us in respect of theirs; for had it been as far removed from us as some of them, it had never been seen at all by us.

(*t*) More accurately as in the following Table.

Satell.

Saturn has no less than five Satellites: The first and innermost revolves about it in one Day and 21 Hours, at the Distance of about 5 Semidiameters and an half of *Saturn* from its Center; the second performs its Course in two Days and 17 Hours, at the Distance of 6 Semidiameters and a Quarter; the third in about 4 Days and an half, and is distant from it about 8 Semidiameters and 3 Quarters; the fourth compleats its Period in about 16 Days, at the Distance of 20 Semidiameters; the fifth and outermost requires about 79 Days to perform a Revolution in, and is 59 Semidiameters of *Saturn* distant from its Center (*u*).

These Satellites, as also those of *Jupiter*, perform their Revolutions about their Primaries from West to East, as their Primaries do about the Sun.

The Orbits of the Secondary Planets are expressed in Fig. 1. by the pricked Circles;

Satell.	Periodical Times.				Dist. in Semid. of <i>Jupiter</i> .
	D.	H.	M.	S.	
1	1	18	27	34	5,667
2	3	13	13	42	9,017
3	7	3	42	36	14,384
4	16	16	32	9	25,299

(*u*) More accurately as in the following Table.

Satell.	Periodical Times.				Dist. in Sem. of <i>Saturn's</i> Ring.
	D.	H.	M.	S.	
1	1	21	18	27	2,10
2	2	17	41	22	2,69
3	4	12	25	12	3,75
4	15	22	41	14	8,70
5	79	07	48	0	25,35

viz.

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viz. that of the Moon at E, those of the Satellites of *Jupiter* at I, and those of *Saturn* at T; the Centers of the Earth, *Jupiter* and *Saturn* being supposed to be in the Points E, I, and T respectively.

As to the Orbits of the Satellites of *Jupiter* and *Saturn*, Astronomers have not observed that they differ from Circles; and whether these Secondaries revolve round their *Axes* or not, has also not been discovered (*x*).

The Secondaries are also Bodies having no Light of their own, being observed to be eclipsed as oft as they enter the Shadows of their Primaries.

Besides its Satellites, *Saturn* is also encompassed with a flat broad Ring, not unlike the Verge of an Hat, but it no where touches the Body of *Saturn*, the Space between it and *Saturn* being equal to the Breadth of the Rim. The Diameter of the Ring, measuring across from one Outside to the other, is to the Diameter of *Saturn's* Body as 7 to 3 (*y*).

The Form of the Ring encompassing the Body of *Saturn*, is expressed in Fig. 1. at RR.

(*x*) The Orbits of *Jupiter's* Satellites lie almost in the same Plane in which *Jupiter* himself moves round the Sun; but those of *Saturn* describe their Orbits nearly in the Plane of the Ring produced, except the fifth, the Situation of whose Orbit deviates a little therefrom. See the Inclination of the Ring in the next Note.

(*y*) The Ring, according to Messrs. *Römer*, *Picard*, and *Hugens*, is inclined to the Orbit of the Earth 31 Degrees: But Dr. *Halley* affirms it to be nearly parallel to the Plane of the Earth's Equator.

The Comets revolve also round the Sun in Orbits, whose Planes pass through its Center ; but they are not confined to any particular Direction, as the Planets are, some moving one Way, and some another : Their Orbits are of a very elliptical or oval Form, so that their Lengths vastly exceed their Breadths ; and they lie not nearly in the same Plane, like those of the Planets, but admit of any Inclination to one another. But more of these in another Place (z). A Portion of the Orbit of a Comet is expressed in Fig. 1. at *abcd* (a).

Those

(z) Chap. XI.

(a) The Claim which the System here laid down has to a Preference to all others, will appear from the following Considerations.

I. The Planet *Venus* moves round the Sun, and not round the Earth. For through a Telescope it sometimes appears with a bright and round Face like the full Moon, and sometimes horned like the new Moon ; (see Chap. VI.) and when it appears full, it is seen near the Body of the Sun, as well as when it appears horned : Which things could not be, did it move either round the Earth alone, or round both the Sun and the Earth, or round neither of them. For did it move round the Earth alone, or round both the Sun and the Earth, it must be seen sometimes in that Part of the Heavens which is opposite to the Sun. Did it move round neither the Sun nor the Earth, it could not appear both horned and full when it is seen near the Body of the Sun : It remains therefore, that it revolves round the Sun, and not round the Earth. Its Orbit therefore, agreeably to the abovementioned System, includes the Sun, and is within the Orbit of the Earth.

II. In like Manner we are assured, that *Mercury* turns round the Sun in a lesser Orbit than that of *Venus*, because it puts on the same Phases with *Venus*, and is never seen so far from the Sun as that Planet is.

III.

Those bright and shining Bodies, which besides the Planets and Comets are dispersed every

III. The Orbit of *Mars* includes both the Earth and the Sun; and the Earth is not in the Center thereof. For this Planet is capable of appearing opposite to, or in any other Situation in respect of the Sun; which it could not be, unless it moved round the Earth: And it always appears full, or nearly so; which it could not do, if it ever came between the Sun and the Earth: It moves therefore also round the Sun. And farther, when *Mars* is in the opposite Part of the Heavens to the Sun, it appears about five times larger than when it is near the Sun; which shews that it is so many times nearer the Earth in one Situation than in the other: The Earth therefore is not in the Center of its Motion. All which is agreeable to the System above laid down.

IV. Since the like is observable both of *Jupiter* and *Saturn*, (though, by Reason of their greater Distance from the Sun and us, the Diversity in their apparent Magnitudes in different Parts of their Orbits is not so great as in *Mars*) it is reasonable to conclude, that these Planets also have both the Earth and the Sun within their Orbits, and that rather the Sun than the Earth is in the Center of the same.

V. Since then the Earth is placed within the Orbits of the Planets *Mars*, *Jupiter*, and *Saturn*, these Planets cannot appear to us to stand still or to go backward, as Observation shews they sometimes do appear, unless the Earth moves: And since, as we have shewn, the Planets *Venus* and *Mercury* revolve about the Sun, and not about the Earth; since likewise the Earth is placed between the Orbits of *Mars* and *Venus*, and the Periodical Time of the Earth, if it does move, is, in point of Duration, between the Periodical Times of those two; it has been thought reasonable to suppose that the Earth revolves in like Manner about the Sun, as the Planets do, and is therefore reckoned in the Number of them.

VI. But what wonderfully confirms this Conclusion, is, that Harmony which upon this Supposition runs through the whole *Solar System*; viz. that the Motions of all the Planets, both Primary and Secondary, are governed and regulated by one and the same Law; which is, that *the Squares of the Periodical Times of the Primary Planets are to each other, as the Cubes of their Distances from the Sun*; and likewise *the Squares of*

every where throughout the Heavens, are the *fixed Stars*. They are termed *fixed*, because they are observed to keep the same Distances from each other in all Ages, having no Motion like that of the Planets or Comets. They are far removed out of the *Solar System*, and shall therefore be considered in a Chapter by themselves.

the periodical Times of the Secondaries of any Primary are to each other, as the Cubes of their Distances from that Primary. Now the Moon, which in the System above laid down is a Secondary of the Earth, in the other Hypotheses is a Primary one; and so the Rule cannot take Place; because her Periodical Time, considered as that of a Primary one, does not agree therewith. Whereas, according to the other Supposition, the Motions of the Heavenly Bodies, as we shall see in its proper Place, are to be accounted for with the greatest Simplicity; and a regular and beautiful Fabrick is thereby exhibited, worthy its divine Architect.

By the *Square* of the periodical Time of a Planet is meant the Time it takes up in revolving about the Sun, or about its Primary, multiplied by itself *Once*; and by the *Cube* of its Distance is meant its Distance multiplied by itself *Twice*. Thus if the Time a Planet moves once round its Orbit in, be 30 Years, then 30 Times 30 is the *Square* of its periodical Time: and if the Distance of a Primary Planet from the Sun, or of a Secondary from its Primary, be 81 Millions of Miles, then 81 Million multiplied by 81 Million, and that again by 81 Million, will express the *Cube* of the Distance of that Planet.

C H A P. II.

Of the fixed Stars.

THE *fixed Stars* are those bright shining Bodies, which, besides the Planets and Comets, are dispersed throughout the Heavens: They are termed *fixed*, as appearing always at the same Distance from each other.

It is observable of the *fixed Stars*, that, when viewed through a Telescope, they appear only as mere Points destitute of all sensible Magnitude. This is supposed to be owing to their very great Distance from us. That they appear larger to the naked Eye than with a Telescope, is ascribed to that Irradiation or Brightness which is observed to surround shining Bodies when seen in the Dark, and which the Telescope takes off (*a*).

From their shining with so strong and sparkling a Light, when at the same Time they are so far off that through a Telescope they appear as mere Points, is inferr'd, that they shine

(*a*) That the Stars would appear of no sensible Magnitude to the naked Eye, were it not for that Irradiation or Brightness mentioned above, is confirmed from their being observed to *twinkle*; which the Planets and other Bodies of sensible Magnitude do not. The Reason of their *twinkling* is, that the least Particle of Dust or Vapour passing between the Eye and them is capable of intercepting their Appearance for a Time, which in larger Bodies it is not.

with their own proper and unborrowed Light : For if they borrowed their Light, they would be invisible to the naked Eye, since the Satellites of *Jupiter* and *Saturn*, which appear of very distinguishable Magnitudes through a Telescope, are so.

There is another remarkable Circumstance relating to the starry Heavens ; that the better Telescopes we make use of, the more Stars appear : From whence it is thought, that all the *fixed Stars* are not placed at equal Distances from us, but that they are every where interspersed at great Distances beyond one another throughout the Universe ; and that probably the different Appearances which they make in point of Splendor and Magnitude may be rather owing to their various Distances from us, than to any Difference in their Magnitudes.

The Distance of the nearest *fixed Star* is computed to be at least 500 thousand Millions of Miles from us ; and their Magnitude is judged to be such, that if they were as near us as the Sun is, they would not appear unlike it (*b*).

Their appearing under various Magnitudes has given Occasion to a Distribution of them into several Orders or Classes : Those which appear largest, are called *Stars of the first*

(*b*) The Method of computing the Distance of the *fixed Stars* is shewn in Chap. V.

Magnitude ;

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Magnitude; those which are nearest to them in Lustre are reckoned of the *second Magnitude*; and so on, till we come to the least Stars that are visible to the naked Eye, which constitute the last Class, and are said to be of the *sixth Magnitude*. For such as are to be seen only by the help of a Telescope, are not included in those six Classes, but are distinguished by the Name of *Telescopic Stars*; the Distribution of the Stars into Classes being made long before the *Telescopic* ones were discovered. But we are not to understand from hence, that all the Stars which are visible to the naked Eye, are reducible to some one of these Classes precisely; for there are scarcely two Stars to be found exactly of the same Size: They are only to be ranked in that Class, to which their Magnitude and Lustre gives them the justest Claim.

There is also another Distribution of the Stars into *Constellations*. The Ancients, that they might the better distinguish the Stars with regard to their Situation in the Heavens, divided them into several *Asterisms* or *Constellations*, that is, Systems of Stars, each System consisting of such as are near each other, And to distinguish these Systems from one another, they gave them the Names of such Men or Things as they fancied the Space they took up in the Heavens represented, or of those whose Memories, in Consideration of some notable

table Exploit, they had a Mind to transmit to future Ages.

Besides the Stars contained within these Constellations, there are some others, which for that Reason are called *Informes*. Of these the later Astronomers have made some new Constellations.

There is also a Division of the Heavens into three Parts; viz. 1. The *Zodiac*, or that Portion of the Heavens in which the Planets would appear to move to an Eye placed in the Sun. The Breadth of this Space depends on the Inclination of the Orbits in which the Planets move to one another, and includes twelve Constellations, commonly called the Signs of the *Zodiac*; viz. *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, and *Pisces*: And because most of these are Animals, that Space is called the *Zodiac*. 2. All that Region of the Heavens which lies on the North Side the *Zodiac*, which contains twenty-one Constellations: And 3. That on the Southern Side, which contains fifteen.

There is a remarkable Tract that goes quite round the Heavens, called the *Milky Way*. This Space has a peculiar Whiteness, occasion'd by an infinite Number of small Stars therein, none of which are to be seen distinctly without a Telescope, yet conspire to render that Part of the Heavens, where they are, much
more

Chap. II. *Of the fixed Stars.* 29

more luminous than the rest. This Tract is in some Parts single, in others double (*a*).

Notwithstanding that seeming Impossibility of *numbering* the Stars, their relative Situations one to another have been so carefully observed by Astronomers, that they have not only been able to number them, but even to distinguish the Places of each in the Heavens, and that with greater Accuracy, than any Geographer could ever point out the Situations of the several Cities or Towns upon the Surface of the Earth. And not only the Places of those few, if I may call them so, which are to be seen

(*a*) *Hugenius*, in the Year 1656, looking by Chance through a large Telescope at three small Stars very close to one another in the Middle of *Orion's* Sword, saw several more as usual: But three little Stars very near one another, together with four larger ones, shone out as it were through a whitish Cloud much brighter than the ambient Sky; which being very black and serene, caused that lucid Part to appear like an Aperture that gave a Prospect into a brighter Region. He viewed it many Times, and found it continued in the very same Place and of the same Shape. But in the *Philosophical Transactions*, N^o 347, there is an Account of a later Discovery of five more such lucid Spots, though less considerable than this of *Hugens*; the Middle of which, we are there told, is at present in II. nineteen Degrees, with South Latitude twenty-eight Degrees, forty-five Minutes; and that it sends forth a radiant Beam into the South-East; as another in the Girdle of *Andromeda* seems to do into the North-East. It is also there remarked, that tho' these Spots are in Appearance but small, and most of them but a few Minutes in Diameter, yet since they are among the *fixed Stars*, as it appears they are by their having no Parallax, they cannot fail to occupy Spaces immensely great, and perhaps not less than our whole *Solar System*; in all which Places, it should seem, there is a perpetual uninterrupted Day.

with

with the naked Eye have been pointed out and registered by them, but even of such as are discoverable only by the Telescope. The first among the *Greeks*, who numbered those which are visible to the naked Eye, and register'd their Places in a Catalogue, was *Hipparchus*. He flourish'd about 120 Years before *Christ*, and number'd 1022 Stars. To this Catalogue the Places of several more have been added by succeeding Astronomers, as *Ptolemy*, *Copernicus*, *Tycho Brahe*, *Ricciolus*, and Dr. *Halley*; which last went to the Island of *St. Helena*, where he reckoned up and set down the Places of such Stars in the Southern Hemisphere as are not visible to us in this.

The last Catalogue is that of *Flamsteed*, who reckons 3000, of which a great Part are *Telescopical*. The Number of Stars which may be seen at one Time with the naked Eye in one Hemisphere seldom exceed a Thousand; which perhaps may appear strange, since at first Sight they seem to be innumerable. But this Appearance is only a Deception of Sight, arising from a confused and transient View. Let a Person single out a small Portion of the Heavens, and after some Attention to the Situation of the more remarkable Stars therein, begin to count, and he will soon be surpris'd to find how few there are therein.

Yet nevertheless, if we consider the Telescopic Stars along with those which are visible
to

Chap. II. *Of the fixed Stars.* 31

to the naked Eye, we may venture to pronounce them infinite; so prodigious are the Numbers that may be seen in a good Telescope.

Those in Mr. *Flamsteed's* Catalogue mention'd above, that are Telescopical, are only the more remarkable ones, such whose Longitudes and Latitudes (that is, their true Situation in the Heavens) it was thought worth while to register and put down. Dr. *Hook*, with a Telescope of 12 Feet, saw 78 Stars among the *Pleiades*, that is, those which are commonly called the seven Stars; and with a longer Telescope saw more (*b*). And in the single Constellation of *Orion*, which in *Flamsteed's* Catalogue has but 80 Stars, there have been seen 2000 (*c*).

There is one Thing more which is very remarkable in the fixed Stars, viz. that some of those which were taken Notice of, and had their Places register'd, have since disappeared. Some have disappeared for a Time: Some new ones have appeared, and afterwards disappeared; particularly a remarkable one in the Constellation *Cassiopeia*, in the Year 1572, which some time after its first Appearance outshone the biggest of the fixed Stars, and in sixteen Months Time by degrees vanished quite away, and was never seen since. In the *Pleiades*, which used to be reckoned 7, there are but 6 to be seen with the naked Eye.

(*b*) *Hook's* Micrograph. pag. 241.

(*c*) *Antonius Maria de Rheita*, Rad. Sydereomist. pag. 197.

From

From the Similitude there appears to be between the *fixed Stars* and the Sun, it is generally supposed by Philosophers, that they are not placed in the Heavens by way of Ornament only, or to supply us with a faint Light in the Absence of the Moon; but that each of them is placed in the midst of a System of planetary Worlds, and that it directs their Motions, and supplies them with Light and Heat, in the same Manner that the Sun does the several Bodies of which the Solar System is composed (*d*).

As to those Alterations in the Appearances of some of the fixed Stars mentioned above, Sir *Isaac Newton* conjectures, that, as it is possible the Sun may sometimes receive an Addition of Fuel by the falling of a Comet into it (the only Use (*e*) they seem to be of) so the sudden Appearance of some of the Stars, which formerly were not visible to us, may be owing to the falling of a Comet upon them, and occasioning an uncommon Blaze and Splendor for some Time. But that such as appear and disappear periodically, and increase by very

(*d*) How the Sun directs the Motions of the Planets will be shewn afterwards.

(*e*) Sir *Isaac Newton* computes, that the Comet which appeared in 1680 approached towards the Sun within less than a sixth Part of the Sun's Diameter; and from thence concludes, that it must have been retarded by the Resistance of the Sun's Atmosphere, which is very large; the Consequence of which is, that it must come nearer and nearer the Sun every Revolution, till at last it falls into its Body.

Chap. III. *The diurnal Motion, &c.* 33

flow Degrees, seldom exceeding those of the third Magnitude, may be such as having large Portions of their Surfaces obscured by Spots, may by revolving about their *Axes*, like the Sun, expose their lighter and darker Parts to us successively.

S E C T. II.

Of the apparent Motions of the Heavenly Bodies.

C H A P. III.

HAVING now laid down the *true* and proper Motion of the Planets, and shewn, as far as the Observations of Astronomers have been able to inform us, the Nature of the Sun, the Planets, and the fixed Stars; we are in the next Place to enquire particularly into their *apparent* Motions, and shew how it comes to pass that these seem to us so different from what they really are. And these are principally of two Sorts, *viz.* those which arise from the Motion of the Earth round its *Axis*, and those which are owing to the Motion of the Earth and Planets about the Sun. The former of these shall be consider'd in this Chapter; the latter in those which follow.

Since

Since then, as we observed above, the Earth turns round its own *Axis* from West to East, every Spectator upon its Surface must necessarily be carried round it the same Way; and consequently those Parts of the Heavens which lie hid towards the East, will by and by come into his Sight, and those which are visible to him will depart out of it towards the West. From hence it is that the Spectator, not being sensible of his own Motion (the Reason of which is, because all things about him move along with him) imagines the whole Heavens to turn round the contrary Way, *viz.* from East to West, every 24 Hours, which is nearly (*f*) the Time the Earth performs one Revolution about its *Axis* in. This may be illustrated in the following Manner.

Let the Circle STV (Fig. 3.) represent the Earth, S the Place of a Spectator, ABC so much of the Heavens as is visible to him in that Situation, and let A be the Place of the Sun or any other of the Heavenly Bodies: When S, the Place of the Spectator, is carried by the Rotation of the Earth about its *Axis* to T, the visible Part of the Heavens will become BAD; and the Point A, which before was just at one Edge of the visible Portion of the Heavens, is now in the midst of it, or directly over the Spectator's Head. Again,

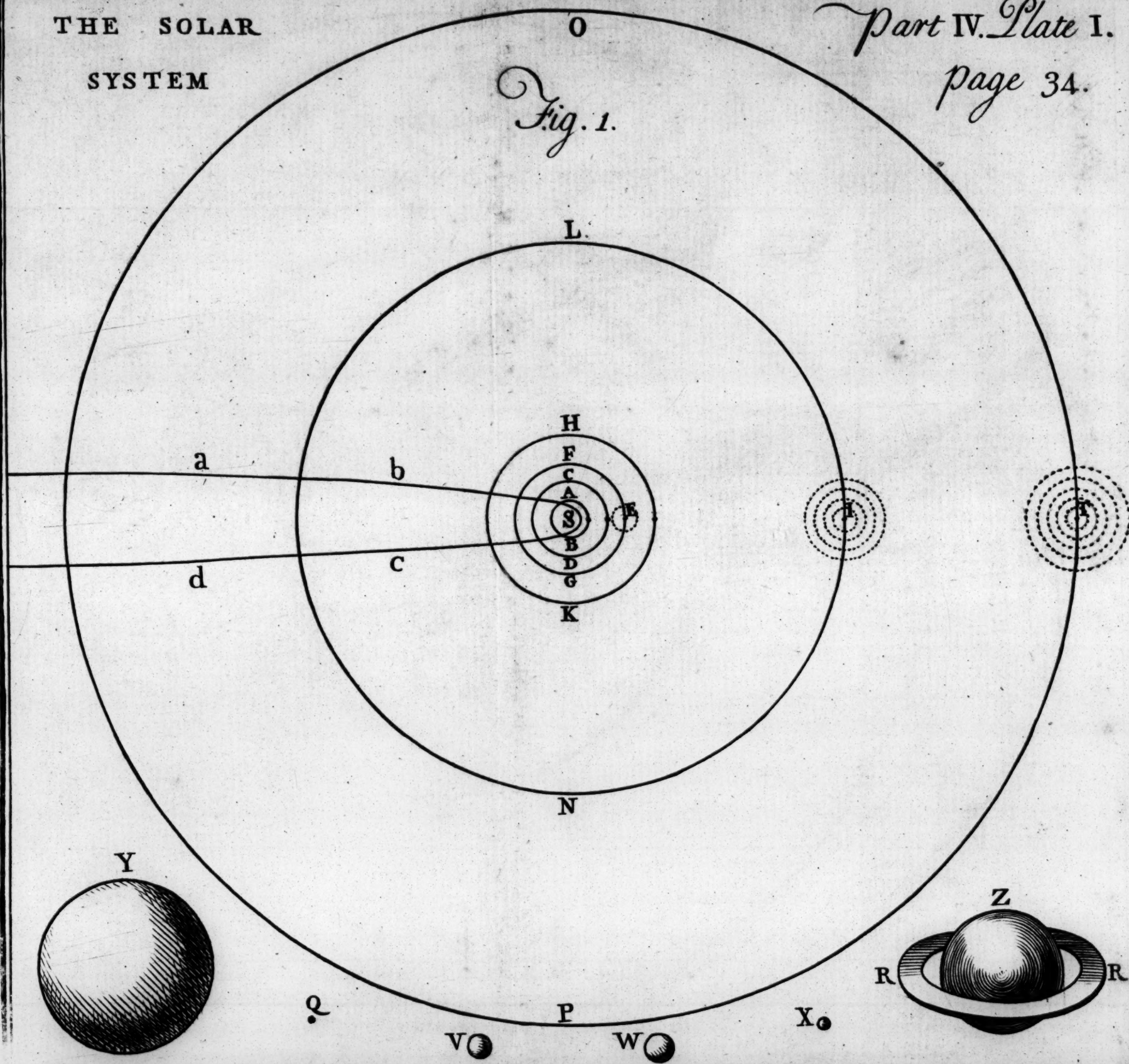
(*f*) The Reason why this is not the *exact* Time, is, because of the Motion of the Earth in its Orbit performed in the mean while; as shall be explained in Chapter the XV.

when

THE SOLAR
SYSTEM

Part IV. Plate I.
page 34.

Fig. 1.



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Chap. IV. *of the Heavenly Bodies.* 35

when the Spectator is got to V, the visible Part of the Heavens is become ADX, and the Point A is got to the other Extremity of it, and just ready to disappear; after which it is seen no more till the Spectator arrives at the Point S again.

From this Motion of the Earth arises that apparent Revolution of the Planets and fixed Stars once in 24 Hours; as also that of the Sun, and therewith the Succession of Day and Night.

That the Days vary from each other both in Point of Length and Heat, is owing to the Motion of the Earth round the Sun, and will be accounted for in the next Chapter.

C H A P. IV.

*Of the apparent Motion of the Sun,
arising from the Earth's revolving
about it.*

AS, when a Person sails along the Sea-Coast, the Shore, the Villages, and other remarkable Places upon Land appear to change their Situation, and to pass by him; and this apparent Change of Place is of two Sorts, (the one is that of Bodies at rest, the

C

Change

Change of whose Places depend solely on that of the Spectator; the other is, that of Bodies in Motion, whose apparent Change of Place depends as well on their own Motion as on that of the Spectator;) so it is in the Heavens: To a Spectator upon the Earth, as it moves along its Orbit, the Sun, the Planets, and the fixed Stars appear to change their Places; which Change in those which are at rest is owing wholly to the Motion of the Earth; in those which move, it is to be ascribed to their own conjointly with that of the Earth: The former Kind shall be considered in this Chapter and the next, the latter in those which follow. To begin with that of the Sun.

As a Spectator in the Sun would see the Earth describing a Circle in the Heavens, which is that where the Plane of the Earth's Orbit (were we to suppose it extended far enough) would cut them; so a Spectator on the Earth, being carried round the Sun therewith, would imagine *that* to move through the same Circle, and in the same Direction, passing from Star to Star, till it has appeared to move quite round the Heavens. This Circle is called the *Ecliptic*, and, from what was observed above of the Zodiac, necessarily passes through the twelve Signs thereof. From hence it is, that the Sun is said to pass through the Signs of the Zodiac once a Year.

To

Chap. IV. *Motion of the Sun.* 37

To illustrate this, let S (Fig. 4.) represent the Sun, ABC the Orbit of the Earth, and DEF the starry Firmament. Then if we suppose the Spectator to be in the Sun, as the Earth moves from A to B, it will seem to move from D to E, describing the Arch DE in the Heavens, and so on; but if the Spectator be upon the Earth, then whilst it moves from A to B, the Sun will seem to describe the Arch GH in the opposite Part of the Heavens: And whilst the Earth is passing from B to C, the apparent Place of the Sun will pass from H to I; and so on through the whole Circle. So that the Sun appears to describe the same Circle among the fixed Stars in a Year, that the Earth would seem to describe in the same Time to a Spectator in the Sun.

Having thus shewn, that the annual Motion of the Earth gives the Sun an apparent one in the *Ecliptic*, we shall now proceed to enquire how the Variety of Days and Nights, as to their Lengths and the different Seasons of the Year, are to be accounted for. In order to which we will premise the three following Observations.

I. As the Earth turns round its *Axis*, there are two Points in its Surface which have no Motion from thence, *viz.* the two Extremities of its *Axis*. These Points are called the *Poles* of the Earth; that which respects the
C 2
North,

North, is called the *North Pole*, the other the *South Pole*. But every other Point of the Earth's Surface describes a Circle, which will be greater or less, as the Point that describes it is farther from or nearer to either Pole; and consequently that Circle, whose describing Point is equally removed from either, will be the largest of all. This Circle is called the *Equator*, by Mariners the *Line*; and its corresponding Circle in the Heavens, or that which the Plane of this Circle, were we to suppose it extended thither, would mark out, the *Equinoctial* Circle; and those lesser Circles described by other Points of the Earth's Surface are called *Parallels* of this.

II. The *Axis* of the Earth, as was observed above, making an Angle with the Plane of its Orbit of 23 Degrees and an half, does the same with that of the *Ecliptic*, those two Planes being coincident one with another. Thus supposing the Line AB (Fig. 5.) to represent the Plane of the *Ecliptic* seen edgewise, and the Circle DFE the Earth; the *Axis* of it is not coincident with the Perpendicular DE, but stands inclined in the Situation FG, supposing the Arch DF to contain 23 Degrees and an half. And,

III. In whatever Part of its Orbit the Earth is, its *Axis* has the same Inclination to the Plane

Plane thereof, and is every where parallel to itself; that is, if a Line be conceived as drawn parallel to the *Axis*, while the Earth is in any one Point of its Orbit, the *Axis* will in every other Situation of the Earth be parallel to the said Line. Thus supposing the Line FG to represent the Situation of the *Axis* of the Earth when at DFG, and to be parallel to the Line HI; then when the Earth is at *dfg*, or any other Part of its Orbit, its *Axis fg* will still be parallel to the same Line HI (*a*).

These things being observed, let us imagine the Plane of the *Ecliptic* to be represented by the Line AB (Fig. 6.) seen edgewise as before. Let the Point S represent the Place of the Sun, and PEpQ the Earth, whose *Axis* let be Pp, and its Poles P and p, the former the North, the latter the South; and let EQ be the Equator, and TC, RN, &c. Parallels thereof. And let us in the first Place consider the Earth in the Situation KL, or directly beyond the Sun with respect to the Eye. It is evident from the Figure, that in this Situation of the Earth, the Sun's Place S is in the Plane of the Equator EQ produced; and consequently, while the

(*a*) N. B. The *Axis* of the Earth deviates a little from its Parallelism every Revolution of the Earth about the Sun, but in so small a Degree that the Consideration of it is better omitted in this Place; it shall be taken Notice of in the next Chapter.

Earth revolves upon its *Axis* Pp, the Sun will appear to describe that Circle, and will enlighten one Side of the Earth from Pole to Pole. Whence half of the Equator, and half of each of its Parallels RN, TC, &c. will be enlightened thereby, while the other half, being on the opposite Side the Earth, will be in the Dark. And consequently every Place on the Surface of the Earth, since it must describe some one of those Parallels as it is carried round by the diurnal Motion, will be as long in the Light as in the Dark; that is, the Days and Nights will then be of equal Length all over the Earth (*b*).

But because the *Axis* of the Earth is every where parallel to itself, as was just now observed; when the Earth is carried by its annual Motion into the Situation MO (which we will suppose to be a quarter of a Circle distant from its former Situation, the Poles of the Earth being still turned the same Way as before) it is manifest that the Sun's Place S is not now in the Plane of the Equator EQ produced, (for that would pass below the Sun towards L) but directly over C, a Point in the Parallel TC; and therefore, as the Earth revolves about its *Axis*, all Places of the Earth

(*b*) When the Sun appears to enter the first Degree of the Sign *Aries*, the Earth is in that Part of its Orbit where these Phænomena happen.

Chap. IV. *Motion of the Sun.* 41

that lie in this Parallel will come directly under the Sun once in 24 Hours, as those that lay under the Equator did in the former Situation of the Earth: And since the Rays of the Sun always enlighten one half of the Globe of the Earth at a Time, they will reach beyond the North Pole as far as F; and on the other Side no farther than I. From whence it follows, that the Tract of the Earth which lies within the Circle FG enjoys continual Day-light, while the Earth is in this Part of its Orbit; and on the contrary, that it is continual Night to that Part of the World that lies within the opposite Circle HI; that is, the Sun will not appear to set in the former, nor rise in the latter. And farther, greater Portions of those Parallels which lie between the Equator and the Circle FG will be in the illuminated Hemisphere than in the dark one, as appears by Inspection of the Figure, in which the latter Hemisphere is distinguished from the other by being shaded; and on the contrary, greater Portions of such as lie between the Equator and the Circle HI will be in the Dark, than in the Light. And hence it is that the Earth being in this Part of the Orbit, the Days are longer than the Nights on the Northern Side the Equator; whereas on the Southern Side, the Nights are longer than the Days; and the Difference between the Lengths of the Days and Nights is so much

the greater, as the Place is more remote from the Equator on either side, because the Difference between the Portions of the Parallels which are in the illuminated Hemisphere, and those which are in the contrary one, are greater the farther they are from the Equator: But to such as live under the Equator, Day and Night are in this Situation of the Earth also equal to one another, one half of the Equator EQ being here also in the illuminated Hemisphere (c).

Let us now conceive the Earth to have moved through another quarter Part of its Orbit, and to be in the Situation KL again, but with this Difference, that it is now on the contrary side the Sun to what we supposed it at first. In this Situation also it is evident, that the Plane of the Equator EQ being produced, will pass through the Sun, and that the illuminated Hemisphere will pass from Pole to Pole as before; for we are still to conceive the *Axis* of the Earth parallel to its first Situation; so that the Sun will seem to describe the same Circle in the Heavens, and the Days and Nights will be equal every where,

(c) In this Situation of the Earth the Sun appears to be in the first Degree of *Cancer*, and to the Inhabitants of the Northern Hemisphere of the Earth introduces the Summer, to those of the other the Winter Quarter.

as

Chap. IV. *Motion of the Sun.* 43

as when the Earth was in the opposite Point of its Orbit (*d*).

But when the Earth has passed through another quarter of its Orbit (which we will suppose it to have done, when it is in the Situation X Y) its *Axis* Pp still remaining parallel to its former Direction, the Place of the Sun will now be vertical to R, a Point under the Parallel RN; so that to all the Inhabitants that live under this Parallel, the Sun will be vertical once in 24 Hours, as it was to those who live under the Parallel TC, when the Earth was at MO; and that Portion of the Earth's Surface which is included within the Circle FG will be wholly in the Dark, and that within the opposite one HI will be wholly in the Light; and lesser Portions of the Northern Parallels will be in the Light than in the Dark, and greater Portions of the Southern. So that the Inhabitants of the Northern Hemisphere will then have their Days at the shortest, and those of the Southern at the longest. Those who live within the Circle FG will have no Day, and those within HI no Night. Tho' to those who live under the Equator, the Days and Nights are in this, as in all other Situations of the Earth, of an equal Length, the Equator

(*d*) In this Situation of the Earth, the Sun enters the first Degree of *Libra*, and makes *Autumn* to us, and *Spring* to those on the other side the Equator.

being

being every where cut into two equal Parts by the Boundary of Light and Darkneſs (*e*).

From

(*e*) When the Earth is in that Part of its Orbit where theſe Phænomena happen, the Sun is then entering the firſt Degree of *Capricorn*, and brings on Winter to us, and Summer to the Inhabitants of the other Hemisphere.

There is another Way in which theſe *Phænomena* may be illuſtrated; and that is as follows. Let S (Fig. 7.) repreſent the Sun, and the Circle ABDD the Earth's annual Courſe round it; EFGH the Earth, P the North Pole, RT the Arctic Circle, IKLE the Tropic of *Cancer*, FNH the Northern half of the Equator; the ſhaded Part the obſcure Hemisphere, the other the illuminated one: And let X be the Place of the Sign *Aries* in the Heavens, Y that of *Cancer*, M that of *Libra*, and Q that of *Capricorn*.

When the Earth is in that Part of its Orbit that reſpects M or *Libra*, the Sun appears in the oppoſite one, *viz.* at X, or in *Aries*; and the Circle GPF, which ſeparates the light Hemisphere from the dark one, paſſes through the Poles of the Earth, and therefore cuts each Parallel of the Equator into two equal Parts, and thereby renders the Days and Nights every where equal.

But while the Earth in its annual Courſe moves from M towards Q, which we will ſuppoſe to be a quarter of a Circle diſtant from it, its *Axis* being directed as before, the Pole P will gradually advance into the illuminated Hemisphere, and that Portion of the Parallels that are therein will continually increaſe until the Earth is arrived there: In which Poſition the Boundary of Light and Darkneſs paſſes through FRH, the Pole P, and all the Space within the Arctic Circle falling wholly within the illuminated Hemisphere; and thoſe Portions of the ſeveral Parallels that are in the Light, will be longer than thoſe which are in the Dark; all which is evident from the Inſpection of the Figure. So that in the Northern Hemisphere of the Earth the Days will be at longeſt, and the Nights at the ſhorteſt.

Again, as the Earth is moving from Q towards X, the Pole P begins to incline towards the Line that diſtinguiſhes Light from Darkneſs, for the like Reaſon that before it receded from it; and therefore thoſe Portions of the Parallels that are in

Chap. IV. *Motion of the Sun.* 45

From this apparent Shifting of the Sun from one Side of the Equator to the other, those Parallels of the Equator, mentioned in the foregoing Explication, have been distinguished from the rest by particular Names, as the two *Tropics*, and the two *Polar Circles*. The *Tropics* are TC and RN: The former is called the *Tropic of Cancer*; because, when the Sun seems to describe that Circle, it then appears to be in the first Degree of that Part of the Ecliptic which is distinguished by the Sign *Cancer*: The latter is called the *Tropic of Capricorn*; because at that time of the Year when it appears to describe that Circle, it is entering the first Degree of the Sign *Capricorn*. The two *Polar Circles* are FG and HI; the former the *Arctic*, the latter the *Antarctic*. These are just as far distant from their respective Poles, as the Tropics are from the Equator; viz. 3 Degrees and an half; this being a necessary Consequence of the Sun's illuminating but one half of the Globe of the Earth at a Time.

in the Light gradually lessen till the Earth arrives there; at which Time that Line will pass through it again, and so make the Days and Nights every where equal.

But afterwards the Pole will begin to fall into the obscure Hemisphere, and so recede gradually from the Light, until the Earth comes to Y; in which Situation not only the Pole, but the whole Arctic Circle will be involved in Darkness; and those Portions of the several Parallels that are in the Light, will be shorter than those which are in the Dark; so that the Days will be at the shortest, and the Nights at the longest.

The

The Days and Nights being of an equal Length, when the Sun appears to be in those Points where the Equator cuts the *Ecliptic*, (*viz.* the first Degree of *Aries* and *Libra*) they are from thence called the *Equinoctial Points*, and the Times of the Year answering thereto the *Equinoxes*; one the *Vernal*, the other the *Autumnal*: And of the *Equinoctial Points*, that where the *Ecliptic* crosses the Equator in its Ascent towards the North Pole, is called the *vernal Equinoctial Point*; that where it crosses it in its Descent towards the South Pole, the *Autumnal*: And a Circle imagined to pass through them and the Poles of the Earth is called the *Equinoctial Colure*.

And because when the Sun seems to describe either of the *Tropic Circles* (that is, when it enters the first Degree of *Cancer* or *Capricorn*) it being about to revert to the Equator, does as it were seem to stand still, neither receding from, nor approaching towards it, these Points are called the *Solstitial Points*; and the Times of the Year answering thereto are termed the two *Solstices*; the one the *Summer*, the other the *Winter*. And a Circle conceived to pass through the Poles of the Earth and these Points is called the *Solstitial Colure*. And lastly, these four Points of the Heavens being those in which the Sun appears to be when the four Seasons of the Year (*viz.* the *Spring*, *Summer*, *Autumn*, and *Winter*) begin,

Chap. IV. *Motion of the Sun.* 47

begin, are distinguished by the Names of *Cardinal Points*.

If a right Line perpendicular to the Plane of the *Ecliptic* be supposed to pass through the Center of the Sun, and to be extended both ways to the Heavens, this Line is called the *Axis of the Ecliptic*; and the two Extremities thereof the *Poles of the Ecliptic*. But this Line is oft conceived to pass through the Center of the Earth instead of the Sun; for they are so near to each other with respect to the Distance of the Heavens, that the Difference is not material.

The Sun being not placed in the Center of those *Ellipses* which the Planets describe in moving round it, but in one of their *Foci*, as was observed above, it follows, that they must be farther from the Sun in one Part of their Orbits than in another. And not only so, but they must also move faster in one Part of their Orbits than in another; because, as has been already demonstrated in Part I. Chap. VIII. page 46. whatever Orbit a Body describes, if it be retained therein by any centripetal Force whatever, it will describe equal *Areas* in equal Times; that is, it will move faster in Proportion as it approaches nearer to the Body it revolves about, and slower the farther it is from it. And accordingly it is observed by Astronomers, that the nearer a Planet

Planet comes to the Sun in its Orbit, the swifter it moves.

The Consequence of this is, that the Sun does not appear to move uniformly along the *Ecliptic*, but in some Days to describe a greater Portion of it than in others. In Winter it moves the fastest, in Summer the slowest; insomuch that it takes up eight Days more in passing from the *Vernal* to the *Autumnal Equinox*, than in its Passage from the *Autumnal* to the *Vernal*; altho' in either of these Intervals of Time it moves through just half of the *Ecliptic* (*f*).

When an *Ellipse* is considered as the Orbit of a Planet, the longer *Axis* thereof is called the *Linea Apsidum*; and if the *Focus* F be the Place of the Sun (*g*), the Point A is called the *Summa Apsis* (*b*), or the *Aphelion*; and the opposite

(*f*) Hence it is, that in one Part of the Year the Sun is farther towards the East, and in another farther towards the West, than it would seem to be, was its apparent Motion in the *Ecliptic* uniform; so that it passes the Meridian sometimes sooner, sometimes later, than the Time at which a Clock that moves uniformly would indicate the Hour of *Twelve*. This is one Reason that Sun-dials are observed to go sometimes slower, and sometimes faster. When the Sun appears too far toward the East, they are too slow; when it appears too much to the Westward, they are too quick. But see this farther explained, Chap. XVI.

(*g*) See Fig. 2.

(*b*) This is the general Name to distinguish that Point of an Orbit, in which a revolving Planet is at the greatest Distance from the Body it revolves about. With respect to a Body revolving about the Sun, it is called the *Aphelion*; but with respect to the Moon considered as revolving about the Earth, it

is

Chap IV. *Motion of the Sun.* 49

opposite one B, the *Ima Apfis*, or the *Perihelion*. The Distance of a Planet from its *Aphe-
lion* is called its *Anomaly*; and this is of
two Kinds, *mean* and *real*: The latter is its
real Distance from its *Aphe-
lion*; the former is
that Distance at which it would be from it,
supposing its Motion in its Orbit to be uni-
form; and the Difference between these two
Astronomers call the *Equation*, or *Prosthaphæ-
resis* of the Planet.

The *Ellipses* the Planets describe do them-
selves turn round the Sun, though with a
very slow Pace, their *Aphe-
lia* respecting some-
times one Part of the Heavens, and some-
times another. In this Age the *Aphe-
lion* of
the Earth's Orbit points towards the former
Part of the Sign *Capricorn*. This is the Rea-
son that the Sun is farther from us in Sum-
mer than in Winter; for the Sun appearing
in *Cancer* in the former Season, the Earth is
then in *Capricorn*, and therefore in its *Aphe-
lion*,
that is, its greatest Distance from the Sun.

The Sun being farther removed from us in
Summer than in Winter, it may be asked how
it comes to pass, that since the Sun is the
Fountain of Heat, as well as Light, our Win-
ters are so much colder than our Summers? In
answer to this it is to be considered, that the

is called the *Apogee*. And the *Ima Apfis* is the nearest Point,
which with respect to a Body moving about the Sun is called the
Perihelion, but with respect to the Moon the *Perigee*.

Rays

Rays of the Sun fall with much less Obliquity upon the Surface of the Earth on our side the Equator in the Summer, than in the Winter; and therefore they not only act more forcibly upon it, but a greater Quantity of them fall upon a given Place. Again; in the Winter the Sun being much lower in the Heavens when at its Meridian Height, than in Summer, its Rays pass through a longer Portion of the Earth's Atmosphere, by which great Part are intercepted, and some by various Refractions and Reflections turned another Way (*i*). And lastly, in Summer the Sun continues with us sixteen Hours, and is absent but eight; whereas in Winter it is with us but eight Hours, and is absent sixteen. All which Things conspire to make a considerable Alteration with respect to Heat and Cold.

If this be so, why is not the Weather hotter when the Sun is in the Tropic of *Cancer*, its Rays then falling with the least Obliquity, and passing the shortest Way through the Atmosphere, and the Days being then at the longest, than it is about a Month afterwards, when the Sun is in the beginning of the next Sign? In answer to this it is to be remembered, that Bodies are not always the hottest at that very Instant the greatest Degree of Heat is applied to them: They require Time

(*i*) See the State of the Atmosphere, Part II. Chap. III.

PART IV. PLATE II. *Pag. 50.*

Fig. 2. P. 34.

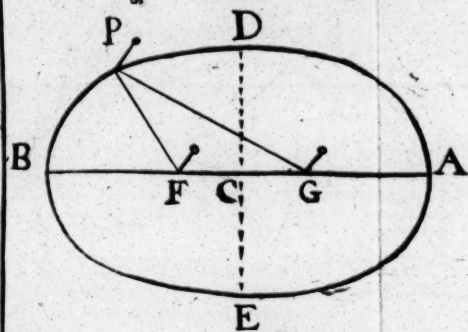


Fig. 5. P. 38.

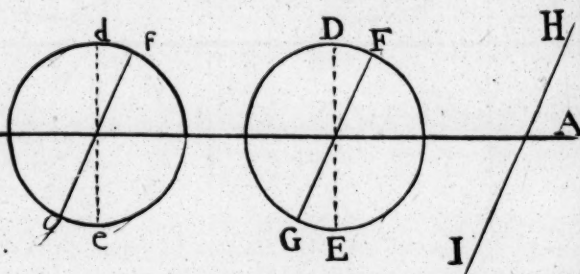


Fig. 3.

P. 34.

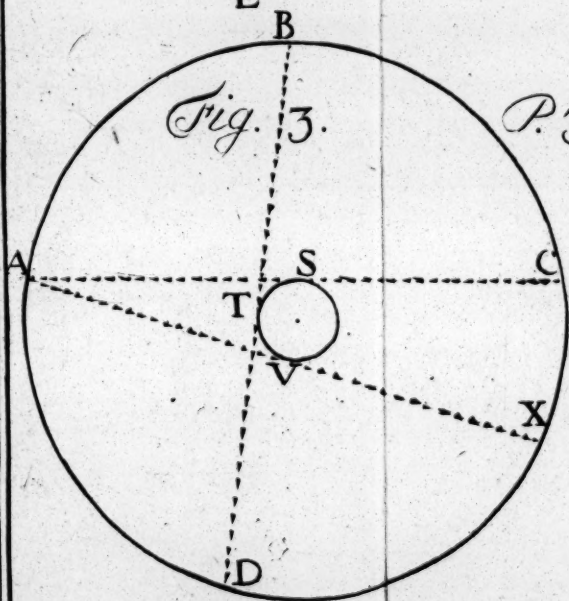


Fig. 4.

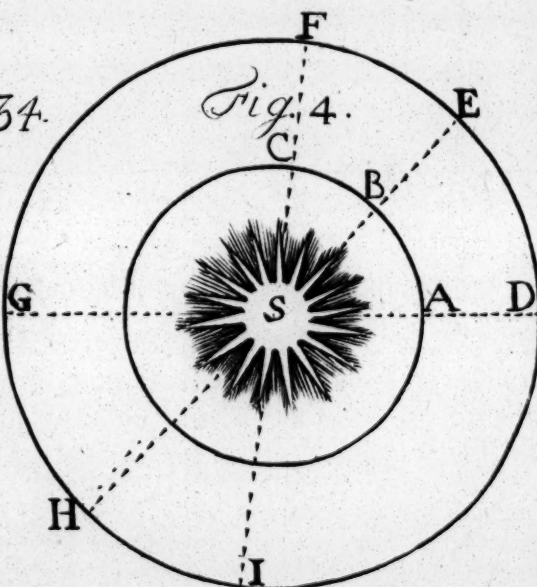
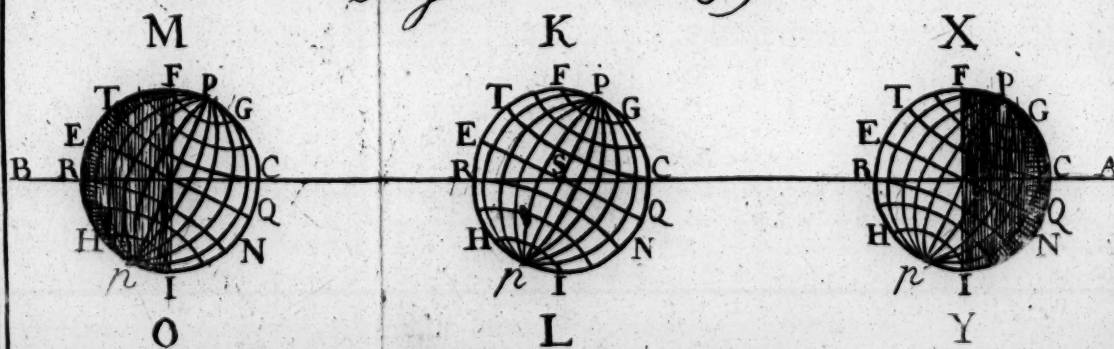


Fig. 6. P. 39.



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Chap. IV. *Motion of the Sun.* 51

to heat as well as to cool ; it is the Length of Time therefore that the Heat is applied to them, as well as the Degree of it, that determines the Quantity of Heat communicated to them.

For the like Reason we commonly find it warmer about one or two of the Clock in the Afternoon, than at twelve, when the Sun is in its Meridian Altitude, and its Rays fall thickest and most forcibly upon the Earth.

The Sun appearing by means of the annual Motion of the Earth (as was illustrated in the beginning of this Chapter) to move from West to East in the Heavens, it happens, that if any Star rises or sets along with the Sun at any time, that Star will after a few Days rise or set before it ; because the Sun's apparent Place in the Heavens will be removed to the Eastward of the Star. So that those Stars which at one time of the Year set with the Sun, and therefore never appear at all, shall at another time of the Year rise when the Sun sets, and shine all Night. And as any one Star shifts its Place with respect to the Sun, and in Consequence of that with respect to the Hour of the Night, so all the rest do ; and from hence it is, that all those Stars which at one time of the Year appear on any one Side of the Pole-Star in the Evening, shall half a Year after, at the same Hour, appear on the contrary Side thereof.

D

C H A P.

C H A P. V.

Of the Earth's annual Parallax, the Nutation of the Poles, and the Precession of the Equinoctial Points.

SINCE the *Axis* of the Earth is always parallel to itself, it is in different Parts of the Year directed towards different Parts of the Heavens; and therefore to a Spectator on the Surface of the Earth, who cannot be sensible of his Change of Place, the apparent Places of the fixed Stars are different, when viewed from opposite Points of the Earth's Orbit. Thus let S (Fig. 8.) represent the Place of the Sun, AB the Orbit of the Earth, and when the Earth is at A, let its *Axis* be directed towards the Point C in the Heavens; then when the Earth is arrived at B, the opposite Point of its Orbit, its *Axis* being parallel to its former Position, will be directed to a different Point in the Heavens, *viz.* to D, as far distant from the former as the Point A is from B, that is, a Space equal to the Diameter of the Earth's Orbit: And therefore to a Spectator on the Surface of the Earth, the Places of the fixed Stars appear removed as far as from C to D, *viz.* by the Quantity of the Angle CBD; or, because the Lines AC and BD are parallel, by the Angle BCA, which is that under which the Orbit of the Earth

Earth would appear from a fixed Star at C. This Angle is by Astronomers called the *Parallax of the Orbis magnus*, or the *Earth's annual Parallax*.

If this Angle could be accurately taken, the Distance of a fixed Star at C, with respect to that of the Sun at S, might be found by the Rules of *Trigonometry* (a). But by reason of the great Distance of the fixed Stars, that Angle is so exceeding small that it cannot be taken with any tolerable Accuracy, even with the best Instruments. It has appeared to some to contain about 47 Seconds of a Minute, according to which the Distance of the Stars must be five hundred thousand Millions of Miles from us; but to others, particularly to *Flamsteed*, who has taken great Pains to observe it accurately, it has appeared to contain but 42 Seconds, according to which the Distance of the Stars must be much greater.

Altho' we hitherto have looked upon the *Axis* of the Earth as being always parallel to itself, yet it is not so, but is subject to a twofold Motion; to the one is owing the Nu-

(a) For the Angle $\Gamma\delta C$ being taken (by Observation, its Equal ACB is had; and the Angle BAC , or SAC , subtends an Arch of the Heavens that measures the Distance between the Place of the Sun and the said Point, and may therefore easily be obtained: And consequently, by the Rules of *Trigonometry*, the Proportion which the Line AC , or the Distance of the Star from the Earth, bears to the Line AB , which is double the Sun's Distance, may be found.

tation of the Poles, to the other the Precession of the Equinoctial Points, which are now to be explained. The first is a Deviation of the Earth's *Axis* from its Parallelism with itself twice in the Year, and a Returning to it as oft. For instance, the Inclination of its *Axis* to the Plane of the *Ecliptic* increases while the Earth is moving from the Solstitial to the Equinoctial, and diminishes as much in its Passage from the Equinoctial to the Solstitial Points. So that the Inclination of the *Axis* of the Earth to the *Ecliptic* is, notwithstanding this, at opposite Points of its Orbit always the same. This Nutation therefore does not in the least interfere with what was said with regard to the Earth's annual Parallax, in which though we supposed that *Axis* always parallel to itself, yet we consider'd it only in opposite Points of its Orbit. This is called the Nutation of the Poles (*b*). The other indeed clashes a little with our former Supposition; but taking a single Revolution of the Earth by itself, as we have hitherto done, it is so small as to be quite insensible, tho' not so when we take many Revolutions together. Accordingly, by comparing several Years together, Astronomers have observed that the *Axis* of the Earth de-

(*b*) As the Inclination of the *Axis* of the Earth is thus twice augmented and twice diminished, while the Earth revolves about the Sun, in like manner it is twice augmented and twice diminished in each Revolution of the Moon about that.

viates

viates a little from the Direction it formerly had ; notwithstanding which Deviation, its Inclination to the *Ecliptic* remains the same. To illustrate the Manner in which this Alteration in its Direction is made, let S (Fig. 9.) represent the Sun, the Circle XEY the Earth, AB the Plane of the *Ecliptic*, EL the *Axis* of it, which we will now suppose to pass thro' the Center of the Earth, and let PQ denote the *Axis* of the Earth. On the Surface of the Earth, and round the Points E and L, let us conceive the two small Circles PTX and QVY to be described. Then if at any Time the Position of the Earth's *Axis* is in the Line PQ, as we have supposed, after several Revolutions of the Earth about the Sun it shall be found in the Situation CD ; and after several more Revolutions it shall be removed into the Situation TV, &c. the one Pole of the Earth describing the Circle PCTX, and the other QDVY : So that notwithstanding this Change in the Direction of the *Axis* of the Earth, its Inclination to the Plane of the *Ecliptic* remains the same, the Poles of the Earth being every where at the same Distance from E and L, the Poles of the *Ecliptic*. But this Motion of the Poles is so very slow, that they do not perform their Revolution in less than 25,920 Years, which is at the Rate of one Degree in 72 Years. It is performed in a Direction contrary to that

of the Order of the Signs, or from East to West. Which is termed by Astronomers a Moving in *Antecedentia*; whereas when a Body moves according to the Order of the Signs, its Motion is said to be in *Consequentia*.

From hence it follows, that the Solstitial Colure, which passes both through the Poles of the *Ecliptic* and of the Earth, must necessarily turn round the *Axis* of the *Ecliptic* in the same Direction, or contrary to the Order of the Signs; and with that the Equinoctial Colure also, because these cross one another at right Angles in the Poles of the Earth. Whatever Star therefore the Equinoctial Points are directed towards at any particular Time, after 72 Years they will not be directed to the same, but to some other Star or Point of the Heavens, situated one Degree towards the West. This Motion of the Equinoctial Points is called their *Precession*. And from hence the Stars seem to move towards the East, and thereby to have their Longitude (which is always reckoned upon the *Ecliptic* from the vernal Equinoctial Point) increased. The Constellations therefore seem all to have deserted the Places allotted to them by the first Astronomers. For Instance, the beginning of the Sign *Aries*, which was near the vernal Equinoctial Point, and gave Name to that Point of the *Ecliptic*, is now advanced about an whole Degree forwards; So that *Aries* is now where
Taurus

Chap. V. *the Equinoctial Points.* 57

Taurus used to be; *Taurus* where *Gemini*, &c. But to avoid Confusion, Astronomers have thought fit to let the several Portions of the *Ecliptic*, where these Constellations were at first observed to be, retain their old Names (c); so that the vernal Equinoctial Point is still reckoned the first *Degree of Aries* (d).

(c) Those Portions of the *Ecliptic*, where the Constellations were at first, are called *Anafra*; those where they now are, are distinguished by the Title of *Stellata*.

(d) The ancient Astronomers observing this Motion of the fixed Stars, and not knowing how to account for it, concluded that they really had such a Motion, and called the Time of an entire Revolution of them, *Annus Magnus*, or the great Year; and imagined that when that great Year was ended, and the Stars reinstated in their proper Places, all Things would begin to be done over again in the World, not only in the same Order and Manner, but by the same Persons they were done before.

The Causes of this Precession, as also of the Nutations above mentioned, will be accounted for when we treat of the Causes of the Motions of the heavenly Bodies.

C H A P. VI.

Of the Phænomena which arise from the Motion of the Earth, and of the inferior Planets, Mercury and Venus conjointly.

SInce the Orbits of *Mercury* and *Venus* are included within that of the Earth, it is evident neither of them can come to an Opposition to the Sun; that is, they cannot appear in the opposite Part of the Heavens with respect to it, but must always accompany it in its apparent Motion through the Ecliptic, just as the Satellites of the primary Planets do in reality accompany them. But whereas they perform their respective Revolutions about the Sun in Periods different from that of the Earth, it is obvious they must be sometimes on one Side of the Sun, and sometimes on the other, with respect to us; sometimes be between the Sun and us, and sometimes directly beyond it. When they are between the Sun and us, they are said to be in their *inferior Conjunction*; when beyond it, in their *superior Conjunction*; and when they appear to be removed the farthest from the Sun, on either Side, they are then said to be in their greatest *Elongation*.

To

Chap. VI. *from the Earth's Motion.* 59

To illustrate this, let S (Fig. 10.) represent the Sun, ABCD the Orbit of one of the inferior Planets, suppose *Venus*; E the Place of the Earth in its Orbit; FG and HI a Portion of the Heavens. Then will A be the Place of its *inferior*, C that of its *superior Conjunction*; in both which Cases it will appear in the Heavens to be at K, which is the apparent Place of the Sun. When it is at B or D, where a Line joining its Center and that of the Earth touches its Orbit, it then appears at I or H, and is said to be in its greatest Elongation, which is about 48 Degrees (a). But this is liable to some Alteration; because, as its Orbit is an *Ellipse*, it is sometimes nearer the Sun, and sometimes farther from it, in its greatest Elongation.

From hence we see the Reason why the Planet *Venus* is sometimes said to be our Morning Star, and sometimes our Evening Star: For after it has passed its *inferior Conjunction* with the Sun, and is ascending towards its *superior* one, it is all that Time on the *Western* Side of the Sun, and therefore necessarily rises

(a) Hence the Distance of *Venus* from the Sun, with respect to that of the Earth from the same, is found. For if in the Triangle EBS, we take the Angle at E, when that at B is a right one, the Proportion between the opposite Sides is had from the known Rules of *Trigonometry*. The Reason why the Angle at B is not always a right one, is, because the Orbit of the Planet is not a Circle, but an *Ellipse*.

every

every Morning before it, and is then our *Morning Star*. On the other hand, while it descends from its *superior* to its *inferior Conjunction*, it is to the *Eastward* of the Sun, and therefore rises and sets after it; and appears to us only in the Evening, after the Sun is down.

Farther, altho' the Motion of the inferior Planets in their Orbits is direct, or according to the Order of the Signs, yet they frequently appear to be *retrograde*, or to move the contrary Way; and sometimes to stand still. When they are near their *inferior Conjunction* with the Sun, they seem to move one Way; and when near their *superior Conjunction*, they appear to go another. And at that Time when their direct or progressive Motion is changing into a regressive one, or the contrary, they seem to stand still. All which may be illustrated in the following Manner.

Let ABCD (Fig. 10.) represent the Orbit of the Planet *Venus*, S the Place of the Sun, E the Earth, FG a Portion of its Orbit, and HI the Sphere of the fixed Stars, as before; and because *Venus* moves faster than the Earth, let us suppose the latter to stand still, and the former to move with the Difference of their Velocities: Then while *Venus* describes that Portion of its Orbit that is represented by DAB, it will seem from the Earth to move from H to I in the Heavens, which is contrary to its real Motion, and to the Order of the Signs; and

Chap. VI. *from the Earth's Motion.* 61

and in moving over the remaining Part of its Orbit, it will seem to pass from I to H. And was the Earth really to stand still, as we have supposed, it would be stationary, or seem to have no Motion either Way, at the Points B and D.

But as the Earth does not stand still, but continues to move on in its Orbit, *Venus* will not seem stationary exactly at the Points B and D; but during its Motion through a certain Part of its Orbit, as LM, where supposing the Lines EL and OM parallel to each other, their Orbits are so inclined to each other, that the Space LM shall be to the Space EO as the Velocity with which *Venus* moves is to that with which the Earth moves; for it is evident, that in that Case the Earth will pass through EO, while *Venus* passes through LM; and therefore it will appear all the Time in the same Part of the Heavens, because the Distance between those parallel Lines is nothing, when compared to that of the Heavens.

There is also another Particular to be taken Notice of with regard to the apparent Motion of the Inferior Planets, *viz.* that they do not seem to describe the *Ecliptic* in the Heavens as the Sun does, but are observed to be sometimes above and sometimes below it. The Reason of this is, that their Orbits are inclined to the Plane of the Earth's, having one half above it and the other below it, on which
account

account they intersect the Plane of the *Ecliptic* in a Line that passes through the Center of the Sun (the Center of the Sun, as was observed above, being in the Plane of each Orbit.) This Line is called the *Line of the Nodes*; and the two Points of the *Ecliptic*, through which it passes when produced to the Heavens, are called the *Nodes*. These Planets therefore never appear in the *Ecliptic* except when they are in their *Nodes*, and in all other Parts of their Orbits seem to be more or less distant from it, according as they are situated with respect to them and the Earth. But if we suppose them to be seen from the Sun, their greatest Distance from the *Ecliptic* will be when they are farthest removed from their *Nodes*.

Their Distances from the *Ecliptic*, as it would appear if seen from the Center of the Earth, is called their *Geocentric Latitude*; as it would appear from the Center of the Sun, is called their *Heliocentric Latitude*.

There is a remarkable *Phænomenon* relating to the inferior Planets, which was not known to Astronomers, till discovered by the Telescope; and that is, that in their different Situations with respect to the Earth, they assume different *Phases*, or Appearances, like those of the Moon. Which may be explained in the following Manner.

Let

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Let A, B, C, D, (Fig. 11.) represent the Situation of one of the inferior Planets (*Venus* suppose) in several Parts of its Orbit, and let S be the Place of the Sun, and I that of the Earth in its Orbit KL: And if from the Center of the Sun to that of *Venus* in its several Stations we draw the Lines SA, SB, &c. respectively; and at right Angles to these the Lines MN, MN, &c. these last Lines will represent Circles upon the Surface of the Planet, separating the enlightened from the obscure *Hemisphere*. And if from the Earth to the Planet we conceive the Lines IA, IB, &c. to be drawn, and at right Angles to these the Lines OP, OP, &c. respectively, these latter will represent Circles separating that Part of the Planet which is turned towards us from that which is not. These we may call *Circles of Vision*. Now it is evident, that when *Venus* is in its inferior Conjunction at A, the Circles MN and OP coincide, and no Part of the Planet besides its unilluminated Hemisphere is turned towards us; in which Case it is invisible, unless it happens at that Time to be in or near the Plane of the *Ecliptic*, which if it be, it appears as a Spot upon the Surface of the Sun. When it has advanced in its Orbit as far as B, there is a small Part of the illuminated Hemisphere included between the Lines OB and BM, on that side the Circle of Vision which is next the

64 *Phænomena arising* Part IV.

the Earth; in which Case, since by reason of its great Distance from us we cannot distinguish the *Convexity* of its Surface, it appears as a flat round Surface, a small Portion of which is illuminated, the rest being dark; and so appears *horned*, like the new Moon (*b*). When it has arrived at C, then is half its illuminated Hemisphere turned towards us, and it appears as a round Surface, half of which is enlightened, and half not. When it has got to D, then is the greatest Part of its illuminated Hemisphere turned towards us, and it appears as a round plain Surface almost wholly enlightened, or what the Astronomers call *gibbous*. When it is in its superior Conjunction at E, the Circles MN and OP coin-

(*b*) The Reason why *Venus* or any other Planet (the Moon suppose) appears *horned*, when a small Portion only of its illuminated Hemisphere is turned towards the Eye of the Spectator, will perhaps in some Measure appear from the Consideration of the 12th Figure. In which let ABCD represent the Planet *Venus*, or any other Body of a globular Form; which that it might the better do, I have drawn some of those Circles which are usually found on Globes designed to represent the Earth. Now if half this Globe be illuminated, and half be in the dark, and if only so much of the illuminated Part as is included within the Space ABCE be turned towards the Eye, the remaining Part AECD being obscure, it is plain, that the illuminated Part ABCE, when the whole Surface ABCD, by reason of the great Distance of the Globe, appears *flat*, must seem to be *horned*. And again, if the Space ABCE was a Portion of the dark Hemisphere, and AECD of the illuminated, 'tis certain that when both are considered as making up one flat Surface, the illuminated Part must appear convex, or *gibbous*, on the Side AEC, which is contiguous to the unilluminated Hemisphere ABCE.

Chap. VI. *from Mercury's Motion.* 65

cide again, and the whole enlightened Hemisphere is turned towards the Earth ; in which Situation the Planet, if not intercepted by the Body of the Sun, appears like the Moon at the *Full*. After which it puts on the same Appearances again, but in an inverted Order, till it arrives at A, where it is *invisible*, as before (c).

What has been here illustrated with regard to *Venus*, is to be understood in Reference to the like *Phænomena* of *Mercury* ; only with this Difference, that the Orbit of *Mercury* being less than that of *Venus*, its greatest *Elongation* is not so great as that of *Venus*, not exceeding about 33 Degrees. Upon which Account this Planet is rarely to be seen with the naked Eye. And as *Mercury* revolves round the Sun in less Time than *Venus* does, its *direct Motions*, *Stations* and *Retrogradations*, as also its Conjunctions with the Sun, succeed one another quicker than those of that other Planet.

(c) Before the Invention of the Telescope, when *Copernicus* revived the old *Pythagorean Hypothesis*, it was objected to him, that, according to that Scheme, *Venus* ought to assume the like Phases with the Moon. To which he replied, that possibly some Means would be found out, whereby succeeding Astronomers would discover that it did so. The first that observed it was *Galilæo*, who thereby greatly confirmed the Truth of the *Copernican System*.

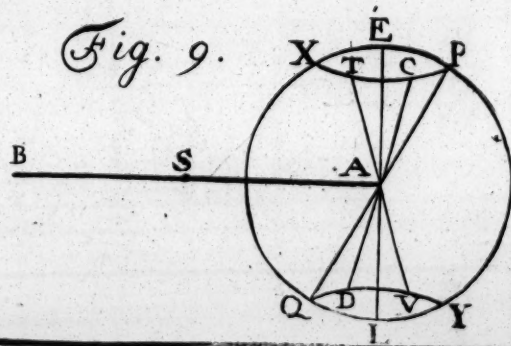
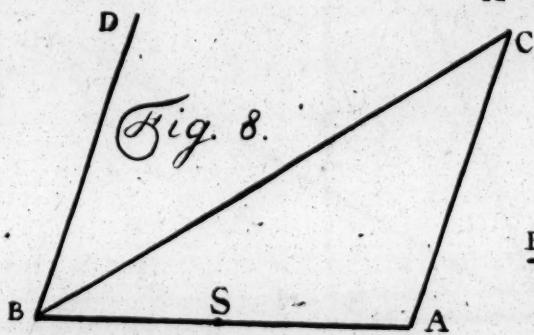
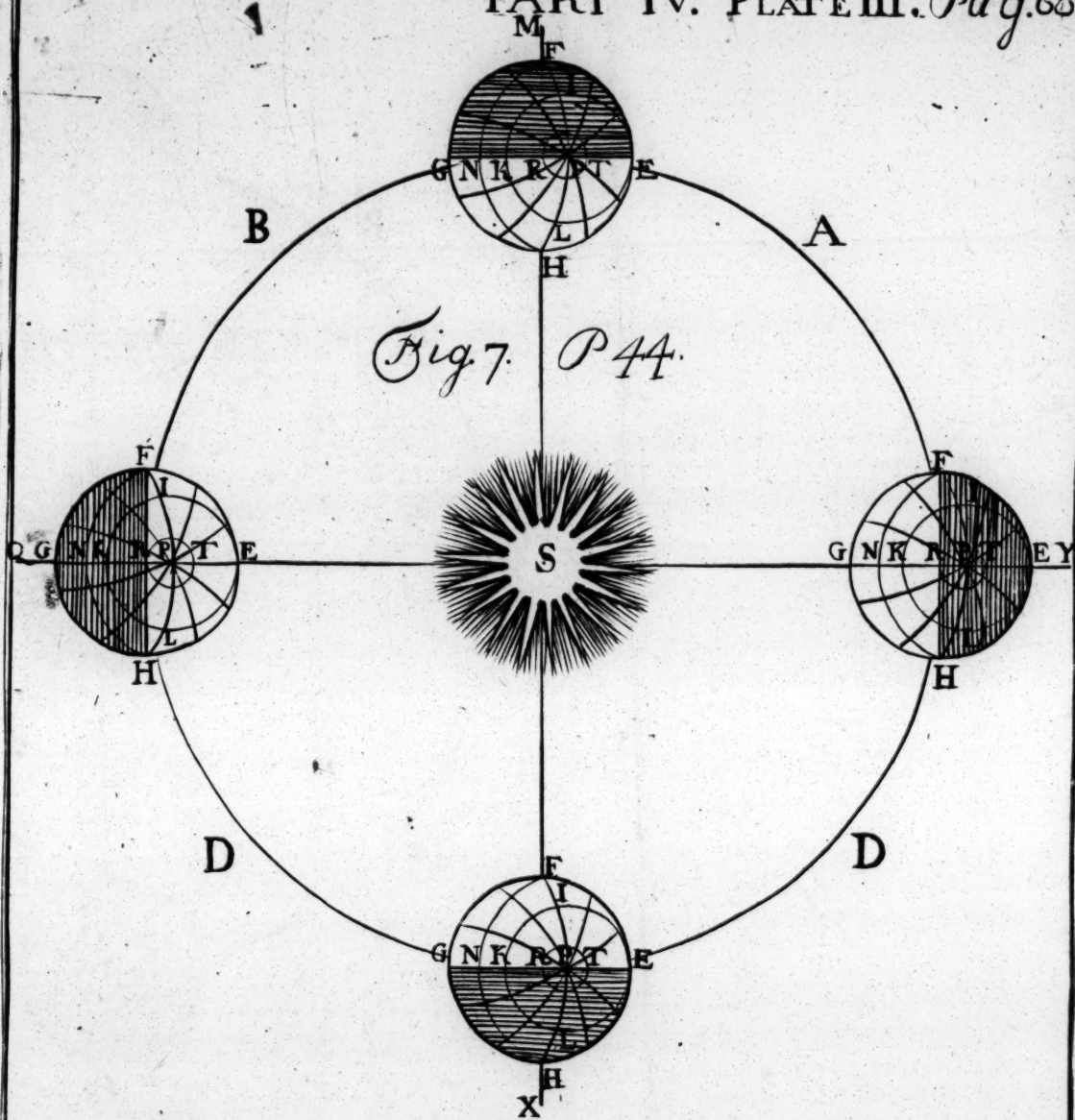
C H A P.

C H A P. VII.

Of the Phænomena which are owing to the Motion of the Earth, and that of the superior Planets Mars, Jupiter, and Saturn conjointly.

THE superior Planets do not always seem to accompany the Sun, as we have shewn in the foregoing Chapter that the inferior ones do; but are sometimes in Conjunction with it, sometimes in Opposition; and are liable, as was observed before of the Planet *Mars*, to be seen at any Distance from it.

To illustrate this, let S (Fig. 13.) represent the Place of the Sun, ACE the Orbit of the Earth, NRM that of one of the superior Planets (suppose *Mars*) and let OP be a Portion of the *Zodiac*. Now since these Planets perform their Revolutions about the Sun in different Times from that wherein the Earth goes round its Orbit, it is possible, that when the Earth is at E, *Mars* may be at M, or it may be at N; in the former Situation, it is in Conjunction with the Sun, in the latter it is in Opposition: Or it may be at R, or at T, or
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Chap. VII. *the Motion of Mars.* 67

in any Situation with respect to the Sun and the Earth whatever.

The superior Planets have this in common with the inferior ones, *viz.* that according as the Earth is situated with respect to them, they sometimes seem to move forwards according to the Order of the Signs, sometimes contrary to that Order; and between the Times of their being thus *direct* and *retrograde*, they seem to stand still, as shall be made evident by the following Illustration.

Things remaining as before in the thirteenth Figure, since the Motion of the Earth is quicker than that of *Mars*, let it be supposed that *Mars* stands still at N, and that the Earth is moving with the Difference of their Motions from B to C: It is obvious, that in this Case *Mars* will seem to pass from O to P, which is contrary to the Order of the Signs; but while the Earth is passing over the other Part of its Orbit, *viz.* CAB, the Planet *Mars* would seem to move back again from P to O, even though it had no Motion of its own. Since then it is moving that Way all the Time, it necessarily seems to move according to the Order of the Signs; tho' with this Difference, that it seems to move faster than it would do were it not for the Motion of the Earth in the mean Time. Analogous to what was observed of the inferior Planets (since *Mars* does not stand still, as we

68 Phænomena owing to Part IV.

have supposed) it will not appear to be *Stationary* when the Earth is at B and C, but during the Motion of the Earth through a certain Part of its Orbit, as LH, which, supposing the Lines LN and HI parallel, is to the Portion of *Mars's* Orbit NI (through which that Planet is moving at the same Time) as the Celerity with which the Earth moves is to that of *Mars*.

What was said in respect of the inferior Planets, that they do not always appear in the *Ecliptic*, holds also with regard to the superior ones. For their Orbits are also inclined to that of the Earth, one Side of them being above, and the other below it; so that those have also their *Heliocentric* and *Geocentric* Latitudes as well as the other, and are never to be seen in the *Ecliptic*, but when they are in their *Nodes*.

What has been illustrated with regard to *Mars*, is applicable to the other two superior Planets, only with this Difference; that the Motion of *Saturn* is more frequently changed from progressive to regressive, than that of *Jupiter*; as also its Oppositions and Conjunctions with the Sun are more frequent. The Reason of this is, because as *Saturn* moves much slower than *Jupiter*, the former is more frequently overtaken and pass'd-by by the Earth than the latter. And as these *Phænomena* happen oftner in *Saturn* than in *Jupiter*, so they

Chap. VII. Sat. and Jupiter's Motion. 69

do more frequently in *Jupiter* than in *Mars* for the same Reason.

So small is the Distance of the Earth, with respect to that of *Jupiter* and *Saturn*, from the Sun, that in whatever Part of their Orbits they be situated with respect to the Earth, that Side of them, as to Sense, which is turned towards the Sun, is always towards the Earth; on which Account they always appear to us as shining with a full Orb. But it is not so with respect to *Mars*; for since that Planet is not much farther removed from the Sun than the Earth, it is obvious that that Side of it which is turned towards the Sun will not always be towards the Earth. For Instance; though when *Mars* is at N, and the Earth at A or E, its illuminated Hemisphere is directly towards the Earth, yet it is turned away from it considerably when the Earth is at B or C; at which time therefore it appears to want something of its usual Rotundity, or to be *Gibbous* like the Moon, a little before or soon after the *Full*.

As the inferior Planets *Mercury* and *Venus* have their respective Elongations from the Sun when seen from the Earth; so the Earth, if we imagine it to be viewed from the superior Planets, hath also its Elongation from the Sun. But this being a Phænomenon, which has not happened to our Astronomers to see, they give it a different Name, and call it when

at the greatest, from a Phænomenon in the superior Planets depending upon it, *the Parallax of the Semidiameter of the Earth's annual Orbit in those Planets*: By which is meant, the Difference between the Place in the Heavens the Planet would appear in if seen from the Center of the Sun, and that in which it would appear if seen from the Center of the Earth, when at its greatest Elongation from the Sun with regard to that Planet. To explain this, let N be the Place of the Planet, and B that of the Earth in its greatest Elongation from the Sun, as seen from that Planet. Then will the apparent Place of the Planet as seen from the Sun (that is, its true or *Helio-centric* Place) be Q, but its *Geocentric* or apparent Place as seen from the Earth, will be O. Now because the Arch QO subtends an Angle (*viz.* QNO) at the Planet which is equal to SNB, *viz.* that which SB, the Semidiameter of the Earth's annual Orbit, subtends, the above-mentioned Difference in the apparent Situation of the Planet is called *the Parallax of the Semidiameter of the Earth's annual Orbit in that Planet (a)*.

This

(a) Hence, as was observed of the inferior Planets, the Distances of the superior ones from the Sun, with respect to that of the Earth from thence, may be found. For in the Triangle SBN, all the Angles being known, the Relation between the two Sides BS and SN is had by the known Rules of *Trigonometry*. The Angle SBN is had from immediate Observation,

as

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This Angle in *Mars* is but 42 Degrees, in *Jupiter* not above 11: In *Saturn* it is but the fourth Part of the greatest Elongation of *Mercury* seen from the Earth, viz. about 6 Degrees. So that since *Mercury* by reason of its Nearness to the Sun is rarely seen by us, in all Probability the Astronomers in *Saturn* (if there be any) do not know that there is such a Body in the Universe as our Earth.

as being subtended by an Arch in the Heavens between the apparent Place of the Planet and that of the Sun. The Angle at N is equal to the Difference between the real and apparent Place in the Planet; the first of which is found by Computation, the latter by Calculation. In *Jupiter*, the best Way of finding that Angle is from his *Satellites*, the Method of doing which shall be shewn when we have considered the Phenomena of those Planets.

C H A P. VIII.

Of the Phænomena of the Moon.

HAVING now finished what relates to the *Phænomena* of the *Primary* Planets, we proceed to consider those which are owing to the Motion of the *Secondary* ones; and first of the Moon, which is the constant and only Attendant of the Earth, and is always so near it, that if the Earth and the Moon were viewed together from the Sun, the

Moon at such Time as it would appear the farthest from the Earth would not seem to be at a greater Distance from it than what is equal to one third Part of the Breadth of the Sun's Body, as it appears to us.

Since the Moon is an opaque spherical Body, and revolves about us in such Manner as to pass between the Sun and the Earth, it is absolutely necessary it should put on different *Appearances*, according to its various Position with respect to the Sun and the Earth; its illuminated Hemisphere being constantly turned towards the Sun the Fountain of Light, and for that Reason sometimes a greater and sometimes a less Portion of it turned towards us. To shew this, little need be said, after so full an Explication of the several Phases of *Venus*, as was delivered in Chap. VI.

When the Moon is between the Earth and the Sun, that is, in Conjunction with the Sun, its whole illuminated Hemisphere is turned from us, in which Station it is said to be *New*; afterwards when by its Motion in its Orbit it emerges out of the Sun's Rays, a small Portion of that Hemisphere which is turned towards the Sun is also turned towards the Earth, and then (as was observed of *Venus*) the Moon, by reason of its great Distance from us, appearing as a flat Surface, one Edge
of

Chap. VIII. of the Moon.

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of that Surface appears brighter than the rest of it, and so the Moon in this Situation appears *horned*; and because it moves from East to West, as was observed above, it at this Time appears to the Eastward of the Sun; upon which account it seems to follow the Sun in its daily Course, rising and setting after it. When the Moon is got to a *quartile Aspect*, as Astronomers call it, (that is, to the Distance of a quarter of a Circle in the Heavens from the Place of the Sun) then is half of its illuminated Hemisphere turned towards us, and half from us, in which Case it appears to us a plain round Surface *half illuminated*, and half not. When it has advanced a little farther in its Orbit, and got farther from the Sun, more of its illuminated Hemisphere becomes visible to the Inhabitants of the Earth, whence it appears like a plain round Surface, somewhat defective of Light on that Side which is from the Sun, and is said to be *Gibbous*. When it comes to an Opposition with respect to the Sun, it then turns the same Side directly towards the Earth that is towards the Sun, at which time it appears to us wholly enlightened, and is therefore called the *full Moon*. And as it performs the other Part of its Revolution, it assumes first a *gibbous*, then an *halved*, afterwards an *horned* Face; and when it arrives at its Conjunction with the Sun, it

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disappears for a while, the obscure Part only being turned towards us, as before.

When the Moon is horned, the darker Hemisphere, or that which is turned from the Sun, has a faint Light diffused over it, sufficient to render it visible. This Light is owing to a Reflection of the Sun's Rays from the Surface of the Earth to that Part of the Moon. For as the Moon when at the *Full*, by means of its enlightened Hemisphere which is then turned towards the dark Side of the Earth, shines upon it, and affords a certain Degree of Light; so the Earth, when the Moon is at the *New*, has its enlightened Hemisphere (or that Side where it is *Day*) turned towards the obscure Hemisphere of the Moon, and enlightens that, but in a much greater Degree than the unilluminated Hemisphere of the Earth (or that Side where it is *Night*) is enlightened by the *full* Moon; the Earth being considerably larger than the Moon. So that as that Body is a Moon to us, our Earth is a much larger and more luminous one to that.

The Time in which the Moon performs one entire Revolution about the Earth from any Point of the Zodiac to the same again, is called a *Periodical* Month. This Month consists of 27 Days, 7 Hours, and 43 Minutes. But the *Synodical* Month, which is the Time the Moon takes up in passing from one Con-
junction

Fig. II.

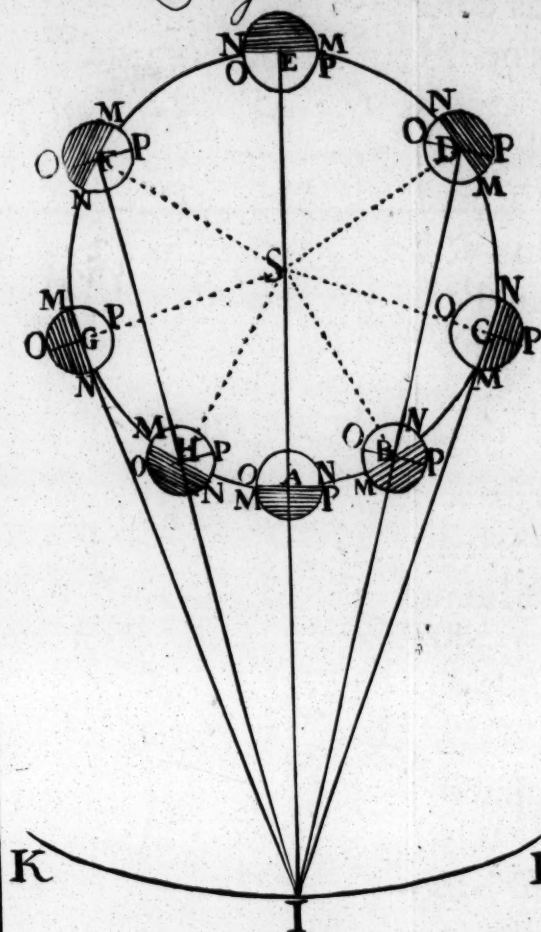


Fig. 10.

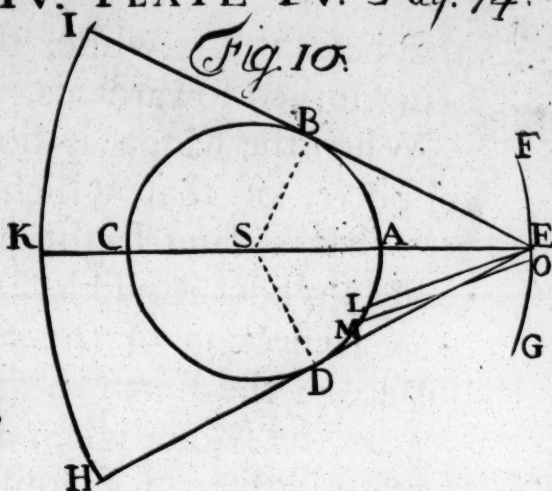
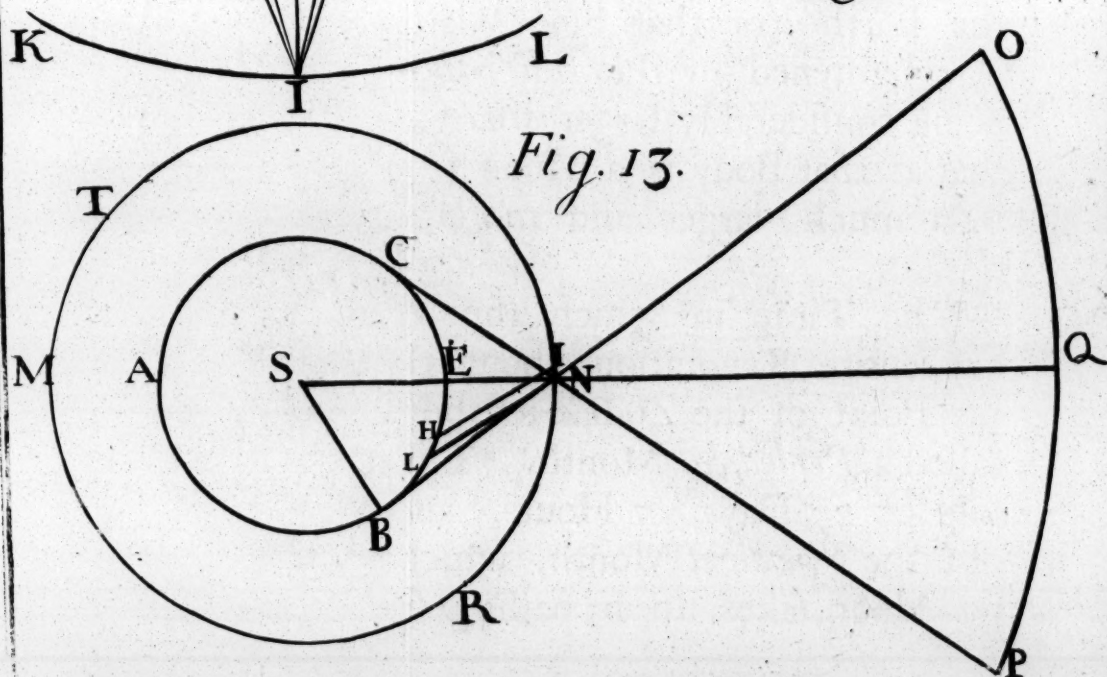


Fig. 12. P. 64.



Fig. 13.



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junction with the Sun to another, is larger than this by about two Days and 5 Hours. The Reason of this is, that while the Moon is passing from one Conjunction to another, the Sun by means of the Earth's Motion in its Orbit, is in appearance advanced towards the East; so that when the Moon comes round to that Part of the Heavens where the Sun was at the foregoing Conjunction, it is not then in Conjunction with it again, but has such a Portion of the Heavens to describe, as takes up so much more time, as, being added to the *Periodical* Month, makes it up 29 Days, 12 Hours, 44 Minutes, and 3 Seconds; which is the *Synodical*. But perhaps the 14th Figure may make this clearer: In which let S represent the Sun, E the Earth moving in its Orbit AB from A towards B, and let MNO denote the Orbit of the Moon, and M its Place therein when in Conjunction with the Sun. Now while the Moon describes its Orbit MNO, let it be supposed that the Earth advances in its Orbit from E to *e*, in which Case the Moon's Orbit will be found at *mno*, and the Point M will be at *m* in the Line *ed*, if we suppose it parallel to ES. And therefore when the Moon comes to *m*, it will have performed an entire Revolution about its Orbit; but will not be in Conjunction with the Sun, but has still the Arch *mp* to describe
before

before it reaches that Point of the Heavens in which the Sun will then appear to be.

The Plane of the Moon's Orbit is not coincident with that of the Earth, but makes with it an Angle of about 5 Degrees, cutting it in two Points opposite to each other, after the same Manner as the Equator and the Ecliptic were observed to cut each other; so that the Moon does not seem to describe the Ecliptic in moving round its Orbit, but one half of the Way it declines towards the *North*, the other half towards the *South*. The two Points where the Moon's Orbit passes through the Plane of the Ecliptic, are called the *Nodes*; and a right Line drawn from one of these Points to the other, will pass through the Center of the Earth, (that being in the Plane of the Moon's Orbit as well as in that of its own) and is called the *Line of the Nodes*. That Node through which the Moon passes from the *South Side* the Ecliptic to the Northern is called the *Ascending Node*; the other thro' which it passes from the *North* to the *South Side* is called the *Descending Node* (a).

The Extremities of the Line of *Nodes* are not always directed towards the same Points of the Ecliptic, but continually shift their Places from East to West, or contrary to the

(a) Astronomers often express the former of these Nodes by this Mark Ω , which they call the *Dragon's Head*; the latter by φ , which they call the *Dragon's Tail*.

Order of the Signs; performing an entire Revolution about the Earth in the Space of something less than 19 Years.

As was observed of the primary Planets, the Moon also moves not in a Circle, but in an *Ellipse*, one of whose *Foci* is in the Center of the Earth. And the *Linea Apsidum* of it is not carried along with the Earth in a Direction parallel to itself, but continually turns round the Center of the Earth thro' which it passes from West to East, or according to the Order of the Signs, in the Space of about 9 Years.

The Moon being subject to the Influence of the Sun as well as the Earth, it is not only liable to various Irregularities in its Motion, but its Orbit is wont to put on different Forms and Motions, all which depend on its Situation with respect to those two Bodies. They are these that follow.

I. The nearer the Moon is to its *Syzygies*, (that is, its Conjunction or Opposition with the Sun) the greater is its Velocity; and the nearer it is to the *Quadratures* (that is, the first and last Quarter) the slower it moves (*b*).

II. It does not describe *Areas* proportionable to the Times, except in the *Quadratures* and *Conjunctions*.

(*b*) This Irregularity was observed by *Tycho Brahe*, who gave it the Name of the Moon's *Variation*.

III. Its Orbit is somewhat more curved in the Quadratures, than it would be if the Moon were only influenced by the Earth; and less curved in the Syzygies, the Moon running off farther from the Earth in the Quadratures than in the Syzygies.

IV. When the Earth is in its Perihelion, (that is, in that Part of its Orbit which is nearest the Sun) the Periodical Time of the Moon is greater than when it is in its Aphelion, or farthest from the Sun: So that the Lunar Months of one Part of the Year exceed those of another.

V. The *Linea Apsidum* of the Moon's Orbit goes forwards when the Moon is in the Syzygies, and backwards when in the Quadratures; but it goes farther forwards than backwards every Time, so that at length it revolves quite round, as observed above.

VI. The Excentricity of the Moon's Orbit is greater and less in every Revolution of the Moon; it is greatest when the Moon is in the Syzygies, and least when it is in the Quadratures: And consequently while it passes from the Quadratures to the Syzygies, it is increasing; and while it passes from the Syzygies to the Quadratures it is diminishing. And if we compare several Revolutions of the Moon together, its Orbit is the most Excentrical when the Situation of it is such, that the *Linea Apsidum* is in the Syzygies, and least

when

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when in the Quadratures. So that when both the Moon and the *Linea Apsidum* of its Orbit are in the Syzygies, its Excentricity shall be half as great again, as when they both happen to be in the Quadratures.

VII. The Line of the Nodes does not move uniformly backwards, that is, contrary to the Order of the Signs; but when it is in the Syzygies it stands still, and moves the fastest of all when in the Quadratures.

VIII. The Inclination of the Lunar Orbit to the Ecliptic (*c*) is also liable to Change; it is greatest when the Nodes are in the Syzygies, and least when they are in the Quadratures.

All these Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion; and also greater when the Moon is in Conjunction with the Sun than in Opposition.

There is another Particular in the Moon's Motion, which is by some reckoned among its Irregularities; which is, that it moves faster in its Perigeon, than in its Apogee; but this is not owing to any Perturbation in its Motion arising from the Influence of the Sun;

(*c*) Some Astronomers measure the Inclination of the Moon's Orbit to the Ecliptic, not from the Ecliptic, but from a Perpendicular to that, and so call its Inclination 85 Degrees instead of 5. And so of the Inclination of the Equator to the Ecliptic; reckoning that 66 Degrees and an half, and not 23 and an half, as is the most usual Way.

but

but is common to all the Planets, and is occasion'd only by its describing equal Areas in equal Times in an elliptical Orbit, as the other Planets do.

All these Irregularities observable in the Motion of the Moon naturally flow from the Supposition, that the Earth and the Moon are retained in their Orbits by Gravity, in the Manner laid down and explained in the Introduction to this Part, as will appear when we have considered the Physical Causes of the Motion of the heavenly Bodies.

The only uniform Motion the Moon has, is its Rotation about its *Axis*, which it performs the same Way, and in the same Time, that it does its Revolution about the Earth. The Consequence of this would be, that if its Revolution in its Orbit were performed with an uniform Celerity, and that Orbit were a Circle having the Earth in its Center, and also if the *Axis* of the Moon were perpendicular to the Plane of that Orbit, the Moon would in all Positions turn exactly the same Side towards the Earth. This is so obvious as to need no Illustration. But since its Motion in its Orbit is not uniform, nor its Orbit a Circle, neither its *Axis* perpendicular to the Plane thereof, the Moon does not keep the same Face exactly towards us, but turns it a little, first one Way, then another; appearing thereby subject to a two-fold

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fold *Libration*, viz. from East to West, and from North to South: To the one of which the two first Causes abovementioned concur; the other is owing to the third. And first to illustrate that *Libration* which is from East to West. Let the Elliptical Orbit BBB (Fig. 15.) represent that of the Moon, let E be the Place of the Earth, and let the Circle ABCD represent the Moon in its Apogee at A; and for the present, let us suppose the *Axis* of the Moon perpendicular to its Orbit. Then because its Motion about its *Axis* is uniform, and its Motion from its Apogee the slowest of all; it is evident, that at the Time when it has revolved a quarter of the Way round its *Axis*, and obtained, suppose the Situation FBCD, it will not have advanced a quarter Part of the Way through its Orbit; and therefore the Face BCD, which was turned towards the Earth at first, cannot now be turned to I the Center of the Ellipse, much less to E the Place of the Earth, but to some other Place, as K, on the other Side the Center: But when it has advanced to G, the same Face BCD will be turned towards E, that was when the Moon was at A; because as it has advanced half through its Orbit, it has advanced half round its own *Axis*. Again, as it moves quicker in that Part of its Orbit, that's near its Apogee, it will have proceeded above a quarter Part of the Way through

through its Orbit, and so will have got into the Situation HBCD suppose, when it has turned a quarter round its *Axis*; and that Part of the Moon which was visible when the Moon was at A, will be turned towards K, as it was when the Moon was at F. And it is observable from the Figure, that the Face BCD, which was turned directly towards the Earth at E, when the Moon was either at A or G, is, when the Moon is at F, turned to the *Right*, or towards the West, with respect to a Spectator viewing it from E; but to the *Left*, or towards the East, when the Moon is at H. And this is the first kind of *Libration* which was to be explained. As to that which is from North to South, it may be thus illustrated. Let HI (Fig. 16.) represent the Plane of the Moon's Orbit, being supposed to be seen edgeways; let E be the Place of the Earth, and let the Circle AMB m represent the Moon, the Line AB being its *Axis*, and M m one of the Parallels of its Equator; and when the Moon is at H, let the Point M be that which is turned directly towards the Earth. Now when the Moon has revolved half round its *Axis*, which will be when it has got to I the opposite Point of its Orbit, the Point M will be found at m ; but because of the Parallelism of its *Axis*, not this, but some other Point, as N, is that which is turned directly towards

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towards the Earth at E; so that to a Spectator at E, the visible Part of the Moon librates from North to South, and returns to its Place once in each Revolution, one of the Poles as A being only visible, when the Moon is in one Part of its Orbit, suppose at I, and the Pole B only when it is at H; because the Circle represented by the pricked Line CD terminates the View of a Spectator at E, in either Situation of the Moon (a).

As to the Surface of the Moon, whoever views it with a good Telescope, will perceive some Parts of it shining very bright, while others are as dark and obscure. The brighter Parts are thought by some to be Land,

(a) It may be worth considering here, what kind of Motion the Earth seems to have to the Inhabitants of the Moon, if any such there be. And first, we will suppose that the Hemisphere, the Moon turns towards us, is always exactly the same. In this Case it is evident, that to such as live in the Middle of that Hemisphere, the Earth must always seem to be directly over their Heads; because a Line drawn from the Center of the Moon through the Place where they are, would pass through the Earth: And to such as live at the Extremity of that Hemisphere, the Earth must always appear in their Horizon. But since the Moon, as seen from the Earth, has these libratory Motions abovemention'd, it is impossible that the Earth should always appear directly Vertical to the one, or in the Horizon of the other; but must seem always nearly at the same Degree of Elevation; continually shifting its Situation from East towards the West, and from West towards the East, and at the same time from North towards the South, and from South towards the North, according as the Libration of the Moon, as seen from the Earth, is made, in that Time.

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the other Water ; because less Light is reflected from Water than from Land. But *Keil* is of Opinion, that some of those darker Places may be occasioned by the Shadows of higher Places falling upon them, and others may consist of a Soil of a darker Colour, that reflects less Light than the rest (*b*). However this be, it is most certain, that there are Mountains in the Moon, and those very high ones. For when the Moon is viewed through a Telescope, the Line which separates the illuminated Hemisphere from the dark one does not appear strait when the Moon is *halved*, or uniformly bent when it is *horned* or *gibbous*, as when we view it with the naked Eye ; but seems jagged, broken, and uneven ; which evidently shews that its Surface is so too : For if its Surface were smooth and even, no Reason could be given why the Light of the Sun should not reach as far in one Place as another. But the Boundary of Light and Darkness in the Moon is not only thus jagged and uneven, but some Parts even within the darkened Hemisphere, that lie near the Edge of it, are enlightened by the Rays of the Sun long before such as lie nearer the illuminated Hemisphere, and continue so till after the Parts about them are in the *Dark*. These therefore can be no

(*b*) See his Astronomy, Lect. X.

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other than the Tops of high Mountains, on which the Sun shines as it does on those on Earth, both before and after it is visible in the Vallies below. The perpendicular Height of a very remarkable Mountain in the Moon, called St *Katharine* (c), has been found by Mensuration to be nine Miles, which is three Times that of the highest Mountain we have upon the Earth (d).

(c) See the Names Astronomers have distinguished the Parts of the Moon by, in a Figure annexed to the Lecture referred to in the foregoing Note.

(d) The Method of measuring the Height of a Mountain in the Moon is this: Let ABDE (Fig. 17.) represent the Moon, C its Center, SA a Ray of Light from the Sun, ABD the illuminated Hemisphere, AED the dark one, F the Top of a Mountain beginning to be illuminated. Then with a Telescope in which there is a Micrometer fixed, let the proportional Lengths of the Lines FA and AD be taken. Then because the real Length of the latter, which is the Diameter of the Moon, is known, the true Length of the former will also be had. Since then the Line SAF is a Tangent to the Moon, the Triangle FAC will be a right-angled one, and FC will be its Hypothenuse, whose Square being equal to the Squares of the other two Sides, it may from thence be also known. Now if from this we subtract the Semidiameter of the Moon, the Remainder is the perpendicular Height of the Mountain.

C H A P. IX.

*Of the Eclipses of the Sun
and the Moon.*

AN *Eclipse* is a Deficiency of Light in some of the heavenly Bodies, occasioned by the Interposition of some other between that and the Sun. Thus when the Moon passes through the Shadow of the Earth, it being by that means deprived of the Sun's Light, loses its Brightness, and is then said to suffer an Eclipse. So when the Moon is interposed between the Sun and the Earth, the Sun being in Appearance deprived of its Light, is said to be eclipsed; though it is the Earth which is properly so, that, and not the Sun, being the Body where the Light is wanting.

One primary Planet is never eclipsed by the Interposition of another; because they are so small with respect to the Magnitude of the Sun, that the Shadow of any one of them ends in a Point before it reaches the Body of another. Thus when the Earth is interposed between *Mars* and the Sun, its Shadow vanishes before it reaches to *Mars*. And even when the Moon is interposed between the Sun and the Earth, if the Earth be in its *Perihelion*, and the Moon in its *Apogee* at

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at that Time, the Shadow of the Moon will not reach the Earth, and so the Moon will not seem to cover the whole Body of the Sun, but appear as a large dark Spot, surrounded with a Circle of Light proceeding from the Limb or Edge of the Sun.

From what has been observed it appears, that there can be no Eclipse of the Moon, but when the Earth is interposed between it and the Sun, that is, at the Time of its Opposition, or when it is *Full*; nor any of the Sun, except when the Moon is between the Earth and the Sun, that is, at the Time of its Conjunction with the Sun, or when it is *New*. And the Reason why these Eclipses do not happen at every New and Full Moon, is, because, as was observed above, the Plane of the Moon's Orbit is inclined to that of the Earth's, so that the Moon is sometimes above the Ecliptic, and sometimes below it, at the Time of its Conjunction with, or Opposition to the Sun, in which Case the Shadow of the one passes above or below the other (*a*).

(*a*) It is not requisite to constitute an Opposition or Conjunction of a Planet with the Sun, that it be in a right Line that passes through the Sun and the Earth; it may be in any Degree of Latitude it is capable of, and is nevertheless said by Astronomers to be in Conjunction or Opposition with the Sun, provided it be in the same Part of the Zodiac, that is, if it have the same Degree of Longitude therewith.

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The only Time therefore in which Eclipses either of the Sun or Moon can happen, must be when the *Nodes* are in or near the Syzygies, that is, when the Line of the *Nodes* points towards the Sun. Now if the Line of the *Nodes* always pointed towards the same Points of the Heavens, or remained parallel to itself, then the Sun appearing by means of the Earth's Motion to run through the Ecliptic once a Year, it is obvious that one Extremity of this Line would point towards the Sun each half Year: But since this Line has a slow Motion contrary to that of the Sun, by which it revolves round in nineteen Years, as observed above, it meets the Sun with one or the other Extremity before it has got quite half round, and so points towards it twice in somewhat less than half a Year: So that it sometimes happens that it points towards the Sun three times within the Space of a Year, which is as oft as possible. And it frequently happens, that the Sun being in its Approach towards one of the Nodes at the Time of the New or Full Moon, and near enough to occasion an Eclipse then, the Moon shall get about to the other Node, before the Sun be too far removed from the former, and so there will be another Eclipse then. Upon which account it is possible there may be two Eclipses, while the Sun is passing by either Extremity of the Line of the Nodes; the one of which (if it
be

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be at the New Moon) will be an Eclipse of the Sun, and the other will happen at the next Full Moon, and will therefore be an Eclipse of the Moon; or if the first be at the Full, the other will be at the New. But if the Moon be within a Day or two of passing thro' one of its Nodes at the Time of its Conjunction, it will pass by the other without suffering an Eclipse there.

When it happens that the Moon is in one of its Nodes at the Time of its Opposition to the Sun, then is it said to be *centrally* eclipsed, its Center being then in a right Line with those of the Sun and Earth. In this Case it is also *totally* eclipsed, every Part of it being hid from the Sun's Rays. But the Shadow where the Moon passes through, is about three times as broad as the Face of the Moon, so that it may suffer a total Eclipse, though it be at some Distance from the Node at the Time of its Opposition: An Eclipse therefore may be *total*, when it is not a *central* One; but an Eclipse of this Kind will be of shorter Duration than the former. Farther, if it be so far removed from the Node, that a Part of its Disk only falls into the Shadow of the Earth, it is said to be a *partial* Eclipse, and is of shorter Duration than the former. If the Moon be above 13 Degrees from the Node, there is no Eclipse.

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The Duration of a *central* Eclipse of the Moon is about 4 Hours; the first of which is taken up in entering the Shadow, two more it continues quite immersed therein, and the fourth is spent in getting out.

What is here said, may be illustrated in the following Manner. Let AB (Fig. 18.) represent a small Portion of the Plane of the Earth's Orbit seen edgewise, CD a Part of the Moon's Orbit, and let the Point where these Lines cross each other be one of the Nodes, and E, E, &c. the Shadow of the Earth passing along the Ecliptic. If the Moon falls into the Shadow at N, it is both a total and central Eclipse; if at O, it is total, but not central; if at P, the Eclipse is only partial. And if the Moon is at Q at the Time of its Opposition, it then passes by the Shadow without being eclipsed at all (*b*).

As all opaque Bodies when illuminated by the Rays of the Sun cast a Shadow from them, so they have also a *Penumbra*, which is an imperfect kind of a Shadow every where surrounding the former, and growing larger and

(*b*) We have hitherto supposed that the Shadow of the Earth reaches the Moon, as it would do, were it not for the Atmosphere of the Earth, which by refracting the Sun's Rays as they pass through it towards the Earth, throws them into its Shadow; which refracted Rays falling upon the Moon render it visible to us, altho' placed within the Limits of the Shadow, and is the Occasion of that reddish Colour, with which the Moon appears at that Time.

larger, as we recede from the Body. As the other is owing to a total Interception of the Rays of Light, this is occasioned by a partial one. The 19th Figure will explain this better.

Let ABCL represent the Body of the Sun, KI the Earth, and MN the Moon; and let the Lines AM, BM, &c. be drawn as in the Figure; then will the Space MNFG, within which none of the Rays can come, be the Shadow thereof. But it is evident that besides this there are other Spaces, *viz.* MDE and MEF, &c. within which but Part of the Sun's Rays can come; and the nearer those Spaces are to the Shadow, the less Light they are capable of receiving. The Light which falls within these Spaces is called the *Penumbra*.

This being premised, we may proceed to consider the different Kinds of Solar Eclipses.

The Moon being smaller than the Earth, and having a conical Shadow, as being also less than the Sun, a very small Part of the Surface of the Earth can be covered by the Shadow of the Moon at the same Time, though, as observed above, the whole Body of the Moon may be involved in that of the Earth; so that an Eclipse of the Sun is visible but to a few Inhabitants of the Earth, whereas one of the Moon may be seen by all that live on that Hemisphere of the Earth that is turned towards it.

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In order to constitute a *central* Eclipse of the Sun, it is not necessary that the Moon should be exactly in the Line of the Nodes at the Time of its Conjunction; for it is sufficient to denominate an Eclipse of the Sun *central*, that the Center of the Moon be directly between the Center of the Sun and the Eye of the Spectator; For to him the Sun is then *centrally* eclipsed. But since the Shadow of the Moon can cover but a small Portion of the Earth, it is obvious this may happen when the Moon is not in one of her Nodes. And not only this, but the Sun may be eclipsed *centrally*, *totally*, *partially*, and *not at all*, at the same time. For instance, when the Moon is interposed between the Sun and the Earth, to those who live where the Center of the Shadow passes (as in the Middle between F and G in the last Figure) the Sun will be *centrally* eclipsed; to those who live within the Shadow, but not in the Center (suppose nearer to F than to G) *totally*; to those who live in the *Penumbra*, as between F and D, or G and I, *partially*, and that more or less as they live nearer or farther from the Shadow; to those who live without the *Penumbra*, as between D and K, *not at all*.

But as the Shadow passes along the Surface of the Earth, different Inhabitants, and those

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those very distant ones, will see the same Kind and Degree of Eclipse, though not at the same Hour (c).

(c) For the Method of predicting Eclipses, see *Keil's Astronomy*, Lect. XIV. Or *Whiston's Astronomical Lectures*, Lect. XII.

C H A P. X.

Of the Phænomena of the Satellites of Jupiter and Saturn, their Eclipses and Occultations: And also of Saturn's Ring.

THE Satellites of *Jupiter* and *Saturn* are subject to the same kind of Motion with the Moon, except that their Orbits, as far as can be collected from the Observations of Astronomers, are circular, or very nearly so; and therefore, if we suppose them to be seen from their Primary ones, they will exhibit nearly the same *Phænomena*.

All the Satellites of *Saturn* revolve about it in almost the same Plane, *viz.* that of its Ring; excepting the fifth, the Plane of whose Orbit deviates a little therefrom. Those of *Jupiter* move also in a Plane that is proper to themselves, and nearly coincident with that

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that in which *Jupiter* itself moves about the Sun.

These Planets, *viz.* the Secondaries both of *Jupiter* and *Saturn*, when viewed from the Earth, appear to have their Progressions, their Stations, and Retrogradations independent of those of their Primaries; that is, when a Primary is progressive, its Secondary may be regressive, or otherwise; and *vice versa*. For instance, when a Secondary is beyond its Primary, that is, in its superior Conjunction with respect to the Earth, it seems to move according to the Order of the Signs; but when it is in its inferior one, it seems to move the contrary Way; and when its progressive Motion is changing into a regressive one, the Planet appears stationary.

To illustrate this; let S (Fig. 20.) represent the Sun, I the Planet *Jupiter* or *Saturn*, BCG the Orbit of one of its Satellites, and let EPF be the Orbit of the Earth. While the Satellite passes through OHB, the superior Part of its Orbit, it will appear from the Earth to move the same Way with its Primary; but while it moves through the remaining Portion BLO, it will seem to go the contrary Way; as is evident if we consider that its Motion in its Orbit is quicker than that of its Primary.

These Satellites and their Primary ones mutually eclipse each other in the same Manner

Chap. X. of Jupiter and Saturn. 95

ner in which it has been shewn that the Earth and the Moon do. But there are three different Cases in which the Secondaries disappear to us. The One is, when a Secondary is directly behind the Body of its Primary with respect to the Earth; this is called an *Occultation* of the Planet: Another is, when it is directly behind its Primary with respect to the Sun, and so falls into its Shadow, and suffers an Eclipse, as the Moon, when the Earth is interposed between that and the Sun: The last is, when it is interposed between the Earth and its Primary; for then it cannot be distinguished from the Primary itself. All which may be illustrated in the following Manner.

While the Planet describes OHB, the superior Part of its Orbit, it is capable of disappearing twice; once when its Primary is interposed between it and the Earth, as when it is at G or K, or some intermediate Point, according as the Earth is situated at that Time; and also at H, where it is involved in the Shadow of its Primary. When the Earth is between F and Q, or F and P, that is, when the Sun is to the Eastward of *Jupiter*, the Eclipse of the Secondary happens first, viz. when it passes through the Shadow at H; and the *Occultation* afterwards, when the Planet lies hid at K. When the Earth is between E and Q, or E and P, the Occul-

tation

66 *Of the Phænomena* Part IV.

tation is first, and the Eclipse afterwards. While it passes through the inferior Part of its Orbit, it disappears also, according to the Situation of the Earth, either at D or C; or when it is between those Points, being directly between the Earth and its Primary, and therefore not distinguishable from the latter. When the Satellite is at L in Conjunction with the Sun, its Shadow falls upon its Primary, and eclipses a Part of its Disk, in like manner as the Moon, in the like Situation with respect to the Earth, eclipses a Part of that (a).

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(a) From the Occultations and Eclipses of the Satellites of *Jupiter*, three notable Problems are determined; viz. 1. The Parallax of the Orbit of the Earth in that Planet, and thereby (as we observed Chap. VII. in the Notes) its Distance from the Sun. 2. The successive Propagation of Light; and 3. The Longitude of Places upon the Earth.

1. The Method of finding the Parallax of the Orbit of the Earth, is this: Let the Interval of Time which lapses between the Middle of an Occultation of one of the Secondaries, suppose at G, and the Middle of its Eclipse afterwards at H, which is the Time the Planet describes the Arch GH in, be observed. The Time then of its whole Revolution being known, that Arch itself may be had, which is the Measure of the Angle GIH, or its Equal EIS, which is the Parallax of the Orbit of the Earth in that Planet.

2. If Light were *instantaneous*, a Spectator at P would see an Eclipse of one of the Secondaries of *Jupiter*, at the same Time that he would do if on the contrary Side the Sun at Q. But it appears from Observations, that when the Earth is at Q, those Eclipses happen sooner, and that when it is at P they happen later, than they ought to do by Computation. Which shews that Light takes up some time in passing from Q to P. From Observations of this kind it has been computed by *Romer*, that Light is eleven Minutes in passing from the

Chap. X. of Saturn's Ring. 97

As to the *Phænomena* of Saturn's Ring, it is observable, 1. That its Inclination to the Plane of the Ecliptic is about 31 Degrees: 2. That the Thickness of it is insensible; and

the Sun to us; which is upwards of 81 Millions of Miles: But it has since been found by the more accurate Observations of others, that it passes over that Space in about 7 Minutes.

3. The Longitude of a Place is its Distance East or West of some other Place; and as Places lie more or less East or West one of another, the Hour of the Day in one is more or less different from the Hour of the Day in the other. That is, as Places differ in point of Longitude, so they do proportionably in their Reckoning of Time. For instance, if a Place lies fifteen Degrees to the Eastward of another, it is one Hour past Mid-day there, when it is but Mid-day at the other: The Reason is, because the Sun is an Hour in passing over each fifteen Degrees of its daily Course. In like Manner, if a Place lies so many Degrees to the Westward of another, the Sun comes an Hour later to that Place than to the other; so that it is but Eleven of the Clock there, when it is Twelve at the other. And so proportionably for any other Distance. If therefore at two Places it be observed at what Hour the beginning of an Eclipse of one of *Jupiter's* Satellites happens at each of them, and if that Hour be not the same in both, the Difference will shew how far those Places lie East or West of one another. Or if, instead of the Observation at one of the Places, an Almanack be made use of, that shews the Time when the Eclipse will happen at that Place, and at that Time be compared with the Time of its happening observed at the other Place, the Differences between those Times, if there be any, will shew the same.

This Method very rarely succeeds at Sea, because the Tossing of the Ship is inconsistent with the Accuracy required in Observations of this kind.

The like may be done by observing the Time when an Eclipse of the Moon happens, or when it comes to a fixed Star: But these are *Phænomena* that are much less frequent than the other.

therefore

therefore when it is so situated, that the Plane of it, if produced, would pass through the Earth, it becomes invisible. Neither is it to be seen when the Plane of it passes between the Sun and the Earth; for then the Sun shines upon that Side of it which is turned from us; in which Case, the dark Side being next the Earth, it is not perceptible to us. But in this Case a black List or Circle appears to encompass the Planet, which in all Probability is no other than the Shadow of the Ring thereon. The Extremities of the Ring which appear on either Side the Planet are by some called its *Anscæ*.

C H A P. XI.

Of the Comets.

THE Comets are opaque, spherical, and solid Bodies like the Planets; and like them perform their Revolutions about the Sun in elliptical Orbits, which have the Sun in one of their *Foci*. The Particulars in which they differ from the Planets, are, that they move in various Directions, some the same Way with the Planets, others the contrary; neither are their Motions confined within the Zodiac, their Orbits admitting of any Inclination

PART IV. PLATE V. *Fig. 9. 8.*

Fig. 14. P. 75.

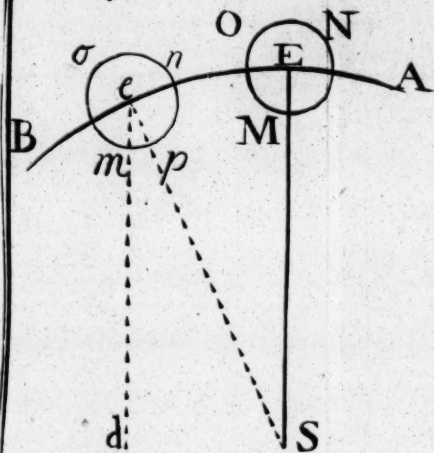


Fig. 15. P. 81.

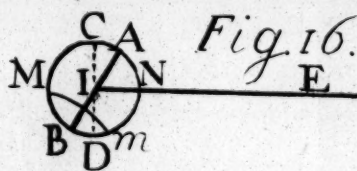
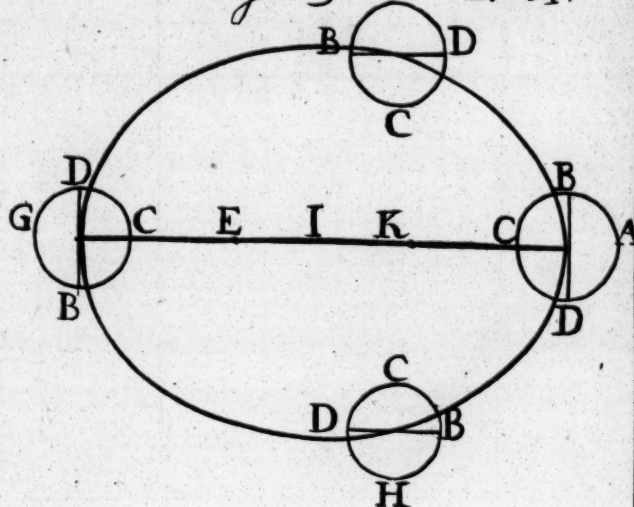


Fig. 16.

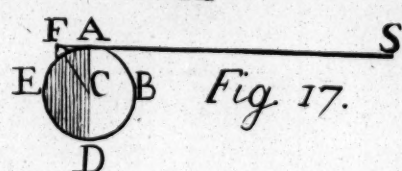


Fig. 17.

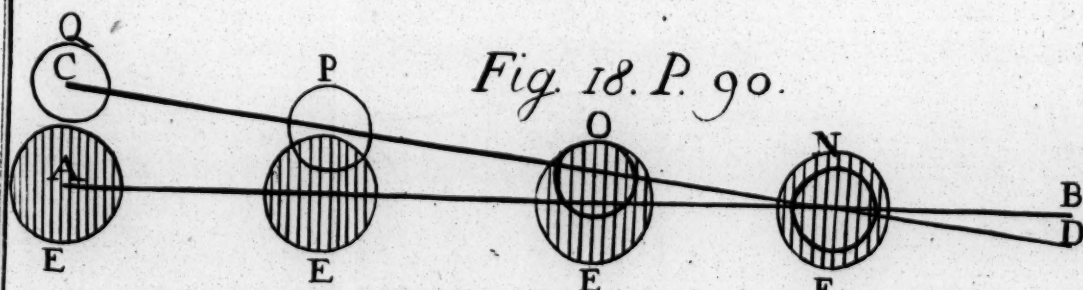


Fig. 18. P. 90.

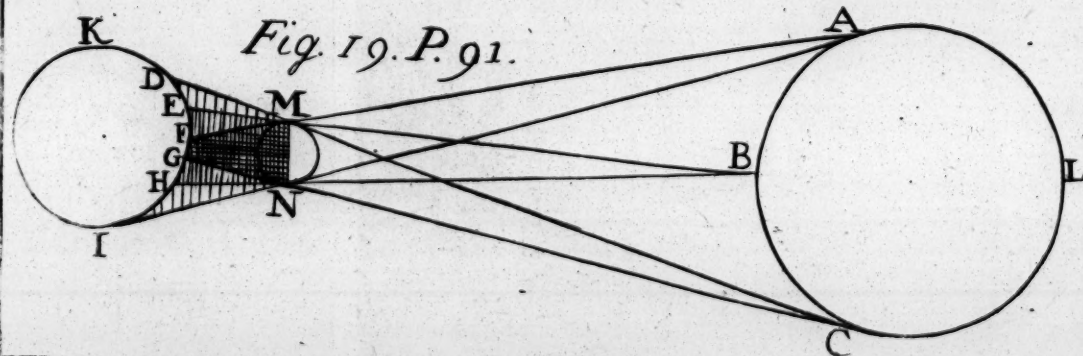


Fig. 19. P. 91.

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tion to the Ecliptic whatever: And the Excentricity of their Orbits is so very great, that some of the Comets perform the greatest Part of their Motion almost in right Lines, tending in their Approach to the Sun almost directly towards it, after which they pass by it; and when they leave it, march off again nearly in a right Line, till they are out of Sight, as if they were hastening back to the fixed Stars; and return not till after a Period of many Years (*a*).

As the Comets approach the Sun, their Motion grows proportionably swifter, for they describe equal *Areas* in equal Times about its Center, as the Planets do. Hence it is, that when they are in their *Perihelia*, that is, at their least Distance from the Sun, their Motion is immensely swifter than when they are in their *Aphelia*, or greatest Distance from it. This is the Cause that they are visible to us for so short a time; and, when they disappear, are so long before they come near enough to be seen again (*b*).

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(*a*) See a Portion of the Orbit of a Comet described, Plate I. Fig. 1. at *abcd*.

(*b*) From the Observations of Astronomers, it seems probable, that the Comet which appeared in the Year 1682 was the same which was seen before in the Year 1607, and the Year 1531, and therefore may be expected again in the Year 1758, after a Period of about 151 Years. And that the great Comet which appeared in the Year 1680 was the same that was seen in the Time of K. Henry I. in the Year 1106, and in 531, and in

The Ancients were divided in their Opinions concerning them, some considering them as wandering Stars, others as Meteors kindled in the Atmosphere of the Earth, subsisting for a Time, and then dissipated: Others looked upon them as ominous Prodigies. But it is put beyond all doubt by the more accurate Observations of the late Astronomers, that they are a kind of Planets. That they are not Meteors, is obvious; for, if they were, they could not bear that vast Heat, which some of them in their *Peribelia* receive from the Sun. The great Comet which appeared in the Year 1680 was within a sixth Part of the Sun's Diameter from its Surface, and therefore must acquire a Degree of Heat intense beyond all Imagination (c).

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in the forty-fourth Year before Christ, in which *Julius Cæsar* was murdered. If so, then the Period of this Comet is about 575 Years. There are between 20 and 30 that have appeared since the Year 1337, but no two Appearances seem to belong to the same Comet, except those abovementioned. See Dr. *Halley's* Synopsis of the Astronomy of Comets, published at the End of *Gregory's* Astronomy, in which there is a Table expressing the Motions of all the Comets that have been hitherto duly observed. The Comet of 1680 was in one Part of its Orbit within half the Sun's Breadth of the Way of the Earth.

(c) One Method by which Astronomers investigate the apparent Course of a Comet, is this: They observe what two Stars are directly one on one Side of the Comet, and the other on the other; which is done by holding up a Thread between the Eye and the two Stars, and extending it in such a manner, as that it shall seem to cross each Star: Then they look out two other

Chap. XI. *Of the Comets.* 101

As to those *Phænomena* of them which arise from the Motion of the Earth; they agree in a great Measure with those of the Planets. For instance, those Comets which move according to the Order of the Signs, a little before they disappear, become more than ordinarily *slow* or *retrograde*, if the Earth at that time be between them and the Sun; but more than ordinarily swift, if the Earth be on the opposite Side: And the reverse of this happens to those which move contrary to the Order of the Signs. This is occasioned by the Motion of the Earth, as was observed of the superior Planets: For when the Earth goes the same Way with a Comet, but with a swifter Motion, the Comet seems *retrograde*; when with a slower Motion, the Comet's apparent Motion becomes slower; and when the Earth moves the contrary way, it becomes swifter.

Few of the Comets are to be seen in their Access to the Sun, but in their Recess appear with long fiery Tails, pointing directly, or nearly so, towards that Part of the Hea-

other Stars in such Situation also, that the Comet shall appear in a Line that passes from one to the other; which are found as before: then they extend a Thread upon the celestial Globe, from one of the two first Stars to the other; and another Thread from one of the two last Stars to the other: And the Point on the Globe where the Threads cross is the apparent Place of the Comet at the Time the Observation was made. This they do daily, and so trace out its apparent Course in the Heavens.

vens which with respect to the Comet is opposite to the Sun. Some are visible before they reach the Sun, and begin to put forth their Tails, which at first are short and thin, seldom exceeding fifteen or twenty Degrees in Length, but grow longer and denser as the Comet comes nearer the Sun. If the Comet passes very near the Sun, it then sends forth fiery Beams of Light every Way. After this it puts forth a Tail forty, fifty or sixty Degrees long, which, as the Comet recedes farther from the Sun, continually diminishes both in Length and Splendor; but is larger and longer at any Distance in its Recess from the Sun, than at an equal Distance in its Access to it.

This great Splendor and Length of the Tails Sir *Isaac Newton* thinks arises from the Heat which the Sun communicates to the Comet as it passes near it; and accounts for it after the following Manner. As the Ascent of the Smoak in a Chimney is owing to the Impulse of the Air, with which it is entangled (for the Air about a Fire being rarefied by the Heat thereof, has its specifick Gravity thereby rendered less than that of the more distant Air: Upon this account it ascends, and carries along with it the Smoak with which it is engaged) in like manner he thinks the Tail of a Comet may rise from the Atmosphere thereof into those Parts which are opposite to the Sun, being carried up by the *Æther* about the
Comet,

Comet, rarefied to a very great degree by the Heat thereof. And thinks his Opinion greatly confirmed by the Appearance of the Tails; for when accurately observed, they are found not to rise always in a Direction precisely opposite to the Sun, but to deviate or incline a little from thence towards those Parts which the Comet has lately left; and not only so, but to be bent into a certain Curvature, the Extremities of the Tails deviating from the true Opposition more in proportion than the other Parts; and to be more dense seemingly, and better defined on the convex, than on the concave Side: And farther, that the longer the Tail is, the more sensible is the Curvature, as being the greatest at the greatest Distance from the Body of the Comet. Upon these Accounts he thinks it evident, that the *Phænomena* of the Tails of Comets depend on the Motion of their Heads, and that the Heads furnish the Matter which forms the Tails. For as in the Atmosphere of the Earth, the Smoak of a Body ascends perpendicularly if the Body be at rest, and obliquely if the Body be moved any otherways than directly upwards or downwards; so in the Heavens, where all the Bodies gravitate towards the Sun, Smoak and Vapour must ascend from the Sun, and rise perpendicularly, if the smoaking Body be at rest; and obliquely, if the Motion of the Body be oblique to the Direction the Vapour ascends

with. And because the Force by which the Vapour ascends is strongest near the Body of the Comet, the Obliquity with which it ascends will be the least near the Body, and greater at greater Distances, and consequently the Column of ascending Vapour will be incurvated or bent towards those Parts the Comet leaves: And because the Vapour in the preceding Side is something more recent, that is, has ascended something more lately, it will be something more dense on that Side, and on that account must reflect more Light, as well as be better defined; the Vapour on the other Side languishing by degrees, and vanishing out of Sight (*d*).

This is the Sentiment of Sir *Isaac Newton*; but I must beg Leave to differ from him in this Matter. For I think it somewhat improbable, on account of the great Rapidity with which Comets move, that the *Æther* diffused throughout the heavenly Regions should gra-

(*d*) Some have supposed, that the Heads of Comets are transparent, and that their Tails are no other than a Beam of the Sun transmitted through them: But were the Heads of Comets transparent, they themselves would scarce be visible. Others, that they arise from the Refraction of the Rays of Light in their Way from the Comets to us: But if so, then both the Planets and fixed Stars ought to have Tails also. *Kepler* ascribed the Ascent of the Tails to the Rays of the Sun carrying the Particles of the Comet's Atmosphere away with them, that is, impelling them into the Regions opposite to it. But we have no Instance of any thing in Nature like this; this is therefore an *Hypothesis* that cannot be supported.

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vitae sufficiently towards the Sun (especially at Distances equal perhaps to twice that of the Earth from it) to cause the Vapours, or the Atmosphere of the Comets, to ascend into those Parts, towards which the Comets are tending. For when a Body ascends in a Fluid, as being specifically lighter than it, its Motion at or near the Beginning of it is very small; so that if the Fluid have but a small Degree of Motion the contrary Way, the Resistance thereof will readily stop the Ascent of the Body, and will carry it the same Way it moves itself. This is the Reason the Air, which causes the Flame of a Candle to ascend, can scarcely blow downwards with so small a Degree of Motion, or (which is all one) the Candle itself can scarcely be lifted up with so gentle an Hand, but the Flame will turn downwards. How is it then likely, that when a Comet moves with an incredible Velocity (*viz.* at the Rate of above a thousand Miles in a Minute, as some of them do at the Time when their Tails rise almost directly before them:) How is it then likely, I say, that the *Æther* should by its Gravity alone raise the Vapour of the Comet with such Force, as to cause it to overcome its Resistance, when that Resistance arises from so great a Rapidity of the Comet? Would it not rather carry it with it the other Way? And if we augment the Density of the *Æther*, and therewith its

Gravity, then the Resistance of it will be the greater; and so the Case will be more desperate still. If we suppose it to be rarer, it will be lighter, and so less able to buoy the Vapours of the Comet up. The Question then is, how are those Tails to be accounted for? My Conjecture is as follows: It is well known that when the Light of the Sun passes through the Atmosphere of any Body (as the Earth) that which passes on one Side is by the Refraction thereof made to converge towards that which passes on the opposite one; and this Convergency is not wholly effected, either at the Entrance of the Light into the Atmosphere, or at its going out; but that beginning at its Entrance, it increases in every Point of its Progress. It is also agreed on all Hands, that the Atmospheres of the Comets are very large and dense. I suppose therefore, that by such Time as the Light of the Sun has passed through a considerable Part of the Atmosphere of a Comet, the Rays thereof are so far refracted towards each other, that they now begin sensibly to illuminate it (or rather the Vapours floating therein) and so render that Part which they have yet to pass through, *visible* to us; and that this Portion of the Atmosphere of a Comet thus illuminated appears to us in the Form of a Beam of the Sun's Light, and passes under the Denomination of a Comet's Tail.

Thus,

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Thus, when a Burning-Glass is exposed to the Sun's Rays, if there be the least Smoak or Dust hovering behind the Glass, the Rays of the Sun, as soon as they are brought near enough together by the refractive Power of the Glass, render so much of the Smoak or Dust as they have yet to pass through, very distinguishable from the rest. Indeed, by the accurate Reflection of the Glass, the Light of the Sun is brought to a *Focus*, wherefore the illuminated Space ends in a Point: But it happens not so to Rays refracted by the Atmosphere of a Body; because the Extremities of an Atmosphere do not refract near so strongly as those Parts which are nearer the Body. Let us now see how this Hypothesis will answer the *Phænomena* of the Tails.

When a Comet has been in its *Aphelion*, by means of its long Absence from the Sun, the Vapours with which its Atmosphere was full, are now condensed and fallen down, the Atmosphere itself grown cold, settled into its natural State, and reduced to a small Compass; therefore as the Comet approaches the Sun, the Tail of it is invisible to us, or at least appears very short: When it comes into the hotter Regions near the Sun, the Atmosphere begins to rarefy and dilate itself, and new Vapours rising up into it, increase the Quantity of Matter therein, on which account the Tail
grows

grows longer. When the Comet is in its *Perihelion*, if it be then very near the Sun, by means of the excessive Heat thereof, not only the hinder Parts of its Atmosphere are strongly illuminated by refracted Light, but the other Parts of it by the direct Light of the Sun, so that the whole Comet, Atmosphere and all, is as it were on Fire; which causes an infinite Number of Beams and Irradiations to be sent out from every Part; in which State the Comet is said to be *Hairy*. Afterwards when the Comet has got to some Distance from the Sun, and this great Illumination ceases, and the Atmosphere is extended by Rarefaction to a monstrous Size, and now more replete with Vapours than ever, which like the Steam of a boiling Cauldron have been continually rising into it, the Rays of the Sun have a great Way to go within it, after they are so far refracted towards each other, as to render the Atmosphere visible. So that the Tail now appears at its greatest Length; but from this Time grows continually shorter and shorter (as the Atmosphere by the Comet's losing its Heat contracts its Dimensions, and the Vapour floating therein subsides on to its Body) till the Comet is out of Sight.

But the greatest Difficulty is yet behind, and that is, to account for the Deviation of the Tail of the Comet from those Parts which are opposite to the Sun, towards those which

the Comet has left. For it will most certainly be thought, that upon this Hypothesis, the *Axis* of the refracted Rays, which is also the *Axis* of the Tail, ought to be in a right Line, that passes through both the Center of the Sun and of the Comet. Now if we consider the exceeding great Rapidity with which a Comet moves, and that the Propagation of Light is not instantaneous, this Difficulty will also vanish. For since Light is propagated in Time, that which passes near the Body of a Comet will not, when it comes at the Extremity of its Atmosphere, enlighten a Portion of it that lies in a right Line drawn through the Center of the Sun and the Place of the Comet where it now is, but where it was at the Instant when that Light passed by it: And therefore the illuminated Point of the Extremity of the Comet's Atmosphere, that is, the End of its Tail, will not be opposite to the Sun, but always in a right Line, that passes through the Center of the Sun and a Point of the Orbit behind the Comet, or which it has lately left; as will readily appear to any one that shall duly attend to this Matter. And because this is true of every other Portion of the Tail, in proportion to its Distance from the Head, the whole Tail will decline from those Parts which are opposite to the Sun, towards those which the Comet has lately left. And thus is the abovementioned

tioned Deviation to be accounted for. But it must not be concealed here, that that very successive Propagation of Light, which occasions this Deviation, is capable of augmenting or diminishing it in Appearance, or even of entirely taking it away, according to the Situation the Comet is viewed from. For, on account of the successive Propagation of Light, all distant Bodies that are in Motion appear not so far advanced as they really are; and the faster they move, and the farther they are from us, the greater is the Difference between their real and apparent Place. Let us then suppose, that the Extremity of a Comet's Tail is farther from the Eye of a Spectator than the Body of the Comet, by a Space equal to the Semidiameter of the Earth's Orbit, then will Light be seven Minutes longer in coming from the Extremity of the Tail than from the Head; and therefore the Tail will appear to be farther behind its true Place than the Comet does, by a Space equal to that through which the Extremity of the Tail moves in seven Minutes; and consequently the apparent Deviation in this Case will conspire with the true one, and render it in appearance proportionably larger. Again, when the Extremity of the Tail is nearer the Earth than the Comet, then Light comes quicker from the former than from the latter, and therefore the Comet appears farther behind its true Place than the

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Chap. XI. *Of the Comets.* III

Extremity of its Tail does, and so the true Deviation appears the less, or not at all, according as the Case may happen.

The apparent Deviation of a Comet's Tail depends also on another Circumstance, *viz.* the Situation of the Spectator with respect to the Plane of the Comet's Orbit; for when a Spectator is in the Plane of that Orbit, the Deviation becomes insensible to him, as not being able to distinguish nicely the exact Distances of the several Parts of the Tail from his Eye: The farther he is removed out of that Plane, the more sensible it becomes.

There is another Particular, *viz.* that the Extremities of the Comet's Tail deviate from the true Opposition more in proportion than the other Parts, so that they are bent into a certain Curvature: And farther, that they are more lucid and distinct on the convex than on the concave Side. All which may be accounted for, if we allow the heavenly Regions the least imaginable Resistance; for in that Case, the Atmosphere of a Comet will be somewhat denser on the Side towards which the Comet tends; and therefore the Light which passes on that Side will suffer a greater Degree of Refraction throughout its whole Progress along the Atmosphere, that is, to the End of the Tail, than such as passes on the other: So that, that Side of the Tail which moves first, will be more lucid and better defined than
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the other: And because this Resistance will make the greatest Alteration in the extreme Parts of the Comet's Atmosphere, the Extremity of the Comet's Tail will deviate from the true Opposition more in Proportion than those Parts which are nearer the Body; so that the Tail (at least that Side of it which moves first) will become convex. And if the Refraction of those Rays which pass on that Side towards which the Comet tends, and on that account are most refracted, be so great as that they shall cross those which pass on the other Side the Comet, before they get out of its Atmosphere, their Refraction being irregular, they will cross the other dispersedly and in several Places, which, as is obvious to conceive, will necessarily give that other Side of the Tail a concave Form, and render it imperfectly defined.

Thus I have ventured to throw in a Conjecture of my own; but propose it rather as a *Quere*, than a Solution; leaving it to be farther examined into by those who may have had better Opportunities of acquainting themselves with *Phænomena* of this Kind. *Valeat quantum valere potest.*

C H A P. XII.

Of the Parallax of the Heavenly Bodies.

THE Difference between the apparent Places of the Heavenly Bodies, when viewed from the Center of the Earth, and when seen from the Surface thereof, is called their *Parallax*. To explain this,

Let AB (Fig. 21.) represent a Portion of the Earth's Surface whose Center is C, HV the Heavens, and let APN be the Horizon of a Spectator at A, and P the Situation of a Planet therein. The Place of this Planet among the fixed Stars, as seen from the Center of the Earth, will be M, which is called its *true Place*: But when view'd by a Spectator upon the Surface of the Earth at A, it appears in the Horizon at N, which is called its *apparent Place*; and the Arch MN, which measures the Distance between the one and the other, is the *Parallax* of that Planet. If the Planet be above the Horizon, as at Q, its true Place as seen from the Center of the Earth is R, its apparent Place S, and its *Parallax* is RS, which is less than before. From whence we see, the more the Planet is elevated above the
 Horizon,

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Horizon, the less is its Parallax; and when it is got directly over the Place of the Spectator, it has no Parallax at all; for when it is at T, its apparent Place in the Heavens is at V, whether it be seen from C or A.

Farther, by how much the more remote a Planet is from the Earth, so much the less is its Parallax, at the same Heights from the Horizon. Thus if the Planet had been at the Distance CD from the Center of the Earth, its Parallax, when in the Horizon at D, would have been NO; and when in the Line AQ produced, as at E, its Parallax would have been measured by the Arch SF, both which are less than the former.

The Angles MPN, and RQS, are called the *parallactic* Angles, supposing the Planet at P or Q, and are equal, the first to the Arch MN, and the other to the Arch RS: They are also equal to the Angle APC, or AQC, which are those under which CA, a Semidiameter of the Earth that passes through the Place of the Spectator, would appear if seen from the Planet.

From hence it is observable, that the apparent Place of a Planet is always lower in the Heavens than its real one, except when the Planet is vertical, or at T.

As Astronomers, by means of the annual Parallax of a Planet, that is, the Parallax of
the

Chap. XII. *the Heavenly Bodies.* 115

the annual Orbit of the Earth, as explained above (*a*), compute the Distance of the Planet from the Sun, with respect to that of the Earth from thence; so by means of this Parallax, they calculate the Distance of the Planet from the Center of the Earth with respect to the Distance of the Surface of the Earth from the same (*b*).

The Sun is also liable to this Kind of Parallax; for it is evident that its Place among the fixed Stars, when seen from the Surface of the Earth, may be different from that in which it would appear if seen from the Center of the Earth. Now could this Parallax be determined with Accuracy, the exact

(*a*) See Chap. VI. and VII.

(*b*) The Parallaëtic Angle APC being known, and the Angle VAN, which is measured by the Arch of the Heavens VN, and which gives PAC, its Complement to two right ones, being taken by Observation, we from thence have PAC, the remaining Angle of the Triangle APC: From whence the Relation, which PC, the Distance of the Planet from the Center of the Earth, bears to CA, the Distance of the Surface of the Earth from its Center, may be determined.

There are several Methods of finding the Parallaëtic Angle; one of which is as follows:

First, let it be observed when the Planet is between two fixed Stars which are both in the same vertical Circle; and afterwards, when these two Stars appear situated in a Line that is parallel to the Horizon, observe how much the Planet appears below them; for the Planet being depressed by the Parallax, will at that time be lower than a Line drawn from one Star to the other; because the fixed Stars are at too great a Distance to suffer any Parallax: The Arch of the Heavens intercepted between this Line and the apparent Place of the Planet is its Parallax; Allowance being made for the Motion of the Planet between the two Observations.

H

Distance

Distance of the Sun from the Earth might be known, and from thence, by the Method explained above (c), that of the other Planets from the Sun. But the Sun's Distance from the Earth is so very great, and of consequence its Parallax is so very small, that the Observations necessary for taking of it cannot be made with sufficient Accuracy. Astronomers therefore have recourse to the Parallaxes of *Mars* and *Venus*; which if they could be found, that of the Sun would also be had: Because the Horizontal Parallaxes of the Heavenly Bodies, as we have just been shewing, are proportionably less as their Distance is greater. Now when *Mars* is opposite to the Sun, it is above twice as near to us, and accordingly its Parallax is above twice as great as that of the Sun: And *Venus*, when in its inferior Conjunction with the Sun, is almost four times nearer than the Sun, and therefore its Parallax is so many times greater. Astronomers find, that the horizontal Parallax of *Mars*, when it is in Opposition to the Sun, does not exceed 25 Seconds, and from thence conclude the Sun's to be about 10 Seconds. With which Parallax of 10 Seconds if a Calculation be made, the Sun's Distance will be found about 81 Millions of Miles from the Center of the Earth.

However, Astronomers need not be solicitous about the Parallax of *Mars*, since Dr.

(c) See Chap. VI. VII.

Halley

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Halley has laid down a Method in the Philosophical Transactions (*d*); whereby the Parallax of the Sun may be had to so great a Nicety, that its true Distance within a 50th Part of the Whole may be determined from it; *viz.* At the next Transit of *Venus* over the Sun's Disk, which will happen on the 26th of *May*, in the Year 1761; when, at five Minutes before Six in the Morning, *Venus* will appear not above four Minutes of a Degree South of the Middle of the Sun's Disk.

(*d*) Numb. 348.

C H A P. XIII.

*Of the Refraction of the Atmosphere,
and the Crepusculum or Twilight.*

WE find by Experience, that when Light enters a transparent Medium obliquely, either denser or rarer than that in which it was before; it does not pass strait on, but its Course is bent at the Point where it enters, which Bending is called its *Refraction*; and if the Medium into which it enters be denser than the other, the Light is then refracted towards a Perpendicular to the Surface drawn through the Point where it enters; but if the Medium be rarer, it is then refracted or bent

the contrary way (*a*). It is from hence that a Staff, having one End under Water and the other above it, appears bent or broke at the Surface of the Water, the Part below the Water appearing above its natural Situation, because the Light that comes from thence is refracted or bent downwards at the Surface of the Water, where it comes out. Thus also the Stars near the Horizon appear above their true Places, on account of the Refraction which the Light that comes from them suffers in passing through the Atmosphere of the Earth (*b*). The Manner of which is now to be explained.

Let then AB (*Fig. 22.*) represent the Surface of the Earth, C its Center, DF a Portion of its Atmosphere, and GI the Sphere of the fixed Stars, and let AH be the Horizon of a Spectator at A, and let there be a fixed Star or a Planet at K; and let KL be a Ray of Light proceeding from hence. Now this Ray passing out of a Vacuum, or empty Space, into the Earth's Atmosphere at L, will be refracted towards the Line LC, that being a Perpendicular to the Surface of the Atmosphere, at the Point where the Ray enters. And since the superior Air is rarer than the inferior, the Ray is

(*a*) See the Reason of this assigned, and the Manner of it largely explained, Part III. Chap. II.

(*b*) This has been already taken Notice of in Part III. Chap. VII. in the Notes.

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continually entring a denser Medium all the way it goes; and so is refracted every Moment downwards towards C, as at first, which will cause it to describe a Curve, as LA, and to enter the Spectator's Eye at A, as if it came from E, a Point above L; and since an Object appears always in that Line in which the Ray passes when it enters the Eye (*c*), the Star will seem to be at G, and therefore higher than its true Place, and sometimes above the Horizon, when its true Place is below it; as in the Case before us, where its Situation K is supposed to be below the Horizontal Line AH.

Thus it has often happen'd, that in an Eclipse of the Moon, when it is directly opposite to the Sun, they have both appeared above the Horizon at the same Time.

When a Star is in the Zenith, it is liable to no Refraction; for then the Light that comes from it to a Spectator's Eye, enters the Atmosphere in a Direction perpendicular to its Surface: But the lower the Star is, the more obliquely the Light enters the Atmosphere; so that the greatest Refraction of all is when the Star is in the Horizon, and it is then 33 Minutes and 45 Seconds; but when the Star is 50 Degrees or more above the Horizon, its Refraction is insensible (*d*).

(*c*) See the Reason of this, Part III. Chap. VII. at the Beginning.

(*d*) See Sir *Isaac Newton's* Table of Refractions. Philosoph. Transact. Numb. 368.

From hence we see the Reason why the Sun and Moon, when near the Horizon, do not appear of a circular Form, as at other Times, but of an oval one, having their longest Diameter parallel to the Horizon. For their lower Parts appear elevated by Refraction more than their upper ones, and consequently nearer together than they ought to do; while their Sides are equally elevated, and keep their just Distance from each other.

It is not with regard to Refraction, as in the Parallax: The latter of which, as explained in the preceding Chapter, depresses the Sun only and the Planets; and that according as they are more or less removed from the Earth; while the former elevates all the Heavenly Bodies alike, at whatever Distance they are removed from us.

Farther; were it not for the Atmosphere, which continually reflects the Light of the Sun from one Portion of it to another, and thereby occasions some of it to enter our Eyes, which way soever they are turned, and also throws it upon those Parts of the Bodies about us which are not exposed to the direct Light of the Sun's Beams, the whole Heavens would appear totally dark; and such Bodies, or such Parts of Bodies, as the Sun did not directly shine upon, would be invisible, and the Stars would appear at Noon-day. But the Atmosphere of the Earth being strongly illuminated by the Beams of the Sun,

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Sun, they are reflected to us, and to other Bodies from all Parts; so that the whole Atmosphere is enlightened, and every thing therein is render'd visible; while the fainter Light of the Stars, which would otherwise appear, is obscured, and eclipsed thereby.

Besides, though the Sun, immediately before it sets, would in this Case shine with the same Brightness as at Noon, yet the Moment it was set, it would be quite dark. Whereas, by means of the Atmosphere, it happens, that although none of the Sun's direct Rays can come to us after it is set, yet we still enjoy its reflected Light for some Time, and Night approaches by Degrees. For after the Sun is hid from our Eyes, the upper Parts of the Atmosphere remain for some time exposed to its Rays, and from thence the whole is illuminated by Reflection. But as the Sun grows lower and lower, that Portion of the Atmosphere which is above our Horizon becomes less enlighten'd, till the Sun has got eighteen Degrees below it, after which it ceases to be illuminated thereby (*e*), till

H 4

it

(*e*) Hence it is, that during that Part of the Year in which the Sun is never about eighteen Degrees below the Horizon, there is a continued Twilight from Sun-setting to Sun-rising. That Part of the Year, in the Latitude of *London*, is while the Sun is passing from about the fifth Degree of *Gemini* to the twentieth of *Cancer*, that is, from about the 15th of *May* to about the 7th of *July*.

Some Philosophers tell us, that the Height of the Earth's Atmosphere may be determined from the Duration of the Twilight, in the following Manner. Let ABD (Fig. 23.) represent the Earth

it has got within as many Degrees of the Eastern Side of the Horizon, at which Time it begins

Earth surrounded by its Atmosphere GEF. Through A, the Place of a Spectator, draw the Tangent AH, passing through the Extremity of the Atmosphere at E, from whence draw the Line ES, touching the Surface of the Earth in B, as also from the Center C, the Lines CA, CE, and CB. Now, say they, since the Crepusculum is owing to the Reflection of the Sun's Rays, by the Particles of which the Atmosphere consists, it must end at such time as Rays of Light coming from the Sun, and passing close by the Surface of the Earth, are reflected from the Top of the Atmosphere to the Spectator's Eye; and therefore knowing how far the Sun is below the Horizon at that time, we may know the Height of the Atmosphere. Thus let S be the Place of the Sun when the Crepusculum ends, SE a Ray of Light passing close by the Earth at B, and reflected from E (a Particle at the Top of the Atmosphere) into the horizontal Line EA, to the Eye of the Spectator at A; then will the Angle HES be the Measure of the Sun's Distance below the Horizon. And because EB and EA are Tangents to the Earth, the Angle ACB at the Center will be equal to HES, or eighteen Degrees; and half of the former, namely ACE, will be nine; from whence EI, the Height of the Atmosphere, which is the Excess of the Secant EC above the Radius CA or CI, is had by Trigonometry: See *Keil's Astronomy*, Lect. 20. But this way of arguing is founded on a Supposition, that Rays of Light cannot come to a Spectator's Eye after more than one Reflection in the Atmosphere; the contrary to which seems evident, as I shall shew in the following manner. Let ABD (Fig. 24.) represent the Earth, A the Place of a Spectator, HO his Horizon, MNR the Portion of the Earth's Atmosphere, and TVW an Arch of the Heavens; and let S be the Place of the Sun less than 18 Degrees below the Horizon; and let the Line SAI touching the Earth be drawn. It is well known, that in this Case so much of the Heavens as is not intercepted by the Earth from the Spectator's Eye at A, will still appear enlightened; that is, of other Words, Light will flow to the Spectator's Eye from every Part in that Portion of the Atmosphere that lies between P and Q; some therefore will come from those Particles that lie within the Space PAE. But these Particles cannot be enlightened by the direct Rays of the Sun; for they necessarily

Chap. XIII. of Twilight. 123

gins to illuminate the Atmosphere again, and in Appearance to diffuse its Light throughout the Heavens, which continues to increase till the Sun be up. This Light, whether in the Morning before Sun-rise, or in the Evening after Sun-set, is called the *Crepusculum*, or Twilight; and is supposed to begin and end when the least Stars that can be seen by the naked Eye (*viz.* those of the sixth Magnitude) cease or begin to appear.

But as the Twilight depends on the Quantity of Matter in the Atmosphere fit to reflect the Sun's Rays, and also on the Height of it, (for the higher the Atmosphere is, the longer it will be before the upper Parts of it will cease to be illuminated) the Duration of it will be various. For Instance in Winter, when the Air is condensed with Cold, and the Atmosphere upon that account lower, the Twilight will be shorter; and in Summer, when the Limits of the Atmosphere are extended by the Rarefaction and Dilatation of the Air of which it consists, the Duration of the Twilight will be greater. And for the like Reason the Morn-

cessarily pass above the Line AE, these therefore themselves are illuminated by reflected Light. Since then these afford Light, and nearly as much as any of the rest, it is evident, that Rays may come to a Spectator's Eye, and in great Plenty, after having suffered two Reflections. And therefore that Demonstration, which is built upon a Supposition that such Rays can suffer but one, must fall to the Ground.

ing

ing Twilights, the Air being at that Time condensed and contracted by the Cold of the preceding Night, will be shorter than the Evening ones, when the Air is more dilated and expanded.

Another Effect that the Atmosphere has upon the Rays of the Sun is, that it intercepts a considerable Part of them in their Progress through it. By which Means it happens, that when the Sun is near the Horizon, it appears less luminous and bright than when it is more elevated; for the nearer the Sun is to the Horizon, the greater is the Portion of the Atmosphere through which the Rays of the Sun must pass to come to a Spectator's Eye. For Instance, in Fig. 23, where ABD represents the Earth, surrounded by its Atmosphere GEF, and A the Place of a Spectator; it is obvious, that when the Sun is in or near the Horizon, suppose at H, its Rays must pass through a greater Portion of the Atmosphere to come to the Spectator's Eye at A, than they would do if the Sun were situated in any Part of the Heavens above the said Point.

C H A P. XIV.

Of the Doctrine of the Sphere.

THE Subject of this Chapter relates chiefly to the Explanation of those Circles which Astronomers, for the better distinguishing the Places and Motions of the heavenly Bodies, conceive to be described within the Sphere of the Heavens. Some of these have already been defined; but that the Reader may have a compleat View of their Definitions all together, I shall repeat them here in their proper Places.

These Circles are of two Kinds, *viz.* the *greater* and the *smaller*.

A great Circle is that which divides ^{A great} the Sphere into two equal Parts, hav- ^{Circle.} ing the same Center therewith.

A less Circle divides the Sphere into ^{A less} unequal Parts, having a different Center ^{Circle.} from that of the Sphere.

These are sometimes called parallel ^{Parallel} Circles, and receive their Denomina- ^{Circles.} tions from some great Circle to which they are parallel. Thus, lesser Circles

run-

running parallel to the Equator, are called Parallels of the Equator; and so of the rest.

The Poles
of a Circle.

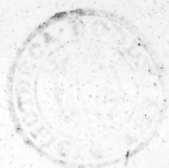
The Poles of a Circle are two Points in the Surface of the Sphere, every where equally distant from the Circle.

Axis of a
Circle.

The Axis of a Circle is an imaginary Line supposed to pass through both its Poles: Or, that passes through its Center, and is perpendicular to its Plane.

Secondaries
of a Circle.

The Secondaries of a Circle are such as pass through both its Poles. These therefore cut it at right Angles in two Points opposite to each other, and are therefore always great Circles.



The Circles of the Sphere are also distinguished into fixed and moveable.

A fixed
Circle.

A fixed Circle is that whose Situation depends not upon that of the Spectator.

A move-
able Circle.

A moveable Circle is that whose Situation depends on that of the Spectator.

Of the former Sort are the Ecliptic and the Equator, with their Secondaries and Parallels.

Of the latter are the Horizon, with its Secondaries and Parallels.

I. Of

I. *Of the Ecliptic, and other Particulars relating to it.*

The Ecliptic is that Circle which the ^{The Ecliptic} Plane of the Earth's Orbit would mark out, were it supposed to be extended to the Heavens. This therefore is that which the Sun seems to describe in its annual Course.

The Axis of the Ecliptic is a right ^{The Axis of the Ecliptic} Line supposed to pass through the Center of the Sun, and to be perpendicular to the Plane of the Ecliptic; and the Points in the Heavens, to which this Axis points, are called the Poles there-^{Its Poles} of; and great Circles passing through these Poles, will be perpendicular to the Ecliptic, and are therefore called its Secondaries, and sometimes Circles of ^{Secondaries of the Ecliptic} Longitude. By Means of these the Place of any Star is referred to the Ecliptic, and its Longitude and Latitude estimated. For the Longitude ^{Longitude of a Star} of a Star is an Arch of the Ecliptic, intercepted between the first Point of Aries, and that where one of these Secondaries that passes through the Star cuts the Ecliptic. And the Latitude ^{Latitude of a Star} of a Star

Parallels
of Latitude.

a Star is the Arch of the same Secondary, intercepted between the Star and the Ecliptic. Upon this account all Circles conceived to be drawn parallel to the Ecliptic, that is, Parallels of the Ecliptic, are sometimes called Parallels of Latitude. The Latitude of a Star is either Northern or Southern, as the Star is placed on this or that Side the Ecliptic.

The Signs
of the
Ecliptic.

The Ecliptic is divided into twelve equal Parts, called Signs. They begin at that Point in which the Sun appears to be at the Vernal Equinox, and are reckoned from West to East. The Names of the said Signs, together with the Characters whereby they are commonly expressed, are these that follow:

Aries,	Taurus,	Gemini,	Cancer,	Leo,	Virgo,
♈	♉	♊	♋	♌	♍
Libra,	Scorpio,	Sagittarius,	Capricornus,	Aquarius,	Pisces.
♎	♏	♐	♑	♒	♓

The Zodi-
ac.

The first six of these are called the Northern Signs, as possessing that Half of the Ecliptic which lies nearest the North Pole of the Earth; the latter, the Southern, as being in that which is nearest the South Pole. The Sun is always seen in some Part of this Circle;

but the Planets, by reason of the Inclination of the Plane of their Orbits to that of the Ecliptic, appear sometimes above it, and sometimes below it, but never deviate from it above seven or eight Degrees on either Side: So that if we imagine a Space furrounding the Heavens sixteen Degrees in Breadth, and extended equally above and below the Ecliptic, this will include all their Ecliptics, if I may so say; that is, all the Circles they would appear to move in when seen from the Sun, and is called the Zodiac.

The Signs of the Ecliptic took their Names, as was observed above, from twelve Constellations situated in the Heavens near those Places. But it must be remembred, that the Signs are different from the Constellations which bear the same Name. For the *Sign* Aries, for Instance, is not in the same Part of the Heavens with the Constellation Aries. The Sign Aries is only thirty Degrees of the Ecliptic, counted from one of the Equinoctial Points; whereas the *Constellation* of that Name is a System of Stars, the most of which are now situated between the first and last Degree of Taurus. And so of the
other

other Signs. These Constellations were in the Infancy of Astronomy situated within the Signs which now bear their Names; but by a slow retrograde Motion of the Equinoctial Points, called the *Precession of the Equinoxes*, as explained above (a), the Constellation Aries has got into the Sign Taurus, and that of Pisces into the Sign Aries; and so of the rest.

II. Of the Equator, and other Particulars relating to that.

The Axis of the Earth. The Axis of the Earth is a right Line supposed to pass through its Center, about which its daily Rotation is performed.

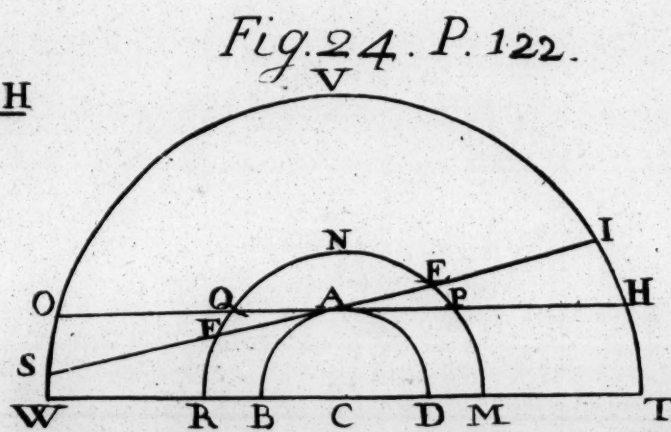
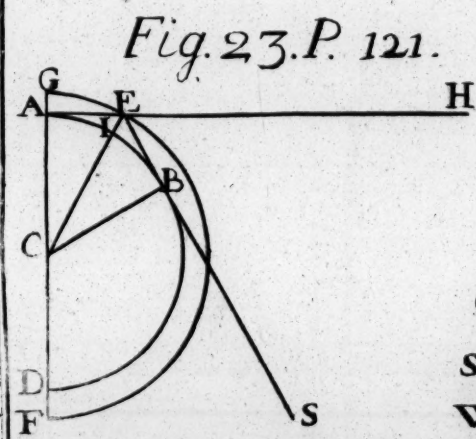
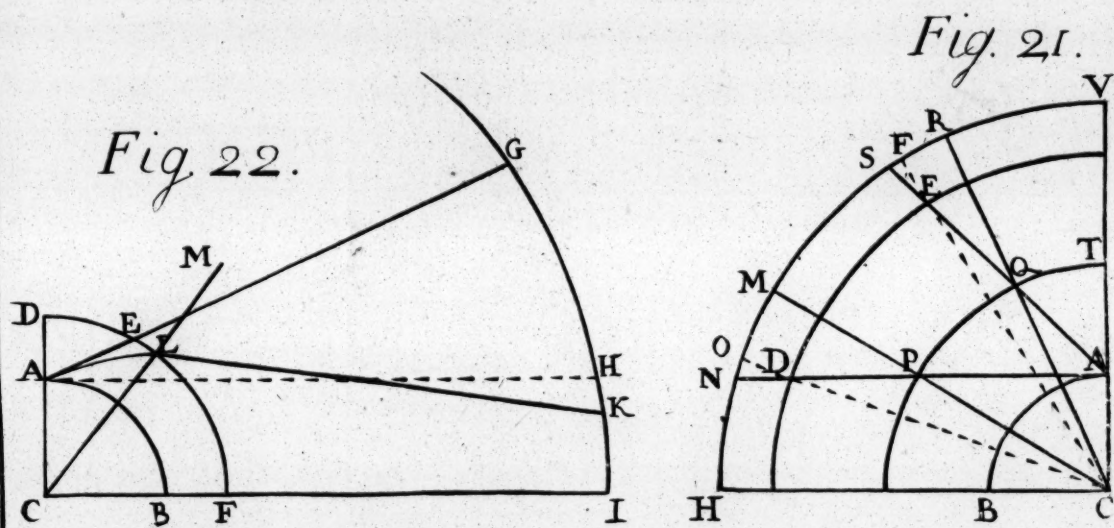
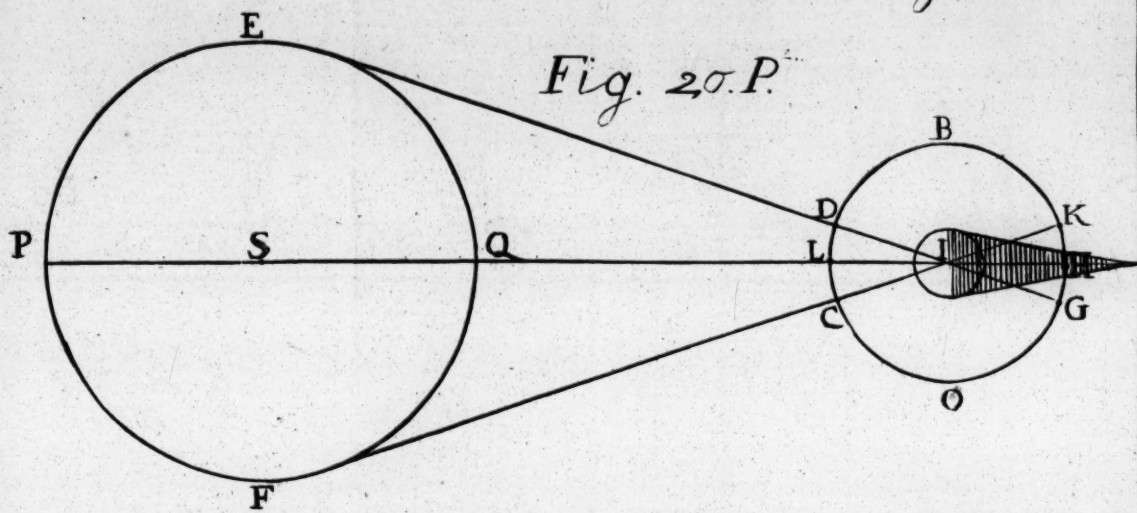
The Poles of the Earth. The Poles of the Earth are two Points in its Surface, where its Axis passes through; the one the North, the other the South Pole.

The Poles of the Heavens. The Poles of the Heavens are two Points therein, where the Axis of the Earth, if produced, would pass through them.

The Equator. The Equator is a great Circle on the Surface of the Earth, equally distant from either Pole thereof. This is by Mariners called the *Line*.

(a) See Chap. V.

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A Secondary of the Equator passing ^{Meridian.} through any particular Place upon the Surface of the Earth, is called the *Meridian of that Place*.

And as the Longitude and Latitude of the heavenly Bodies are determined by the Ecliptic and its Secondaries; so by the Equator and its Secondaries, Places upon the Earth's Surface are distinguished as to their Longitude and Latitude.

The Longitude of a Place is then an ^{The Lon-} Arch of the Equator, intercepted be- ^{gitude of} ^{a Place.} tween the first Meridian (*b*), and that which passes through the Place.

I

The

(*b*) By the first *Meridian* is meant that which passes through some remarkable and well-known Place, which Geographers pitch upon to estimate the Longitude of other Places from. The ancient Geographers called that the first Meridian which passed through the most Western Part of the then known World; and so reckoned the Longitude Eastward from thence. And as they looked upon the World to be longer from West to East than from North to South, they gave the Name of Longitude to the longer Dimension, and that of Latitude to the shorter. But when it was discovered that the Earth was round, and consequently that there was no extreme Point of Longitude, Geographers began to neglect this Method, and every one chose a first Meridian according to his Fancy and Inclination. Thus, an *English* Geographer shall tell you the first Meridian passes through *London*; a *Frenchman*, that it passes through *Paris*, &c.

This

The Latitude of a Place.

The Latitude of a Place is an Arch of the Meridian, intercepted between the Place and the Equator; and is either Northern or Southern, as the Place lies on this or that Side the Equator.

The Equinoctial.

The Equinoctial and the Equator are different, the former being a Circle in the Heavens, which the latter would mark out if supposed to be extended thither.

Northern and Southern Hemispheres.

The Equinoctial divides the Heavens, as the Equator does the Earth, into two equal Portions, called the Northern and Southern Hemispheres.

Right Ascension and Declination.

And as by the Secondaries of the Equator, Places upon the Earth are referred to the Equator; so by these of the Equinoctial, the Places of the heavenly Bodies are referred to the Equinoctial. But as an Arch of the Equator, intercepted between the first Meridian and that which passes through any Place, is the Longitude of that Place; an Arch of the Equinoctial, intercepted between the first Point of Aries and that where it is cut by a Se-

This arbitrary Way of reckoning the Longitude from different Places makes it necessary, whenever Mention is made of the Longitude of any Place, to mention also the Place from whence that Longitude is counted.

condary

Chap. XIV. *of the Sphere.*

133

condary of the Equinoctial that passes thro' the said Body, is called its Right Ascension. And an Arch of the same Secondary, intercepted between the same Body and the Equinoctial, is called its Declination, which is also either Northern or Southern, as it happens to be in the one or the other Hemisphere.

From hence the Secondaries of the Equinoctial are sometimes called Circles of Ascension and Declination.

The Equinoctial Points are those where the Equator cuts the Ecliptic; and are so called, because when the Sun is in these Points, it is also in the Equator, and therefore makes the Days and Nights of an equal Length all the World over. The Solstitial Points are those Points in the Ecliptic where the Equator is most distant from it; and are so called, because when the Sun is in them, it can scarce be perceived to alter its Distance from the Equator for some Time.

There are two of the Secondaries of the Equinoctial more remarkable than the rest, called the *Colures*; the one passes through the Equinoctial Points, and is called the Equinoctial Colure; the other is placed at right Angles with this, and passes through both the Poles

of the Equator and of the Ecliptic too, and is called the Solstitial Colure.

Hour Circles.

Secondaries of the Equinoctial drawn through each 15th Degree thereof, will divide it into 24 Parts; each of which, since the Earth revolves once round its Axis in 24 Hours, will answer to one Hour; that is, each of them (supposing the Heavens to revolve as they appear to do) will succeed into the Place of a foregoing one in the Space of one Hour; and are for that Reason called Hour Circles. And the Inhabitants of any Part of the Earth, whose Meridian coincides with any one of these, will differ in their reckoning of Time one Hour from those whose Meridian is coincident with the next; and so on.

Tropics and Polar Circles.

Of the Parallels of the Equator, four are more remarkable than the rest, and distinguished by particular Names, *viz.* the Tropics and Polar Circles.

The Tropics.

The Tropics are those Circles which the Sun in its greatest Declination seems to describe by its diurnal Motion; and are therefore so far removed from the Equator, the one on the one Side, and the other on the other, as is the Inclination of the Equator to the Ecliptic, *viz.* 23 Degrees and an half. They are called Tropics, because when the Sun arrives

rives at either of them, it returns back again towards the Equator.

That which lies in the Northern Hemisphere is called the Tropic of Cancer, ^{Tropic of Cancer, and Tropic of Capricorn.} because it touches the Ecliptic in the first Degree of that Sign. The other, the Tropic of Capricorn, for a like Reason.

The Polar Circles are those which the Poles of the Ecliptic seem to describe ^{The Polar Circles.} by the diurnal Motion of the Earth; and are therefore each of them at the same Distance from the Poles of the Earth, that the Tropics are from the Equator. That which lies in the Northern Hemisphere is called the Arctic Circle, from *Arctus*, the Bear, ^{Arctic and Antarctic Circles.} a Constellation near that Place; the other being in Position opposite to this, is called the Antarctic Circle. For the same Reason the North Pole of the Earth is sometimes called the *Arctic Pole*, ^{Arctic and Antarctic Poles.} the other the *Antarctic*.

By means of the Tropic and Polar Circles (or rather by certain imaginary Circles upon the Surface of the Earth corresponding to them, and called by the same Names) the Surface of the Earth is divided into five Parts, called *Zones*. The one of which is called the torrid Zone, and is that Space or Tract ^{Torrid Zone.}

Frigid
Zone.Tempe-
rate
Zones.The Am-
phiscii.

The Afcii.

The He-
terofcii.The Peri-
fcii.

of the Earth which is comprehended between the two Tropics. The Ancients imagined this Tract of the Earth to be uninhabitable, on Account of the Heat of the Sun there. There are two frigid Zones, the one is that Portion of the Earth's Surface which is included within the Arctic Circle, the other within the Antarctic. The remaining two are the temperate ones, lying one on one Side the Equator, and the other on the other, between the frigid and the torrid ones.

The Inhabitants of these Zones are distinguished by the different Direction of their Shadows. Thus, they that live in the torrid Zone are called Amphiscii; because their Noon Shadows are cast sometimes towards the North, and sometimes towards the South, according as the Sun at Noon is to the Northward or the Southward of them. But when the Sun passes directly over their Heads at Noon, they cast their Shadows neither the one way nor the other, and are called Afcii.

The Inhabitants of the temperate Zones are called Heterofcii, because they never cast their Meridian Shadow but one way. Such as inhabit the frigid Zones are called Perifcii, because,
by

by reason that the Sun is sometimes above their Horizon for a Day or more without setting, their Shadows turn quite round them.

The Inhabitants of the Earth have also been distinguished into three Sorts, with regard to their Situation with respect to each other. They who live under the opposite Points of the same Parallel of the Equator, are called with respect to each other, Pericæci. Pericæci.

These have the same Seasons of the Year; but it is Midnight to the one, when it is Noon or Mid-day to the other. Others are called Antœci, as Antœci. living under the same Semimeridian, but in opposite Parallels. These have Day and Night at the same Time, but different Seasons, it being Summer with one when it is Winter with the other. Others are Antipodes, with respect to Antipodes each other, that is, they live in opposite Parallels, and also in opposite Semimeridians; so that they are on the opposite Points of the Globe; and therefore have both their Days and Nights, and their Seasons of the Year reversed, the one with respect to the other.

III. Of the Horizon, and other Particulars relating to it.

The sensible Horizon.

The rational Horizon.

Zenith.
Nadir.Vertical
Circles, or
Azimuths.

The Horizon is a Circle in the Heavens, whose Plane touches the Surface of the Earth, in the Place of the Spectator. This Circle separates that Part of the Heavens which is visible from that which is not; and is called the sensible Horizon, in contradistinction to another Circle parallel to this, whose Plane passes through the Center of the Earth, and is called the rational Horizon. The Distance between these two Circles is so small with respect to that of the fixed Stars, that it is usually neglected, and the Circles are supposed to coincide.

The Poles of the Horizon are two Points, the one of which is exactly over the Head of the Spectator, and is called the Zenith; the other directly under his Feet, and is called the Nadir.

The Secondaries of the Horizon pass through the Zenith and Nadir, and are called Vertical Circles or Azimuths: Its Parallels, Almicantarath, or commonly Almicantars (c).

(c) These are Terms introduced into Astronomy by the ancient *Arabian* Philosophers, and still retained.

Among

Among the Vertical Circles of the Horizon there are two more remarkable ones, *viz.* the Meridian, and the prime Vertical, (for the Meridian of any Place is a Secondary of the Horizon of that Place, as well as of the Equator.) The Meridian, as observed above, passes through the Zenith and the Poles of the Equator. This cuts the Horizon in two Places, which are called the North and South Points thereof. The prime Vertical cuts the Meridian at right Angles in the Zenith and Nadir, and by intersecting the Horizon marks out the East and West Points; and therefore these two Circles cut the Horizon into four Quarters, each of which is supposed to be divided into eight, the whole therefore into thirty-two, which are called the Points of the Compass, of which the four first mentioned, *viz.* East, West, North, and South, are called, by way of Eminency, the Cardinal Points.

The Altitude of a Star is an Arch of a Vertical Circle that passes through it, intercepted between the Star and the Horizon. If the Star be below the Horizon, it is then called its Depression.

When a Star is upon the Meridian of any Place, it is said to culminate, or to be in its Meridian Altitude.

The

The Azi-
muth of a
Star.

The Azimuth of a Star is an Arch of the Horizon, intercepted between the North or South Point thereof, and the Vertical that passes through the Star; and is either Eastern or Western according to the Situation of the Star.

The Am-
plitude of
a Star.

The Amplitude of a Star is an Arch of the Horizon, intercepted between that Point where the Star rises or sets, and the East or West Point of the Horizon; and is either Northern or Southern. If the Amplitude be taken from the rising of the Star, it is then called its rising Amplitude; if from the Place where it sets, its setting Amplitude.

Rising and
setting
Ampli-
tude.

The Mid-
heaven.

That Point of the Ecliptic through which the Meridian of any Place passes, is called the Mid-heaven. And that Point of the Ecliptic which is distant a quarter of a Circle from the Points where it cuts the Horizon, is called the *Ninetieth Degree*.

Ninetieth
Degree.

The Ele-
vation of
the Pole.

The Height of the Pole above the Horizon is called its Elevation. And this is equal to the Latitude of the Place; for either of them being added to the Distance between the Zenith and the Pole, makes just a quarter of a Circle (d).

The

(d) From hence arises the Method of measuring the Cir-

The Inhabitants of the Earth are again distinguished by the Sphere they are said to live in. Thus, such as live under the Equator, have their Horizon passing through both the Poles of the Earth, and cutting the Equator and all its Parallels at right Angles, and from thence are said to live in a right Sphere. ^{Right Sphere,} The Property of which Sphere is this, that the Days and Nights are of an equal Length all the Year long, and the Stay which any of the heavenly Bodies make

Circumference of the Earth. For, let two Places be pitched upon, lying North and South of each other, such that the Elevation of the Pole in one shall exceed its Elevation in the other by one Degree suppose, then measure the Distance between the two Places in Miles: Then as one Degree is to the Circumference of a Circle, or 360 Degrees, so is that Number of Miles to the Circumference of the Earth. By this Means the Circumference of the Earth is found such, that the Length of one Degree therein is equal to 69 *English* Miles. This Measure was first found out by our Countryman *Norwood*, and afterwards attempted by the *French* Mathematicians, who twice applied themselves to it by the King's Command, and discovered almost the same Measure that *Norwood* had done before. They took the Length of a paved Causey of near seven Miles, that lies between *Jeu's Village* and *Invisum* in *Picardy*, by actual Mensuration; and then by a Survey from thence with the most accurate Instruments, they took the Distance between *Malciwin* and *Surdon*, which lie almost in a strait Line from North to South; in both which Places they took the Height of the Pole, and from thence computed the Circumference of the Globe. The Mathematicians set over this Work were *Monsieur Picard* the first time, and *Monsieur Cassini* the second.

above

above the Horizon is equal to that which they make below it. The Reason is, that all the Parallels of the Equator, in which the heavenly Bodies seem to perform their diurnal Revolution, are cut into two equal Parts by the Horizon.

Oblique
Sphere.

Such as live between the Equator, and either Pole, have their Horizon crossing the Equator and all its Parallels, less or more obliquely, as they live less or more distant from the Equator. Hence these are said to live in an oblique Sphere, and have all the Parallels of the Equator cut into unequal Parts by their Horizon, so that neither the Sun nor any of the heavenly Bodies, unless they be in the Equator, make an equal Stay above and below their Horizon.

Parallel
Sphere.

To those who live at the Poles of the Earth, if any such there be, the Equator is parallel to, or rather coincident with the Horizon, and all its Parallels are parallel to it; from whence this is called a Parallel Sphere. The Property of this Sphere is, that it is Day for half the Year together, and Night for the other half. For the Equator and Horizon being parallel, so long as the Sun continues on the same Side the Equator,

Chap. XIV. *of the Sphere.*

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so long must its Stay be on the same Side the Horizon, and consequently so long is it Day at one of the Poles, and Night at the other. And the fixed Stars do not rise or set in this Sphere, because they do not remove their Places from one Side of the Equator to the other, as the Sun and Planets are observed to do.

It is obvious, that in a right Sphere, the same Point of the Equator which comes to the Horizon with any Star, will also come to the Meridian with the same; because in this Sphere both the Horizon and the Meridian become Secondaries of the Equator. Now an Arch of the Equator intercepted between this Point and the first of Aries, is called the right Ascension of a Star. Right Ascension of a Star. But in an oblique Sphere it will not be so, but some other Point different from that which comes to the Meridian with it, will rise with it, because of the Obliquity of the Horizon to the Secondaries of the Equator. And the Distance between this Point and the first of Aries, is called the oblique Ascension of a Star. Oblique Ascension. And the Difference between its right and oblique Ascension is termed its Ascensional Difference. Ascensional Difference.

In

Circle of
perpetual
Appari-
tion.

Circle of
perpetual
Occulta-
tion.

Parallels
and Cli-
mates.

In an oblique Sphere there is a Parallel of the Equator, so far distant from the Pole as is its Elevation above the Horizon, the Stars that are comprehended within which seem to revolve about the Pole without ever rising or setting. This is called the Circle of perpetual Apparition. There is another opposite to this, and at the same Distance from the other Pole, called the Circle of perpetual Occultation; because the Stars that are included therein never appear above the Horizon.

The ancient Geographers were wont to distinguish the different Regions of the Earth by Parallels and Climates, in such manner, that in going from one Parallel to another, the Days at the Summer Solstice should be increased a quarter of an Hour. So that supposing the Equator to be the first Parallel, under which the Days are always 12 Hours long, the next should be where the longest Day of the Year should be 12 Hours and a Quarter long, &c. And two such Parallels made one Climate, till we come to the Polar Circles. And from thence to the Poles they reckoned the Climate such, that the Stay of the Sun above the Horizon was a whole Month

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Chap. XIV. *of the Sphere.*

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Month different in one from what it was in the next contiguous one.

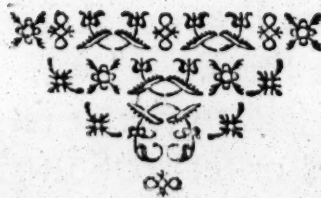
As the Year of the Ancients was not precisely adapted to the annual Motion of the Sun, as we shall see hereafter, it happened that the Seasons did not always fall on the same Part of their Year, so that it was not accurate to denote the Time of the Year by the Day of the Month. It was therefore usual with them to distinguish the Seasons by the Cosmical, Achronical, and Heliacal rising and setting of the Stars. And be-
The poetical rising and setting of the Stars.
cause we meet with these Distinctions most frequently among the Poets, they are called the Poetical Risings and Settings of the Stars.

A Star is said to rise or set cosmically, when it rises or sets at Sun-rising : And when it rises or sets at Sun-setting, it is said to rise or set achronically. A Star rises heliacally, when it emerges out of the Sun's Rays, and is got so far from it as to become visible : And it is said to set heliacally, when it first immerges into the Rays, and becomes invisible (e).

(e) *Kepler* will have it, that these Words are to be taken in another Sense, so as that to rise or set cosmically shall signify the same as to ascend above, or descend below the Horizon ; but that to rise or set achronically, is the same as to rise or set in the Sun's

Oppo-

Opposite, or in the other Achron, or Extreme of the Night: In which Sense *Ptolemy*, and to this Day most Astronomers say, a Planet is achronical, when it is opposite to the Sun, and shines all Night; so that to rise achronically is, as above, the same as to rise when the Sun is setting; but to set achronically, is to set when the Sun is rising, which is commonly called the Cosmical Setting.



A
COMPENDIOUS SYSTEM
OF
Natural Philosophy,
With NOTES
Containing the MATHEMATICAL
DEMONSTRATIONS, and some
occasional REMARKS.

PART IV. Continued.
CONTAINING THE
DESCRIPTION
OF THE
ORRERY *and* GLOBES,
The PRINCIPLES of
CHRONOLOGY;
Together with the Physical Cause of the Motion of the
HEAVENLY BODIES.

By J. ROWNING, M. A.
Rector of ANDERBY in *Lincolnshire*, and late
Fellow of MAGDALEN College in *Cambridge*.

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COMPTON'S SYSTEM

Natural Philosophy

OF THE

PHYSICAL

SCIENCE

OF THE

UNIVERSITY

OF

OXFORD

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C H A P. XV.

The Description and Use of the Orrery and the Globes.

I. **T**H E Orrery is a Machine designed to represent at one View the real Motions of the Heavenly Bodies. In order to this, in the middle of a large Circle designed to represent the Ecliptic, is fixed a Globe representing the Sun. Next to the Sun a small Ball representing *Mercury*. Next to this is *Venus* represented by a larger. At a greater Distance still from the Sun, is the Earth represented by an Ivory Ball. Round the Earth, at some Distance from it, is a Ring to express the Orbit of the Moon, making an Angle with the Circle that represents the Ecliptic, and thereby shewing the Inclination they have to each other in the Heavens, and also the Line of the Nodes. Within this Ring is a small Ivory Ball representing the Moon, and having a black Cap or Case, which always covers that Hemisphere which is turned from the Sun; and thereby distinguisheth the enlightned Part from the dark one. Beyond these is seen *Mars*, then *Jupiter* and its four Moons, and outermost of all is *Saturn* with its Ring and five Moons.

U

All

All these are fixed upon small Stems which severally represent their Axes, each of which has its peculiar and proper Inclination to the Plane of that Circle which we said was to represent the Ecliptic: And when the Machine is put in Motion, all these Bodies move round that which represents the Sun, and at the same time both that and all those which represent such of the Planets as have been observed to have a Rotation about their Axes, turn round upon the said Stems, and in their proper Times. The Moons also revolve about their Primaries at the same Time, and the Ring that represents the Orbit of the Moon has likewise its proper Motion, whereby that of its Nodes is also expressed.

These Bodies are all kept in Motion by Wheelwork conceal'd in a Case underneath, and turned by a small Winch in the side of the Case; one Turn answering to a Revolution of the Ball which represents the Earth, about its Axis; and consequently to 24 Hours of Time. So that 27 Turns causes the Sun to turn once about its Axis, 365 carries the Earth about the Sun, 27 the Moon about the Earth, &c. So that not only the Motion of the Heavenly Bodies may be represented, but their Situations with respect to each other may be shewn for any Time either past or to come; (a)

(a) This and the next Paragraph must be understood with some Limitation; Machines of this kind being sufficiently adapted to common Use, though not accurate enough for this purpose; none of the Machines of this kind which have yet been made, being so contrived as to shew this with Accuracy.

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by giving the Winch a proper Number of Turns backwards or forwards as the Case requires.

There is usually also a Lamp in a Case, with one or more convex Glasses in the side of it, to throw a strong Light directly forwards; this Lamp being put in the Place of the Sun, and that side of the Case which has the Glass in it directed towards the Earth, it being also contrived to turn according to the Motion of the Earth, a strong Light is thereby continually thrown upon it, and the Moon, in whatever part of its Orbit it is, and so not only the Times in which the Eclipses of the Sun and Moon, that is, properly of the Earth and Moon will happen, are shewn, but they themselves are duly represented. (a)

II. As

(a) Artificers generally erect upon the Ecliptic some Semicircles, to represent some of the principal Circles of the Heavens. But this is wrong, and tends to Confusion; because these Circles being only imaginary, and arising from the *apparent* Motions of the Heavenly Bodies, ought to have no Place in the Orrery: Which as it expresses their *real* Motions, serves to *explain*, yet is by no means intended to *represent* their *Phænomena*; the Phænomena and real Motions being in several Cases directly contrary to one another. To be particular, they have in some affixed the two Extremities of a Semicircle, which they call a moveable Horizon by two Hinges to the Ecliptic, at the first Degree of *Aries* and *Libra*; this is as if all Horizons necessarily passed through these Points, which is impossible, because there are as many Horizons as Points upon the Earth. So that the putting on this Circle is not only an Impropriety common to the rest, but is attended with a necessary Blunder. A learned Expositor of the Orrery says, 'when this Machine is set to any Latitude, (for they have a Contrivance for that too) set the moveable Horizon to the same Degree upon the Meridian, and you may

II. As the Orrery is a Machine proper to represent the real Motions of the Heavenly Bodies, so the Globes are adapted to their apparent Motions: And that I may give the Reader a just and adequate *Idea* of them, I shall premise the following Description of a Machine, which though imaginary, will greatly tend to the true Conception both of their Nature and Uses.

In Order to this, let us conceive a large hollow Sphere representing the inside of the Heavens, and on the inner Surface of this Sphere let the Sun, Moon and Stars, be supposed to be delineated in their proper Places; and let the Sphere be put into a Frame, and hang there in such Manner, as that it may be capable of turning about an Axis: And let it be so contrived, that either end of the Axis may be raised or lowered at Pleasure.

- form an Idea of the respective Altitudes, or Depressions of the
- Planets above or below the Horizon, according to their respective
- Positions, with regard to the Meridian.

Now the Idea which the Motion of the Orrery in this Position gives me, is that the whole Earth is one half of the Year above the Horizon, and the other half below it. For the Ball which represents it in the Orrery is manifestly so, while it performs its Revolution about the Sun.

Again, some Artists, in order to make the Orrery represent the several Phases of the Moon, make one half of the Ball that represents her black; in which case they are obliged to contrive it so, that the same side of the Moon shall not be always turned towards the Earth; which is contrary to her real Motion, or that the dark side shall sometimes be turned towards the Sun, which is absurd.

In

In the midst of this Sphere, let there be a small Ball to represent the Earth, but so fixed there, as not to turn round with the Sphere, and let a Spectator be supposed to be placed on the uppermost Point of this Ball; and to prevent the lower half of the Sphere from being visible to him, let a Plane, that is, a thin broad Substance, a Plate of Brass for Instance, be supposed fixed on that Point of the Ball where the Spectator stands, and to reach on all sides as far as the inner Surface of the Sphere: This will represent the Horizon of that Spectator; because it separates the upper half of the Sphere, which we are to suppose visible to this Spectator, from the lower, which we must suppose to be out of his Sight. Then, as is most obvious, by turning this Sphere once round, while the Ball, the Spectator and the aforesaid Plane remain unmoved, the rising and setting of the Sun, Moon and Stars, by their passing by the Edges of this Plane, will be duly represented: (a) And those Stars which do not go down so low as the Plane (as is evident if one End of the Axis of the Sphere be above it, some will not,) will be those which never set to a Spectator on that Part of the Earth, which corresponds with the

(a) The Reader is to suppose here, that in this case the Sphere is not hung in its Frame in such manner, that one Extremity of its Axis shall be directly over the Spectator; because in that case none of the Heavenly Bodies in turning the Sphere, will pass by the Edge of the Plane.

upper-

uppermost Point of the Ball; those which during the whole Revolution, do not rise so high as to come above it, will be those which never rise.

Again, if we suppose a Circle as large as the inside of the Sphere will admit of, to be placed within it, in such manner as to have the Axis of the Sphere passing through two opposite Points of it, and also to have one Point of it directly over the Head of the Spectator, and the opposite one directly under his Feet, and to remain unmoved while the Sphere turns: This will represent the Meridian of that Spectator; and so the Culminations of the Heavenly Bodies, that is, their passing the Meridian in the Heavens, will be exhibited by their passing by this Circle.

Farther, if we also imagine an Index like the Hand of a Clock, to be fixed upon one of the Extremities of the Axis of the Sphere, in such manner as to turn with the Sphere, and to point continually to a Circle fixed on the Frame, with the 24 Hours delineated upon it; and if this Index be set to point to the Hour 12, when the Sun's Place upon the Sphere for any given Day, is at the Circle that represents the Meridian, then by turning the Sphere this way or that, till the Place of the Sun or any of the Stars comes to the Plane abovementioned, the Index moving along with the Sphere, will point to the Hour at which they rise or set that Day: And if we observe to what
Hour

Hour the Index points, when any of them crosses the Meridian within the Sphere, the Time of their Culmination, or Southing, will be expressed: And so by this Machine all Problems relating to the rising, setting and southing of the Sun, Moon and Stars, may be solved. The Reason why the Index must be set to point to the Hour 12, when the Place of the Sun is brought to the Meridian, is that it may point to that Hour, which the real Sun indicates, when the Heavens stand in a Position corresponding with that of our imaginary Sphere.

But since by means of the annual Motion of the Earth round the Sun, the Sun continually shifts its apparent Place in the Heavens in such manner, as to seem to describe a great Circle once a Year; it is evident that its Place cannot be delineated on the inside of this Sphere as we have supposed, unless it be for a Day or a few Hours, during which Time its change of Place is so small that it may be neglected. In like manner the Planets are continually changing their apparent Places in the Heavens, by means of their Motions in their respective Orbits; neither therefore can their Places be assigned them, unless for a very small interval of Time. Let us then imagine both the Sun and the Planets to be represented by little Balls adhering to the inner Surface of the said Sphere; but let it be so contrived, that each Ball shall shift its Place with every Turn of the Sphere,
just

just so much, and in the same Direction that the Sun or other heavenly Body the Ball represents, appears to shift its Place in the Heavens in one Day: And let the Index shift with the Sun, so that it may always point to 12, when the Place of the Sun comes to the Meridian.

Then, as is most obvious, will our Machine be so adapted, that upon turning it round continually, the Change of the Time of the rising and setting of the Sun, its different Meridian Altitude, the vicissitude and length of Day and Night, the alteration of the Time of the rising and setting of the fixed Stars and of the Planets, for so many Days running, as is the Number of Turns given to the Globe, will all be exhibited to View: And consequently by turning the Sphere 365 Times round, the Succession of the Seasons will be represented throughout the Year with abundance more, or rather will all the remaining Phænomena of the Heavens.

This Machine, as hitherto described, serves only to exhibit the Phænomena of the Heavens as they appear on one Point of the Earth's Surface, *viz.* that which is represented by the uppermost Point of the Ball, or that where the Spectator was supposed to stand. But if we imagine the Plane which was put as a Boundary to the Spectator's Sight, to be applied to some other Point of the Ball and fixed there, then the Motion of the Sphere will exhibit the Phænomena, as they would appear to a Spectator

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Spectator on that Part of the Earth, and so for the rest; or which comes to the same thing, if while the Plane remains unmoved, one of the Poles of the Sphere be elevated above it as many Degrees as the corresponding Pole in the Heavens is above the Horizon of a Spectator, on any Part of the Earth; then will the Revolution of the Sphere represent the Phænomena as they appear to that Spectator. For it is the same Thing whether the Pole be lifted up from the Horizon, or the Horizon put down below the Pole. (a)

Now this Machine (except in some few Particulars, in which it vastly excels the celestial Globe) is exactly the same with it as to its Uses. Thus, the Globe turns upon an Axis, either End of which may be raised or depressed at Pleasure; and on one of them is an Index that turns with the Globe, and points to a fixed Circle that has the Hours upon it. There is no Ball in the middle, as we supposed in the Machine, but the Place of the Spectator is al-

(a) By means of an Orrery to represent the real Motions of the heavenly Bodies, and such a Machine as above described, to shew afterwards how the apparent Motions of those Bodies depend on, and are connected with their real ones, the Elements of Astronomy might be taught much better than by any thing that has as yet been contrived; or rather, if I may so say, the Science itself might thereby be exhibited to View. For the Orrery, though it admirably well illustrates some Things, particularly the Eclipses, and in general what relates to the Motions of the Planets about the Sun, yet it shews the Appearances or Phænomena arising from thence to a Spectator, supposed to be at rest upon the Surface of the Earth, but in a forced and unnatural Way.

ways supposed to be there; and instead of the Plane we supposed to touch that Part of the Ball on which the Spectator stands, and to be extended every way as far as the inner Surface of the Sphere, there is a broad wooden Circle on the outside of the Globe, (which is Part of the Frame of it) exactly where that Plane would pass, was it continued beyond the Surface of the Globe, and so answers the Uses of that Plane. In the next Place, instead of the Circle we supposed fixed within the Sphere and passing through the *Zenith* and *Nadir* of the Spectator, and through which the Axis of the Sphere passes, there is a brazen Circle on the outside of the Globe, and in which it swings. And instead of the Stars being on the inner Surface, they are here on the outer one, and the Constellations are delineated in their proper Forms. Then as to the Places of the Sun and Planets, Artificers are content to leave them quite out, and only draw a great Circle representing the Path of the Sun in its annual Course through the Heavens, *viz.* the Ecliptic, and distinguish it into 12 Parts by the Signs thereof, and each Sign into 30 less, which are the Degrees thereof. When therefore a Problem is to be determined by the Globe, requiring that the Sun should be delineated in its proper Place, you are to look out the Place of the Sun for the given Time in some astronomical Tables, Ephemeris or Almanack for that Purpose, or on the broad Surface of the
Horizon

Horizon of the Globe, where there is usually a Table shewing the Sun's Place in the Ecliptic for every Day of the Year, and then to mark its Place on the Line which we said represented the Path of the Sun upon the Globe. In like manner, if a Problem is to be solved requiring the Place of any of the Planets, the Place of the Planet for the Time given must be looked out in some Ephemeris or Almanack, of which there are many publish'd yearly by *Parker*, and others, for that Purpose, containing Tables expressing the Longitude and Latitude of each Planet for every Day of the Year; and marking the Place of the Planet upon the Globe, by sticking on a Patch, or the like, and setting the Index according to the Place of the Sun for the given Time, and turning the Globe about, the Index will shew the Time of that Planet's coming to either Side of the Horizon, to the Meridian, or to any Height above or below the Horizon, &c.

If then the Reader will form to himself a true Idea of the Machine I have described, and of the manner in which such a Machine is capable of representing the apparent Motions of the heavenly Bodies, he cannot fail of having a thorough Notion of the Nature of the celestial Globe; the main Difference between them being only this, *viz.* that what we have supposed in the one to be on the Inside, is either actually on the other on the Outside, or

to be fixed on, or to have its Place marked there, as Occasion requires. The uses both of the celestial and terrestrial Globe, as to particular Cases, will be further illustrated by the Instances at the End of this Chapter, after we have considered some other Particulars relating to each.

The Circles delineated on the celestial Globe are usually, besides the Ecliptic above mention'd, the Equinoctial Circle at an equal Distance from either Pole, and two Parallels of it, *viz.* one on either Side, to represent the apparent diurnal Motion of the Sun, when at its greatest Distance from the Equator, called the Tropics; and at the like Distance from either Pole, two more Parallels call'd the polar Circles. Both the Ecliptic and the Equinoctial are divided into Degrees, beginning at the Point that represents the vernal Equinox, which is one of those where they cross each other; the Equinoctial into 360, but the Ecliptic into 12 Signs, distinguished by their proper Characters, every one of which is subdivided into 30 Degrees (*a*). There are also six Secondaries of the Equator, and as many

(*a*) As to the Divisions of the Ecliptic, it is to be observed, that the Degrees belonging to each Sign are not to be found where the Constellation whose Name it bears is delineated, but where the Character peculiar to that Sign is placed. The Reason of which is, because the first Degree of *Aries*, from whence all the rest are counted, being where the Equinoctial cuts the Equator, is not in this Age in any Part of the Constellation that bears that Name, as was accounted for above.

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of the Ecliptic cutting it in the first Degree of each Sign; two of which are also Secondaries of the Equator, *viz.* the Equinoctial and Solstitial Colures. Further, the Globe is so hung in its Frame, that that Side only of the brazen Meridian which has the Degrees marked upon it, passes through the Axis of it, which Side only, in all Cases where that Meridian is considered, is therefore supposed to represent it. Upon the broad Surface of the wooden Horizon, besides the innermost Circle divided into its proper Degrees, there is a Table shewing what Degree of the Ecliptic the Sun is in for every Day of the Year, as was observed above; in order to which the 12 Signs of the Ecliptic, with the Degrees of each, and the 12 Calendar Months, are so placed over-against one another, both according to the *Julian* and *Gregorian* Account, that is, according to the old and new Stile, as that the Days of the one may duly answer to the corresponding Places of the Sun upon the other. There is also a Circle, representing the Winds or Points of the Compass.

There is also belonging to the Globe a *Quadrant of Altitude*. It is a long and narrow Plate of Brass, made so thin and plyant, that the whole Length of it may be applied close to the Surface of the Globe, and having the Degrees upon it, (a) serves to measure the Distan-

(a) That Edge, which has the Degrees nearest it, is called its *graduated Edge*, and is the only Part of it that is considered in the using it.

ces of Places on the Globes; or being fixed on to the uppermost Point of the Meridian of the celestial Globe, (for which Purpose it has a Screw at one End) and having the other End thrust in between the Horizon and the Globe, measures the Height of a Star, or other Heavenly Body above the Horizon, &c. From which latter Use it has its Name.

The terrestrial Globe differs from the celestial principally, in that instead of the Constellations, the several Countries of the Earth are delineated thereon; and the Circles on this Globe are usually the Ecliptic, and the Equator, on which are numbered the Degrees of Longitude, (a) the two Tropics, the two polar Circles, Secondaries of the Equator cutting it at every tenth Degree, and Parallels also of it at every tenth Degree on each Side. There are also drawn upon it Rhumb Lines, or those which a Ship, keeping the same Course, that is, steering continually in the same Point of the Compass, describes upon the Surface of the Sea. The Property of a Line of this kind, is that it cuts all the Meridians it passes through, under the same Angle, and therefore is a kind of Spiral continually approaching towards some Point. This is called a *Loxodromic Curve*.

(a) On some Globes, the Degrees of Longitude are number'd both ways from the first Meridian, till you come to the opposite Point, and the Longitude is accordingly distinguished into East and West.

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The Lines of Variation described upon the terrestrial Globe, shew the Direction of the magnetic Needle, in those Places in which it does not point directly towards the North. For it is observed, that the Needle in some Parts of the World, declines from the North to the East, and in others to the West.

Further, on the Pedestal of some Globes there is a Mariner's Compass fixed, by the help of which, the brazen Meridian of the Globe may be set North and South. (*a*)

After this Description of the Globes, and general Idea of the Use thereof, it may now be proper to exemplify the latter by the following Instances.

And first of the celestial Globe.

Problem I. *To rectify the Globe to the Latitude of any Place upon the Surface of the Earth.*

If the Place be in the Northern Hemisphere of the Earth, slide the Meridian within the Notches of the Horizon, till there be as many

(*a*) The Armillary Sphere comes the nearest of any we have to that imaginary Machine described above, for it is hollow, and has a Ball in the middle to represent the Earth; but then in all other Respects it is like a Globe, except that the Surface is cut away every where, (unless where a Circle is described) that you may see the Ball in the middle. There is one Circle left broad enough to represent the Zodiac, on which the Places of the Sun and Planets may be marked, as Occasion requires.

Degrees

Degrees of the Meridian between the North Pole and the Horizon, as are equal to the Latitude of the Place (*a*). If the Place be in the Southern Hemisphere, the Southern Pole must be raised to a like Height above the Horizon.

PROB. II. *To find what Stars never rise or never set in any Place or Latitude given.*

The Globe being rectified to the given Latitude of the Place by Problem the first, those Stars which pass not under the wooden Horizon of the Globe, during its whole Revolution, never set to a Spectator in that Latitude: And such as do not come up above it, never ascend above that of the Spectator, and so never are seen to rise.

(*a*) The Reason of this is, because to a Spectator, upon any Part of the Earth, the Elevation of the Pole is equal to the Latitude of the Place; and the Globe is said to be *rectified* by this Operation, because, by this means, that Point of the little Ball, (for such we must always suppose to be in the middle of the Globe, to represent the Earth) that corresponds to the Place of the Spectator upon the Earth, is brought to the uppermost Point thereof; and so the Horizon thereof becomes coincident with the Plane of the wooden Horizon that surrounds the Globe, and the Poles of the Globe having the like Situation with respect to the uppermost Point of the Ball, that the Poles of the World have with respect to the Place of the Spectator; it is evident that the Revolution of the Globe is fitly disposed to exhibit that of the Heavens, as it would appear to that Spectator.

PROB.

If the Day given be according to the *Julian* Account, or old Stile, then in the wooden Horizon of the Globe, in the *Julian* Kalendar, otherwise in the *Gregorian*, find the Day given, over-against which in a Circle, which contains the several Signs and Degrees of the Ecliptic, is that Degree in which the Sun is in that Day. Though it may be found more accurately by an Ephemeris calculated for that Year. Then in the Ecliptic, delineated on the Surface of the Globe, look for the same Degree, and that is the Place of the Sun upon the Globe for that Day.

Having rectified the Globe according to the Latitude, (by *Prob. 1.*) and found the Sun's Place in the Ecliptic (by *Prob. 3.*) bring the latter to the graduated Edge of the brazen Meridian, and holding the Globe in that Position,

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set the Hour Index to 12 at Noon; (a) then turn the Globe till the Sun's Place in the Ecliptic comes to the eastern Part of the Horizon, and the Index will point to the Hour at which the Sun rises; and if you turn the Globe till the Sun's Place cuts the western Side of the Horizon, the Index will shew the Time of its setting.

PROB. V. *To find the Hour of the Day, by the Height of the Sun.*

First, by the Help of a Quadrant or some other Instrument for that Purpose, observe how many Degrees the Sun is elevated above the Horizon. Then the Globe being rectified to the Latitude of the Place, and the Sun's Place in the Ecliptic found, and the Hour Index duly placed, that is, so as to point to 12 at Noon, when the Sun's Place is under the Meridian; and screwing the Quadrant of Altitude on to the Meridian at the Zenith, (*i. e.*) at so many Degrees from the Equator, as is the Latitude of the Place. Turn the Quadrant of Altitude and the Globe backwards and for-

(a) Because when the Place of the Sun is in the Meridian, the Globe represents the Position of the Heavens, in respect of the given Place, as they are at Noon, or 12 o' Clock of the Day given. And consequently, if the Hour Index be put to 12, when the Globe is in this Position, as it turns with the Globe, it will point to the Hour that corresponds with any other Position of the Globe that Day.

wards,

wards, till the Sun's Place in the Ecliptic lies under that Degree on the graduated Edge of the Quadrant of Altitude, which answers to the Sun's Height found as above, then will the Hour Index point out the Time of the Day. (a)

PROB. VI. *To find the Hour of the Night, by the Height of a Star.*

The Height of the Star being found by an Instrument, the Globe rectified, and the Quadrant of Altitude fixed, as in the foregoing Problem, move the Globe and the Quadrant, till the Star comes under its graduated Edge, at that Degree which expresses the Height of the Star above the Horizon, then will the Hour Index point to the Hour of the Night.

PROB. VII. *To find the Place of a Star or Planet upon the Globe, its Longitude and Latitude being given.*

Lay the first Degree of the Quadrant of Altitude, upon that Degree of the Ecliptic, which expresses the given Longitude, and the ninetieth Degree thereof on the Pole of the

(a) Because when the Sun's Place is brought to this Point of the Globe, it is so many Degrees above the Horizon, as the Sun itself is above that of the Spectator.

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Ecliptic

Ecliptic, (that is, where its Secondaries cross each other upon the Globe) either Northern or Southern, as the given Latitude is North or South: Then look upon the Quadrant for the Degree of Latitude given, and the Point of the Globe which is under that, is the Place of the Star or Planet sought. (a)

P R O B. VIII. *To represent the Appearance of the Heavens at any Time of the Night, that is, to shew the Situation of the fixed Stars at that Time.*

Rectify the Globe, and set the Hour Index to the Sun's Place in the Ecliptic, and turn the Globe till the Index points to the Hour of the Night. (b)

Prob.

(a) The Reason of this is, because the Quadrant of Altitude in this Situation, represents a Secondary of the Ecliptic, an Arch of which intersected between the Star and the Ecliptic is its Latitude; and an Arch of the Ecliptic between the Point where it is intersected by a Secondary passing through the Star, and the first Degree of *Aries*, is its Longitude.

(b) For then the Situation of the Stars upon the Globe, with respect to the Horizon and Meridian, will be similar to that of the Stars in the Heavens; *viz.* those which appear on the Eastern Side the Globe in the Horizon, will then be rising, and those in the Horizon at the other Side will be setting, those which are under the Meridian, will then be culminating, &c.

And if by means of a Compass upon the Pedestal of the Globe, you set the brazen Meridian North and South, so that the South

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P R O B. IX. *The Day of the Year and the Latitude of the Place being given, to find the beginning of the Morning, and End of the Evening Twilight.*

The Globe being rectified to the Latitude of the Place, and the Hour Index set to the Place of the Sun in the Ecliptic, elevate that Degree of the Ecliptic which is diametrically opposite to the Sun's Place, 18 Degrees above the West Side of the Horizon, (*a*) and the Hour Index will shew the Time of Day-break

Part of it shall respect the South Part of the Heavens, the Situation of the Stars upon the Globe, with respect to the Points of the Compass, will also be similar to that of the Stars in the Heavens at that Time; so that by comparing the Situation of the Stars upon the Globe, with those in the Heavens, you may easily make yourself acquainted with all the Stars, that are visible in this Part of the World.

If the Situation of the Moon or the other Planets is also to be represented, their Places in the Zodiac for that Time must be assigned them by Problem the last, their Longitude and Latitude being first found by an Ephemeris for that Purpose.

(*a*) Because then the Place of the Sun will be so many Degrees below on the other Side, in which Situation of the Sun, we observed above, that the Twilight begins. The Quadrant of Altitude will shew when the Point opposite to that of the Sun is 18 Degrees above the Horizon. If this be tried for any Time while the Sun is passing from about the 5th Degree of *Gemini*, to the 20th of *Cancer*, that is, from about the 15th of *May*, to about the 7th of *July*, it will be found that there is no Beginning or End of Twilight, the Sun never being 18 Degrees below the Horizon (in the Latitude of *London*) during that Time.

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or the Beginning of Morning Twilight. And if the same Point of the Ecliptic be raised 18 Degrees above the East Side of the Horizon, the Index will point out the Time when the Evening Twilight ends.

P R O B. X. *To account for the Phænomena of the Harvest Moon.*

About the autumnal Equinox, when the Moon is at or near the *Full*, it rises almost at the same Hour for several Nights together; which Phænomenon is called the *Harvest Moon*. (a) In order therefore to shew by the Globe how this comes about, let us suppose the Place of the Sun to be in the first Degree of *Libra*, as it always is at that Equinox, and that of the Moon to be in the Point opposite to the Sun, or the first Degree of *Aries*, because the Moon must be considered as being at the *Full*. In this Case it is obvious that the Moon will rise when the Sun sets, which will be at Six of the Clock; because both Luminaries are in the Equator and at opposite Points. Now since the Moon performs an entire Revolution in its Orbit, (which we will at present consider as coincident with the Ecliptic) in the Space of one Month, it therefore advances forwards 12 Degrees each Day, and consequently

(a) Mr. *Johnson* calls this Phænomena *Luna autumnalis*, and is the only Author that I know of that has accounted for it. *Quæstiones Philosoph.* Edit. 2da. Page 124.

the next Night it will be in the 12th Degree of *Aries*. Let us then enquire at what Time the Moon will rise this Night. To this End, rectify the Globe to the Latitude of the Place, bring the Sun's Place, which is now the second Degree of *Libra*, to the brazen Meridian, and put the Hour Index to 12; and if the Globe be turned till the Place of the Moon, *viz.* the 12th Degree of *Aries*, cuts the eastern Side of the Horizon, we shall find the Hour Index will point nearly to the Hour of Six, which, as observed above, was the time of the Moon's rising the Evening before. And thus if we investigate the Time of the Moon's rising for several Nights together, both before and after the Full, it will be found much the same in all.

The Reason of this is, that about the Time of the full Moons, which happen at this Part of the Year, the Moon being in the ascending Signs of the *Zodiac*, appears in its nocturnal Course to describe a Parallel of the Equator one Night, much nearer the North Pole than another, and so rises every Night more and more to the Northern Parts of the Horizon, and that considerably, as passing quite from the Southern to the Northern Tropic in a Fortnight's Time. Now it is easy to conceive that the nearer any of the Heavenly Bodies is to the North Pole, the sooner that Body (*cæteris paribus*) ascends the Horizon: Thus, if the Body be within a little of the *Circle of perpetual Apparition*, that Body is no sooner set but it rises again. In every Lunation there-fore

fore, while the Moon is passing from the Southern to the Northern Tropic, some Minutes are on that Account to be deducted from those by which it would rise later and later each Night, did the Plane of its Orbit coincide with that of the Equinoctial; and as much is to be added while she passes from the Northern to the Southern Tropic. And for the like Reason in the former Case something is to be added to the Time it would set at; and in the latter a like Quantity is to be deducted.

Now about the time of the autumnal *Æ*-quinox, the Sun being then in *Libra* or thereabouts, the Moon is in the midst of the ascending Signs, when at the *Full*. Hence it is that the *full* Moons at that Time of the Year, rise so little later the one than the other.

In like manner, if any one thinks it worth while to observe it, he will find that the *new* Moons in the Spring, rise nearly at the same Hour for several Nights successively, while the full Moons shall rise one later than another, by a greater Difference than at any other time of the Year. The Reason is, because at the time of the new Moons, which happen at that Part of the Year, the Moon is in the ascending Signs, but at the full, in the descending ones. (a)

(a) It is here to be observed, that this Phænomenon is not the same as to Degree, in different Years; because as the lunar Orbit is inclined to the Ecliptic 5 Degrees, and the Line of its Nodes is continually revolving round, the Inclination of that Orbit to the Equator will be sometimes 10 Degrees greater than at others. So that the Moon does not hasten to the North, or descend to the South in each Revolution with equal Pace.

Prob.

PROB. XI. *To determine the Time of the Year in which a Star rises or sets Cosmically, or Acronically.*

Having rectified the Globe to the Latitude of the Place, bring the Star to the Horizon on the Eastern Side of the Globe, and observe what Degree of the Ecliptic rises with it. Look for that Degree in the wooden Horizon, and right against it in the Kalendar, you will find the Month and Day when that Star rises cosmically (*a*). And if you bring the Star to the Western Side of the Horizon, the Degree of the Ecliptic which cuts the Eastern Side of the Horizon in that Situation of the Globe, will give the Day of the Month when the said Star sets cosmically (*b*). So likewise against the Degree which sets with the Star, you will find the Day of the Month of its *Acronical* setting; and if you bring it to the Eastern Part of the Horizon, the Degree of the Ecliptic which then cuts the Western Part of the Horizon will be the Sun's Place when the Star rises *Acronically*.

(*a*) For that will be the Day when the Sun and the Star rise at the same time, which is its cosmical Rising.

(*b*) The cosmical setting of a Star being its setting at Sun-rise.

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PROB.

PROB. XII. *To find the Time of the Heliacal rising and setting of a Star.*

Having rectified the Globe to the Latitude of the Place, bring the Star to the Eastern Side of the Horizon, and apply the Quadrant of Altitude to the Western Side in such manner that its twelfth Degree may cut the Ecliptic, if the Star be of the first Magnitude (*a*). Then will the Point of the Ecliptic opposite to that which is cut by the Quadrant, be 12 Degrees below the Horizon. Look for this Degree on the wooden Horizon, and over-against it you will have the Day of the Year, when the Star rises Heliacally. To find the Heliacal setting, bring the Star to the Western Side of the Horizon, and turn the Quadrant of Altitude about to the Eastern Side, till the 12th Degree of it cuts the Ecliptic; then that Degree of the Ecliptic which is opposite to this Point, is the Sun's Place at that Time of the Year when the Star sets Heliacally.

(*a*) Because a Star of the first Magnitude being above the Horizon, may be seen when the Sun is 12 Degrees below it. If the Star be of the 2d Magnitude, that Degree of the Ecliptic must be taken, which is 13 Degrees below the Horizon when the Star rises, if of the 3d, then the 14th, &c.

PROB.

PROB. XIII. *The Day of the Month, the Time of the Day, and the Latitude of the Place being given, to find thereby the Hour of the Day, according to the Babylonian, Italic, and the Judaical Way of reckoning.*

1. To find the *Babylonian* Hour (which is reckoned from the Time of the Rising of the Sun) having found the Time of Sun's rising in the given Latitude, the Number of Hours between that and the Time given, is the Hour of the Day according to the *Babylonian* Way of Reckoning.

2. To find the *Italic* Hour (which is reckoned from Sun setting,) first find the Time at which the Sun sets on the given Day. The Time elapsed from that to the Time given, will give the *Italic* Hour sought.

3. To find the Hour according to the *Jewish* Way of Reckoning (one of which Hours is a twelfth Part of the Time that the Sun continues above the Horizon, and they are reckoned from Sun rising,) first find the Hour the Sun sets at, this being doubled will give the Number of Hours the Sun continues above the Horizon on the Day given. Then as that Number of Hours is to 12, so is the Number

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of

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of Hours since Sun rising, to the *Judaical*
Hour required.

Thus in the Latitude of *London*, when the
Sun rises at 4 of the Clock and sets at 8; if the
Hour given be 5 in the Evening, the *Babylon-
nic* Hour will be the 13th, the *Italic* the
21st, and the *Jewish* Hour will be Nine and
three Quarters.

PROB. XIV. *To find the Place of any
of the Heavenly Bodies upon the
Globe, from their right Ascension
and Declination.*

Bring that Point of the Equator which ex-
presses the right Ascension given, to the brazen
Meridian, and look upon the Meridian towards
the North or South Pole, according as the De-
clination is North or South, for the Degree of
Declination given, under which is the Place
sought (a).

(a) For in this Case the Meridian is made to represent a Se-
condary of the Equator passing through the Place, an Arch of
which lying between the Place and the Equator is the Declina-
tion, and the Distance between the Place of the Equator through
which it passes, and the first Degree of *Aries*, is the right Ascen-
sion.

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Of the Terrestrial Globe.

PROB. XV. *From the Longitude and Latitude of a Place, upon the Surface of the Earth being given, to find the Place upon the Globe.*

This Problem is much the same with the last, only instead of the celestial, we use the terrestrial Globe; and instead of reckoning the Longitude from the first Degree of *Aries*, the Point from which right Ascension is reckoned, it must be counted from the first Meridian.

Bring the Degree of the Equator, which numbers that of the Longitude given, to the brazen Meridian, upon which count towards the North or South Pole, according as the Latitude is North or South, till you come to the Number of the Latitude, under which is the Place sought. (a)

(a) In some Globes, the Degrees of Longitude are numbered both Ways from the first Meridian, till they meet in the opposite Point, and the Longitude is distinguished into East and West as observed above: In such Case it must be known which Kind the given Longitude is of.

PROB.

PROB. XVI. *To determine the Difference of Time in different Places.*

Find the Longitude of each Place, by the Converse of the last Problem, and reduce the Difference into Time; which is done by allowing an Hour for every 15 Degrees, and so proportionably for lesser Parts, the Number of Hours, &c. thus found, will be the Difference between the Time of the Day at one Place, and that at the other (a).

PROB. XVII. *To find the Distance between two given Places, and the Angle of Position, that is, the Bearings of them, or the Situation of one from the other, with respect to the Points of the Compass.*

Rectify the Globe to the Latitude of one of the Places, and bring that Place to the brazen Meridian; then fix the Quadrant of Altitude

(a) The Reason of this is, because as the Earth by turning round its Axis, brings all Places to the Sun once in 24 Hours; that Place which lies 15 Degrees to the Westward of another, comes to it an Hour later than the other; and so the Inhabitants of this Place are an Hour later in their reckoning of Time than at the other Place. And the contrary holds of such as live towards the East.

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to the uppermost Point of the Meridian, and putting the lower End of it between the Horizon and the Globe, slide it about till it passes through the other Place, and the Number of Degrees upon the Quadrant between Place and Place, turned into Miles by reckoning 60 to each Degree, will give the Distance between them; and the Number of Degrees upon the Horizon, between the Meridian and the Foot of the Quadrant, will give the Bearing of the Second, or its Situation from the first, with respect to the Points of the Compass.

P R O B. XVIII. *To find a Meridian Line, partly by the Help of the celestial Globe.*

Set the Hour Index to the Place of the Sun in the Ecliptic, and turn the Globe till the Pole Star, that is, the Star which is the nearest the Axis of the Globe, comes under the Meridian, either above or below the Pole; above, if you find that the Index in that Situation of the Star, points to one of the Hours of the Night, otherwise below; and mark what Hour the Index points to. Then at that Hour of the Night hang up two Plummets, letting their Weights hang in Water to prevent their vibrating, in such manner that the String of one of them may be directly between the Pole Star and the String of the other. Then will a Line
drawn

drawn from one String to the the other, or the Shadow of the one when it falls upon the other, be a Meridian Line. And this I take to be a much more accurate, as well as an easier Method, than the common one of fixing up a Pin upon a Plane, drawing a Circle about the Foot of the Pin, and observing where the Extremity of the Shadow of that Pin enters the Circle in the Morning and goes out of it in the Afternoon, and drawing a Line from the Foot of the Pin through the middle Point between those Places for a Meridian Line. For this latter Method requires more Care and Trouble to do it accurately, than they who have not tryed it, would imagine.

C H A P. XVI.

Of the Equation of Time.

WHereas Time constantly passes on with an uniform and regular Flux, it is evident there is no possible Way accurately to measure and compare the several Intervals of it one with another, but by the Motion of some Body whose Progress is as uniform and regular as itself. To this End it was natural to pitch upon the Sun, whose Motion is obvious to all, and seemingly regular enough for that Purpose. But by the Circumspection and accurate Observations

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servations of Astronomers, it is discovered not to be so; and that neither the Days, nor even the Hours themselves, as measured by the Motion of the Sun, are of an equal Length. And this upon two Accounts, as will appear from the following Considerations.

I. A natural Day, or 24 Hours, is that interval of Time, in which the Sun seems to pass from the Meridian of any Place to the same again: Had therefore the Earth no Motion but about its own Axis, and consequently the Sun no apparent one along the Ecliptic, all the Days would be of an equal Length, as depending only on the Motion of the Earth about its Axis, which is uniform: But since the Earth is constantly moving in its Orbit, the same Way it turns about its Axis, the Sun is always in appearance advancing forwards. When the Earth therefore is turned round, and a Spectator upon its Surface brought again to the Place he was at the foregoing Day at Noon, it is not yet Noon with him, the Sun in appearance having advanced to the Eastward in the mean time; so that the Spectator must be carried farther still, *viz.* so many Degrees as the Sun has advanced that Way, before he can be brought to the Meridian the Sun is in (*a*). Now by Reason of the unequable and

(*a*) This is the Reason that the Length of the Day is not exactly equal to the Time, in which the Earth performs its Rotation about its Axis, which is, as was observed in Note (*f*), Chap. I. but 23 Hours 56 Minutes and 4 Seconds.

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irregular Motion of the Earth in its Orbit, or, the irregular apparent Motion of the Sun along the Ecliptic depending thereon, the Number of Degrees the Sun advances Eastwards each Day, is sometimes greater and sometimes less; and consequently, upon this Account, the Days must differ one from another in Length; and therefore the Hours also, since each Day is supposed to consist of 24, be it long or short.

II. Was the apparent Motion of the Sun through the Ecliptic uniform and regular, yet as the Ecliptic is oblique to the Equator, and therefore equal Portions of the one do not correspond to equal Portions of the other; the Motion of the Sun, when referred to the Equator, would not be uniform; or, which is the same Thing, those Points of the Equator which come to the Meridian with the Place of the Sun on different Days, would not be at equal Distances one from another; (*a*) which as to the Time of the Sun's appearing in the Meridian, or its shewing the Hour of the Day upon a Dial, is the same as if it had an irregular Motion along the Equator; and therefore must cause it to render the Days unequal among themselves.

As these Causes are independent of each other, they sometimes conspire together; sometimes they are contrary the one to the other.

(*a*) This is easily tried upon a Globe, where if we bring every tenth Degree of the Ecliptic to the brazen Meridian, we shall find that each tenth Degree of the Equator, will not come thither with it.

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The Time which an uniform Motion would mark out, is called *true Time*; and that which is indicated by the Sun, is *apparent Time*; and the Difference between these is termed *the Equation of Time*. When the abovementioned Causes have continued for some time, to render the apparent Motion of the Sun too quick or too slow, this Difference is considerable. About the 13th of *October*, it is the greatest of all; at which Time the Sun is 16 Minutes and 11 Seconds too fast.

C H A P. XVII.

Of the Division of Time.

THE several Parts of Time are principally distinguished by Days, Hours, Weeks, Months, Years, Cycles, Periods and *Æras*. Of which in their Order.

By a *Day*, according to the most natural and obvious Sense of the Word, is meant that Space of Time during which it continues to be *Light*, and thus is opposed to that wherein it is *Dark*, which is called the *Night*. But this Space of Time being somewhat vague and indeterminate, the Time between the Sun's Rising and Setting is usually looked upon as the *Day*, and the Time which lapses from its Setting to its Rising again, the *Night*.

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But the Word Day is frequently taken in a larger Sense, so as to include also the Night, and to denote the Time of an whole apparent Revolution of the Sun round the Earth. In this Sense it is called by some a natural Day, and by others an artificial one; but what some call the Artificial, others call the Natural. To avoid which Confusion, it is usual to call it in the former Sense, simply the Day, and in the latter a *Nuchthemeron*; by which Term that Acceptation of it is aptly denoted, as being a Word that implies both Day and Night (a).

The *Nuchthemeron* is divided into 24 Parts, called Hours, which are of two Sorts, *Equal*, and *Unequal* or *Temporary*. The common Division of the equal Hour is into Halves and Quarters: But Astronomers, and those who are more accurate in their Account of Time, divide the Hour into sixty Parts, called Minutes, and these again into sixty Parts called Seconds, &c. And here we may observe, that the Word Minute is taken in a double Sense, either to denote the sixtieth Part of an Hour, and is therefore by Way of Distinction sometimes called an *Horary* Minute; or else to signify the sixtieth Part of a Degree, in which Case it may be called a *graduary* Minute, or Minute of a Degree.

Different Nations and People begin their Day at a different Hour. Thus the *Egyptians*

(a) A Greek Compound of νύξ Night, and ήμέρα Day.

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began their Day at Midnight; from whom *Hipparchus* introduced that Way of reckoning into Astronomy, and *Copernicus* and others have followed him. But the greatest Part of Astronomers reckon the Day to begin at Noon, and so count 24 Hours till the Noon of the next Day, and not twice twelve, according to the vulgar Computation.

The Method also of beginning the Day at Midnight prevails in *Great-Britain*, *France*, *Spain*, and most Parts of *Europe*.

The *Babylonians* began their Day at Sun-rising, reckoning the Hour immediately before its Rising again, the 24th Hour of the Day. From whence the Hours reckoned in this Manner are called the *Babylonic*.

In several Parts of *Germany*, they begin their Day at Sun-setting, and reckon on till it sets the next Day, calling that the 24th Hour. These are usually called the *Italian Hours*.

The *Jews* also began their *Nuchthemeron* at Sun-setting, but then they divide it into twice twelve Hours as we do, reckoning 12 for the Day, be it long or short, and 12 for the Night (a). So that their Hours continually

(a) According to this Way of reckoning, the Hours of the Day mentioned in Scripture are to be understood. The *Jews* also divided the Night into four Quarters, called *Watches*, each Watch containing three of their Night Hours. And distinguished them sometimes according to the Order of their Succession. Thus we find in Scripture Mention of the *second*, *third* and *fourth* Watch, sometimes the first Watch was called the *beginning of the Watches*; the second, the *middle Watch*, as not being ended till Midnight; and the fourth, the *Morning Watch*. The first also was sometimes termed the *Evening*; the second *Midnight*; the third, the *Cock-crowing*; and the fourth, the *Dawning of the Day*.

varying

varying with the Length of the Day and Night, the Hours of the Day were longer than those of the Night one half of the Year, and the contrary the other. From whence their Hours are called *Temporary*: These at the Time of the Equinoxes become equal, because the Days and Nights are so then. The *Romans* also reckoned their Hours after this Manner, as do the *Turks* at this Day (a).

A Week consists of the seven Days in most Countries, called after the Names of the Planets as above. But because Easter Week

(a) This kind of Hours are also called *Planetary*, because the seven Planets were anciently looked upon as presiding over the Affairs of the World, and to take it by turns each of these Hours, according to the following Order, *Saturn* first, then *Jupiter*, *Mars*, the *Sun*, *Venus*, *Mercury*, and last of all the *Moon*. Hence they denominated each Day of the Week from that Planet whose Turn it was to *reign* or preside the first Hour of the *Nuchthemeron*. Thus assigning the first Hour of *Saturday* to *Saturn*, the second will fall to *Jupiter*, the third to *Mars*, and so the twenty-second of the same *Nuchthemeron* will fall to *Saturn* again, and therefore the twenty-third to *Jupiter*, and the last to *Mars*. So that on the first Hour of the next Day, it will fall to the *Sun* to preside. And by the like Manner of reckoning the first Hour of the next will fall to the *Moon*, of the next to *Mars*, of the next to *Mercury*, of the next to *Jupiter*, of the next to *Venus*. Hence the Days of the Week came to be distinguished by the Latin Names of *Dies Saturni*, *Solis*, *Luna*, *Martis*, *Mercurii*, *Jovis*, and *Veneris*; and among us by the Names of *Saturday*, *Sunday*, *Monday*, *Tuesday*, *Wednesday*, *Thursday* and *Friday*. For, as *Saturday*, *Sunday* and *Monday* plainly denote the Day of *Saturn*, the *Sun* and the *Moon*; so *Tuesday*, *Wednesday*, *Thursday* and *Friday*, denote the Day of *Tuifco*, *Woden*, *Thor* and *Friga*, which are the *Saxon* Names respectively answering to *Mars*, *Mercury*, *Jupiter* and *Venus*. *Well's Chronol. Chap. II.*

was

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was formerly esteemed the first or principal Week of the Year, and each Day thereof was a *Feria* or Holyday; hence the several Days were distinguished among the primitive Christians, by the Names of *Feria prima*, *secunda*, &c. that is, the first, second Holyday, &c. But the Sunday, or *Feria prima*, was otherwise stiled by them the *Lord's Day*, as being the Day of our Lord's Resurrection (a).

Months are distinguished principally into two Sorts; *viz.* *Solar* and *Lunar*. The Solar Month is either *Astronomical* or *Civil* (b). The Lunar Month is divided into *Periodical*, *Synodical* and *Civil*. The *Astronomical* Solar Month is the Time the Sun takes up in passing through one of the Signs of the Ecliptic.

The *Periodical* and *Synodical* lunar Months have already been observed; the former to be equal to the Revolution of the Moon about the Earth, *viz.* 27 Days 7 Hours and 43 Minutes; the latter, the Interval of Time that lapses between two succeeding Conjunctions of the Moon with the Sun, *viz.* 29 Days, 12 Hours, 44 Minutes and 3 Seconds.

(a) The Hebrew Word which in the old Testament is rendered *Week*, signifying a Collection of seven, is there sometimes used for seven Years; and in Conformity thereto, our *English* Word *Week* in those Places must be understood to imply not a *Week of Days*, but a *Week*, or *System of seven Years*.

(b) A Month or Year is denominated *Civil*. from its being of common Use in any Republic or Society of People.

Now

Now because in common Use it would be inconvenient to have the Months to consist of odd Pieces of Days; it was usual among the *Jews, Grecks and Romans*, (till *Julius Caesar* made an Alteration) in order to adapt their Months to the apparent Motion of the Sun, to have their Months consist of 29 and 30 Days alternately, one of these Numbers being about half a Day above, and the other about half a Day less than the Synodical Month; so that by this Means the same Appearances of the Moon would happen nearly on the same Day of the Month for a long Time together. This Month thus adapted to the last mentioned lunar Month, is called the *Civil* lunar Month; and is still made use of by the *Turks*.

The *Egyptian* Months consisted of 30 Days each, 12 of these make a Year wanting 5 Days, which they added to the End of the Year, and from thence called them *Epagomenæ*, or Supplemental.

Now because 12 civil lunar Months want 11 Days of a Year, it is evident that the several Seasons of the Year must in Time fall upon different Months; to prevent which, *Julius Caesar* ordained, that the Month should not be taken from the Motion of the Moon, but from that of the Sun; that they should consist alternately of 30 and 31 Days, and that *February* should contain 29, and every fourth Year 30. But it happened afterwards that the Name of the sixth Month, which was formerly *Sextilis*,

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Sextilis, was in Honour of the Emperor *Augustus* changed for that of his, as that of the foregoing Month *Quintilis* had before been, in Honour of his Father *Julius*; and therefore since this Month consisted but of thirty Days according to the Institution of *Julius Caesar*, one more was added to it; to make it equal to the foregoing one, that the Honour paid to the Son might not seem to fall short of that which was paid to the Father. Upon this, the next Month, *viz. September*, was reckoned but 30, the next 31, and so on; that the alternate Order abovementioned might not be destroyed, till *January*, which, according to that Order, ought to have had but 30. But this Month being so named in Honour of *Janus*, the God of Time, it was thought improper to rob the God of a Day, wherefore *February* was reduced to 28, and every fourth Year to 29. And so the Months came to stand as they are in our Calendar, from whence they are called *Calendar Months*. These are properly *civil Solar Months*.

The principal Division of the Year is also into two kinds, *viz. Solar* and *Lunar*; each of which are again divided into *Astronomical* and *Civil*.

The Astronomical Solar Year is of two kinds, *viz. Tropical* and *Sydereal*. The former is the Space of Time which lapses while the Sun appears to move from either of the Solstitial or

Equinoctial Points to the same again. Now by Reason of the continual Precession of the said Points, whereby they do as it were meet the Sun, this kind of Year is something less than the *Sydereal* one; which is the Time in which the Earth performs an entire Revolution in its Orbit, or in which the Sun seems to pass from any fixed Star to the same again. The former consists of 365 Days, 5 Hours, 48 Minutes, and 57 Seconds; the latter of 365 Days, 6 Hours, 9 Minutes, and 14 Seconds, and is also called *Periodical*, or *Anomalistical*. The Astronomical Lunar Year consists of 12 Synodical Months; and is therefore 354 Days, 8 Hours, 48 Minutes, and 36 Seconds.

The *Civil Lunar Year* is also of two kinds, *viz. moveable* and *fixed*; the moveable civil Lunar Year consists of 12 civil Lunar Months, which being but 354 Days, falls short of a Year by 11 Days, so that the beginning of this Year varies with respect to the Seasons, till it runs through them all, which it does in about 32 Years Time; and from hence it is denominated *Moveable*, or the *wandering Lunar Year*; As the *Turks* now make Use of the civil Lunar Month, so they also do of this civil Lunar Year.

In order to render this kind of Year *fixed*, that is, to prevent the Seasons from shifting from one Part of it to another, the *Greeks*, and the *Romans*, till *Julius Caesar's Time*, after every three Years added a Month of thirty-three

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three Days, which they called *Embolimaus*, or the *Intercalary* Month.

But because this Method is not sufficient, (for the civil Lunar Years fall short of the Astronomical Solar Years above 11 Days,) others adapt such Intercalations to them as do in a manner render the beginning of them fixed to the same Part of the Solar Year. Hence these Years properly intercalated are called *fixed* Lunar Years, as also from their Dependency on the Motion both of the Sun and Moon, *Luni-Solar* Years. Years of this kind are used by the *Jews*, and the Clergy of the Church of *Rome* in their Ecclesiastical Affairs.

The civil Astronomical Year is also either *moveable* or *fixed*. The moveable one was in Use among the *Egyptians*, and is from thence called, the *Egyptian* Year. It consisted of 365 Days, and therefore fell short of the tropical Year, with which all the Seasons return, near six Hours; so that four such Years are less than four Astronomical Solar ones, by almost a whole Day, and therefore in 1460 Years, the beginning of the Year would *pass* through all the Seasons.

Although the *Romans* did not use this kind of Year, yet theirs, taking one Year with another, was of the same Length (*a*); *Julius Cæ-*

(*a*) For having as observed above, six Months of 29 Days each, and six of 30, and an *Intercalary* Month of 33 Days, at the End of every third Year; their Years consisted one with another of 365 Days.

far therefore to *fix* this Year, when he rectified the Calendar, and threw out the civil Lunar Months, to make up the six Hours which this Year wants of the true Astronomical Solar Year, and with which only the Seasons keep pace, ordered that every fourth Year should have an intercalary Day, *i. e.* that a Day should be added to the Calendar, over and above those it contain'd the other three, or that every fourth Year should contain 366 Days. Hence it was that as we observed above, *February* came to have 29 Days every fourth Year. But he did not add it to the End of the Month, but ordered that the sixth of the Calends of *March*, which according to our way of Reckoning is the 24th of *February*, should be reckoned twice. Upon which Account that Year was wont to be called *Bissextilis*, and is still retain'd by us under the Name of *Leap Year*. From hence, Years reckoned after this Manner are called *Julian Years*.

But however the Year was not rendered *fixed* by this means, for it was now too long by about eleven Minutes, and therefore varied from the Sun about a Day in 131 Years. So that from the Time of the *Nicene Council*, (in which the Rule was established for the Time of celebrating *Easter*) to the Year 1582, in which Pope *Gregory* the XIIIth undertook a farther Reformation of the Calendar, there was found to have rose a Variation of 10 Days, the vernal Equinox being observed to fall on
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the 11th of *March*, which at the Time of the *Nicene Council*, fell on the 21st. The Pope therefore in order to bring back the Equinoxes to the Time of the Year they fell upon at the Time of that Council, took those ten Days out of the Kalendar by ordering the 5th of *October* 1582, to be reckoned the 15th, which necessarily removed the following Equinox from the 11th to the 21st of *March*. And to prevent the Seasons of the Year from going backwards for the future, he ordered that every hundredth Year, which according to the Form instituted by *Julius Caesar*, was to be a *Bis-sextile* or *Leap Year*, should be a common one, and consist only of 365 Days, but that since too much would be taken away by this Means, every fourth hundred Year should remain *Bis-sextile*.

This Method being established by Pope *Gregory*, is called the *Gregorian Account*, or the *New Stile*, as being new in comparison of that established before by *Julius Caesar*, which is now called the *Old*; and is observed in all Places where the Papal Authority is acknowledged; as also, towards the End of the last Century was received by many of the reformed People in *Germany*. In *Great-Britain* and *Ireland*, and in some reformed Nations of the Northern Parts of *Europe*, the old *Julian* Form is still retained. But above an hundred Years being now lapsed since the abovementioned Reformation of the Calendar, and the hundredth

hundredth Year not being made a common one, as in the *Gregorian* Method; the *Gregorian* Account differs now from the *Julian* one Day more than it did at its first Institution; so that that Day which according to the *Old Stile* is the first of the Month, is the 12th of the same according to the *New*.

As the Form of the Year in different Nations is various, so also is the Beginning of it. Thus the *Jews* begin their Ecclesiastical Year with the *new* Moon of that Month whose *full* Moon happens next after the vernal Equinox. The Church of *Rome*, with the *Sunday* that falls upon the said *full* Moon, or that happens next after it; that is, with the Feast of the Resurrection of our Lord. The *Grecians* began theirs with the new Moon that happened next after the Summer Solstice. And the *Romans* anciently began theirs with the new Moon next after the Winter Solstice. The *Venetians*, *Florentines*, *Pisans* in *Italy*, and the Inhabitants of *Triers* or *Treves* in *Germany*, make the *vernal* Equinox the Beginning of their Year. The ancient Clergy made the 25th of *March*, or the Feast of the Blessed Virgin, the Beginning of the Year; which Method the Church of *England* still retains; and according to our Law the Year begins here too, though in Things that do not require a *legal Date*, we, with the neighbouring Nations, look upon the first Day of *January* to be the Beginning of the Year.

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The *Persians* still retain the *Egyptian Year*, and the Period in which the first Day of it passes through all the Seasons of the Astronomical Solar Year, viz. 1460 Years, they call the *Sothiacal Period*, or *great Dog-Star Year*; as being reckoned to begin in that Year in which the *Dog-Star* (or *Sothis*) rises Heliacally on the first Day of the Month *Thoth*, which is the first Day of their Year.

A *Cycle* is a continual and successive Revolution of a certain Number of Years. The Cycles are four, viz. the Cycle of the Sun, the Cycle of the Moon, the Cycle of *Dionysius*, the Cycle of the Indiction.

In order to apprehend the Foundation and Nature of the Cycle of the Sun, it must be observed that to each Day of the Year is assigned in the Calendar, some one of the first seven Letters of the Alphabet A, B, C, D, E, F, G, the first Letter A being affixed to the first Day of *January*, to the second B, and so to G. After which the Letters are repeated again in the same Order, A falling to the 8th, B to the 9th, &c. By which means, because the common Year contains 52 Weeks and one Day over, the last Day of *December* has also the Letter A affixed to it. The Letters being appropriated in this manner to each Day of the Year, it follows, that which ever of the Letters belongs to any particular Day in the first Week of *January*, that Letter will belong to the
same

same all the Year long. But since the same Letter, viz. A, is affixed to the last, which is affixed to the first Day of the Year, it is obvious, that at the End of one Year and beginning of the next, two succeeding Days will have the same Letter A; so that if *Monday* had an A the last Year, *Tuesday* has it this, and *Monday* the G: And therefore the Letters will move backwards with respect to the Days of the Week; for Instance, that Day which has G one Year, will have F the next, after that E, then D, and so on to A; and consequently the same Day will run through the seven Letters in seven Years Time. But before the seven Years are expired, a Leap Year will intervene, having one Day more than common, which Day is added at the 24th of *February* (a), and has the same Letter that the 24th has, so that in this Case, as there was a Change or Regression of the Letters, with respect to the Days of the Week, upon the Account of two Letters coming together at the End of one Year, and the beginning of the next, there will be another like Change in *February* also; and therefore, as in the former Case, the same Letter would have run through all the Days of the Week in seven Years; this second Change happening once in four

(a) Some say it is added after the 28th, and that it has the same Letter with that. From this Uncertainty there arises a Difficulty in fixing the Day of the Celebration of the Festival of St. *Matthias*, which has occasioned a great Dispute among the Learned.

Years,

Years, the Course of the Letters is so oft interrupted, that it does not become the same again, till after four times 7, or 28 Years.

Thus, suppose a Leap Year begins with a *Sunday*, it will end with a *Monday*; in which Case, A and G will be the *Dominical* or *Sunday* Letters, because A is appropriated to the first of *January*, and G to the 30th of *December*: But such another Year cannot return till after 28 Years, as any one that tries will readily find; the Consequence of which is, that those two Letters cannot be *Sunday* Letters again in one Year, till after that space of Time. And because the same *Sunday* Letters do return after every 28 Years, that Term is called the *Cycle of the Sunday Letter*, or otherwise, though not so properly, *the Cycle of the Sun* (a).

The Cycle of the Moon is a Revolution of 19 Years; after which Time, as was anciently supposed, the new and full Moons would fall upon the same Days of the Year again (b).

This Cycle is therefore of great Note in the

(a) The ninth Year before our Saviour's Birth, was the first of this Cycle; from whence may easily be known what Year of it the present is. The first of this Cycle is always Leap Year, and has for its Dominical Letters G and F. And the Letter A is as already observed, always affixed to the first Day of *January*. From which *Data* may at any Time be computed what is the *Sunday* Letter for the present Year, and from thence what Day of the Week each Day of the Year falls on.

(b) This is also called the *Metonic Cycle* from *Meton*, the Author of it.

Christian Church, a Method founded upon this Supposition, being established by the Fathers of the *Nicene* Council, for finding the Time of celebrating *Easter*, and the other moveable Feasts of the Church for any Time to come.

In order to understand which, we must observe, that as the Christian Passover, or Festival of *Easter*, succeeded the *Jewish* one, the Time of its Celebration was regulated by that upon which the *Jewish* Passover was at its Institution, by God himself appointed to be kept, which was on the fourteenth Day of the first Month, according to the manner of reckoning among the *Jews*, *Exod.* 12. Now the *Jewish* Months being Lunar ones, each Month began upon the Day of the new Moon, or however at the Time of its *Heliacal* rising, and so the full Moon fell upon the fourteenth of each Month. And further, that Month was called the first of the Year, whose full Moon or fourteenth Day, either fell upon the vernal Equinox, or was the first that succeeded it. And at the Time of the *Nicene* Council, when the Rule for finding *Easter* was drawn up, the Equinox fell, or was thought to fall, on the 21st of *March*. But because our Saviour's Resurrection, which the Festival of *Easter* is design'd to commemorate, happened on a *Sunday*; it was ordered that it should not be kept on the full Moon, but the *Sunday* following. Accordingly the Rule then established for finding *Easter*, and which is still made use of by
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the Church, runs thus; *Easter-Day is always the first Sunday after the first full Moon, which happens next after the One and twentieth Day of March. And if the full Moon happens upon a Sunday, Easter-Day is the Sunday after.*

The Foundation of the Rule being thus explained, it remains now to shew the Meaning of it, which is not so obvious as at first Sight it appears to be.

It is to be observed then, that in order to find on what Days of the Year the new and full Moons would happen for the Time to come, the ancient Method was to observe on what Day of each Month, the new Moons fell in each Year of the Moon's Cycle, and to the said Days respectively to set the Number of the Year in their Calendar. Thus, observing that the new Moons in the first Year of the Cycle fall on *January* the 23d, *February* 21, *March* 23, &c. to those Days they affixed the Number 1. And in like manner observing, that, in the second Year, they fell on *January* 12, *February* 10, *March* 12, &c. to each of them they put the Number 2; and so for each Year of the Cycle. And on Account of their great Usefulness, or because they were wrote in Letters of Gold, the Number thus set to the Days on which the new Moons fell, were called the *Golden Numbers* (a).

(a) In the Table for finding *Easter* for ever in some Common Prayer-Books, they are stiled Prime Numbers, probably as being placed in the Prime or first Column of the Calendar.

The Numbers being thus affixed in the Calendar, it is only looking for the Golden Number of any Year; and over-against it you have the Day in each Month of that Year, in which the new Moon happens.

Thus, at the Time of the *Nicene* Council the Time of the new Moon's might be accurately enough found, but not at this Time, because the new Moons do not happen at the same Time of the Year every 19 Years, but fall short an Hour and half, which in 304 Years comes to a Day, so that now the new and full Moons fall almost 5 Days sooner than they are shewn to do by the Golden Numbers, affixed to the Calendar.

However, as no proper Authority has interven'd to alter the Method of finding the *Easter* full Moon by the Golden Numbers, establish'd at the *Nicene* Council, that Method is still retained in the Church. And therefore to understand the abovementioned Rule aright, it must be supposed, that by the full Moon is meant the Time of the full Moon as found by the Golden Numbers affixed to the Calendar in the Common Prayer Book, and not by an Almanac, or by Astronomical Observation (a).

(a) The first Year of Christ was in the 2d of this Cycle.

By the abovementioned Method the true new Moon may at any Time be found; if we deduct five Days in Consideration of the abovementioned Deficiency in the Moon's Cycle, and suppose the new Moon to fall five Days sooner than that Method shews it to do.

By

By multiplying the Cycle of the Sun and Moon together, we have a third Period, containing the Properties of both the other two, so that at the End of this Cycle not only the Days of the Week have the same Letters belonging to them, and so fall on the same Days of the Month, but the new and full Moons also fall on the same Days of the Year, so that the Time of celebrating *Easter* and the other Festivals of the Church, return to the very same Days of the Year. From whence this is called *the great Paschal Cycle*, and sometimes from its Author, the *Dionysian Cycle* (a). It consists of 532 Years, and the first Year of it was 457 Years before Christ.

The *Cycle of the Indiction* is a Revolution of 15 Years made use of by both the *Greeks* and *Romans*, probably upon some political Account. It has no Relation to the Motions of the Heavenly Bodies. The Nativity of our Saviour fell upon the 4th of this Cycle.

By multiplying the three Cycles of the Sun, Moon, and Indiction together, arises a Term of 7980 Years, called the *Julian Period*; which as it began several hundreds of Years before the Creation, and is not now ended, and in all probability will not, so long as the World stands, is of singular Use in Chronology, as

(a) This is also by some called the *Victorian Period*.

taking

taking in all Events, both past, present and to come.

This Period was invented by *Joseph Scaliger*, and began 4713 Years before Christ.

An *Æra*, otherwise called an *Epoch*, is a Continuation of Time beginning from some certain Point, as from a Root, and continually proceeding forwards without beginning again.

That which is of principal Note among Christians, is the *Æra* of Christ.

The Author of this *Æra* was *Dionysius Exiguus*, who flourished about 500 Years after Christ. He began it on the Annunciation of the Virgin *Mary*, or the 25th of *March*: the 25th of *March*, as he supposed, immediately preceeding the Nativity of our Lord: but it is now generally thought that our Saviour was born the *December* before that, wherefore others, beginning the *Æra* from the Nativity it self, or the 1st of *January* following it, reckon almost a quarter of a Year before those who adhere to the *Dionysian* Account.

The *English* and *Irish* still adhere to the *Dionysian* Account in their ecclesiastical and civil Affairs, which all the rest of the World, and even they themselves in common Account, have laid aside for that which begins at *January* the first. This *Æra* is frequently reckoned backwards, as well as forwards: Thus, it is usual to say an Event happened so many Years before Christ.

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The first Year of this *Æra*, agreeably to what was observed above, answers to the 4714th of the *Julian* Period.

There are several *Epochs* or *Æra's* of less Note, which I shall but just mention; this Chapter having already run out to a much greater Length, than I at first intended.

The *Æra* of the *Creation* according to the *Greek Church*, and which is now of Use in some Eastern Nations, begins before the *Julian* Period, *viz.* 5508 Years before Christ. Tho' others, among whom are the greatest Astronomers, begin it something less than 4000 Years before our Saviour's Birth.

The *Æra* of the *Destruction of Troy*, according to *Dionysius Halicarnassens* and *Diodorus Siculus*, begins 1181 Years before Christ.

The *Æra* of the *Olympiads*, or *Olympic Games* of Use among the *Greeks*, 776 before Christ.

The *Æra Urbis conditæ*, or of the *Building of Rome*, made Use of by the *Romans*, according to *Varro*, began 753 Years before Christ; according to the *Fasti Capitolini*, 752.

The *Æra* of *Nabonassar* made Use of by the *Chaldeans* and *Egyptians*, famous among Astronomers, as consisting of *Egyptian* Years, which are disturbed by no Intercalation. It begins 747 Years before Christ.

The *Æra* of the *Death of Alexander the Great*, 324.

The *Æra* of the *City of Antioch*, 49.

The

The *Æra* of the *Julian Reformation of the Calendar*, 45.

The *Æra Actiaca*, 30.

The *Dioclesian Æra*, 284 Years after Christ.

The *Æra* of the *Hegira*, or *Flight of Mahomet*, used by the *Turks* and *Arabs*, 622 after Christ.

The *Æra* of *Tesdegird* or *Persian Æra*, 632 after Christ.

N. B. A Year in the *Æra's* of the Death of *Alexander*, and of *Nabonassar*, consists but of 365 Days; and a Year of that of the *Hegira* is only 354.

As the Subject of this Chapter, though founded in *Astronomy*, relates principally to the Science of *Chronology*, of which this *Compendium* is not designed to treat; I have little more than thrown a few Definitions together, leaving it to those who would see more on the Subject, to consult such Authors as have wrote more largely thereon; as *Gregory*, *Keil*, *Wells*, *Whitby* on the Common-Prayer, and many others.

C H A P.

C H A P. XVIII.

Of the Forces necessary to retain Bodies revolving in circular and other Orbits.

And first of circular Orbits.

IN Order that a Body may move round in the Circumference of a Circle, it is requisite, that as soon as the Body begins to move, some Power or Force continually act upon it, in such manner as to make it bend its Course every Moment towards the Center of the Circle; because, as is obvious to conceive, it would otherwise only move right forwards (*a*); for in Considerations of this kind, we do not suppose the revolving Body to be affected even by its own *Weight*, or any other moving Cause whatever, except that whereby its Motion is made circular: And because the Circumference of a Circle is every where equally distant from the Center thereof, it is obvious there must be the most exact *Adjustment* imaginable between the Power that retains the Body in the Circle, whatever that Power be, and the Endeavour the Body has to move right on in a straight Line: For in Case the former be an *over-balance* for the latter, it will bring the Body *nearer*

(*a*) See Part I. Chap. IV.

and *nearer* to the Center of the Circle it should describe; and on the other hand, if it be too *weak*, it permits it to move *farther* and *farther* off. So that the Body, instead of a Circle, will describe some other Figure, which will be of this or that Form, according as the Force the Body moves with, compared with the Power which bends its Course, is greater or less; and not only so, but as this latter Force (which is sometimes the Cause) acts more or less powerfully, as the revolving Body comes nearer or goes farther from the Point it revolves about. And here we may have recourse to a Fact obvious to every one, which will in some Measure illustrate what we have been speaking of. Let a Ball be hung up by a String: This Ball, when drawn a little to one Side, will endeavour to return back to the lowest Point, as if there were some Power seated in that Point to draw it thither from all Sides. But the Body may have a Cast given it sideways in such Manner, as that by Virtue of the Cast, and by Virtue of its Tendency to the lowest Point together, it shall describe a Circle parallel to the Horizon; or instead of that it may be made to describe other Figures, as Ellipses suppose, which will be of various Forms according to the Force and Direction of the Cast given to the Ball. And if we suppose several Balls hung up, which when removed to *equal* Distances from their lowest Points, should tend thither again with *unequal* Forces, and these Forces

Forces urging them by different Laws and in different Manners, the Figures described by them, when thrown Sideways with this or that Force and Direction, will be proportionably different one from another. Now in this Supposition, the Cast which is given to the Ball Sideways, corresponds with the Endeavour a Body revolving in the Circumference of a Circle or other Figure, has to move right on continually, and so to leave the Center it ought to revolve about; and the Endeavour, Struggle or Tendency the Ball when drawn to one Side, has to move towards the lowest Point, or rather the Power which is, as it were, seated in that lowest Point, and occasions the abovementioned Endeavour or Tendency in the Ball to move thither, corresponds with the Power we supposed above necessary to retain a Body in the Circumference of the Circle or other Figure it describes, and which by bending the Course of the Body inwards, that is, towards the Center of the Figure, prevents it from going off in a straight Line.

We are therefore in the next Place, to enquire what Forces or retaining Powers are necessary to bend the Course of Bodies in such Manner that they shall describe this or that Orbit (*a*); and since the Planets both primary

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(*a*) For unless we can *measure* the Forces we assign, and say they are exactly equal to the Effects we suppose them to produce, it would be in vain to assign them at all. And this is what makes the Difference between true and false Philosophy.

The

and secondary, as also the Comets, describe Orbits either circular or elliptical, as observed above, we will confine ourselves chiefly to the Consideration of the Forces necessary to retain revolving Bodies in such Orbits. This Subject has been already touched upon in the first Part of this *Compendium* (a), under the Title of *Centripetal and Centrifugal Forces*; but little more is taken Notice of there, than what was requisite to shew the Possibility of such a Motion as we are speaking of, and to explain the Terms; which are these that follow, *viz. Centripetal Force*, or that whereby the revolving Body is prevented from going on in a right Line: The *Centrifugal Force*, or that by which it endeavours to go on in such a Line. These, like Action and Reaction, are equal in all Cases, and therefore called by one common Name, *Central Forces*: That whereby the Body continues its Motion, is the *Projectile Force*; and the Time in which it revolves once

The former by a mathematical Scrutiny, (as it were by a kind of Mensuration) finds that the Causes assigned are justly adapted to the Facts accounted for; while the other, by a conjectural kind of Method, assigns Causes which perhaps are plausible at first Sight, but when measured by the Rules of Geometry, are found either too small or too great for the Effects they are supposed to produce.

This was the Case of *Des Cartes's Vortices*, the Doctrine of which captivated the whole Philosophical World, and continued to do so, till they were weighed in that unerring Balance, and thereby discovered to be inadequate to the Purposes for which they were contrived.

(a) Chap. VIII.

round,

round, is called the *Periodical Time*. The several Cases proper to be considered, shall be comprised in the following Propositions.

P R O P O S I T I O N I.

When two or more Bodies revolve at *equal* Distances from the Center of the Circle they describe, but with *unequal* Velocities, the central Forces necessary to retain them, will be to each other, as the Squares of their Velocities. That is, if one revolves twice as fast as the other, it will require four times the retaining Force the other does; if with three times the Velocity, it will then require nine times the Force to retain it in its Orb. &c. (a)

N. B. We do not here consider the Magnitude of the revolving Body, because we suppose the Power at the Center, to act upon every Part of it alike, so that the larger it is, the more forcibly in Proportion it is acted upon; and therefore it is the same thing, whether it be large or small.

(a) Let BDF Fig. 25. represent a Circle whose Center is S, draw ABC a Tangent to the same at the Point B, and let there be a Body moving from A towards C, and let it be supposed to be destitute of Weight, or whatever else might alter its Course. In this Case it would describe the right Line ABC; but when it comes at B, let it begin to be acted upon by some central Force, or retaining Power seated in S, such as shall *bend* its Course at B from a rectilinear to a circular one, and acting upon it afterwards in every Instant of Time in like manner, shall cause it to describe the Circle BDFB. Let us then proceed to estimate the Quantity of Force exerted in producing one of these *Bendings*; in order to which let us conceive a Point of the Circle as D to be

be the very next to the Point B, though taken at some Distance in the Figure to avoid Confusion of Lines. Parallel to the Diameter BF draw the Line CD, and Parallel to the Tangent BC the Line DG, draw also the Line BD, which will be the Diagonal of the Parallelogram CG. Now it is well known that supposing a Body would move from B to C, and by Virtue of some other Force acting upon it at the same Time, would move from B to G, it will neither move to C nor G, but to D; that is, its straight Course will be *bent* into a circular one at B, for that is what is meant by its moving from B to D when B and D are contiguous, as is here supposed. BG therefore is the Space over which the revolving Body would move by that Action of the central Force which causes the Bend at B, or any other Point of the Circle. And as BG is equal to the Space over which the revolving Body would move by Virtue of the central Force, it will always be proportional to the Force it self. The Value of that Line is therefore now to be determined. Join the Points D and F, then (by 31. El. 3.) the Triangle BDF will be rightangled at D, and consequently (by 8 El. 6.) $BG : BD :: BD : BF$, from whence $BG = \frac{BD^2}{BF}$; but when B and D

are contiguous (as is here supposed) the Difference between the Chord BD and the Arch BD vanishes, so that $BG = \frac{\text{the Arch } BD^2}{BF}$

Having thus found a Quantity proportionable to the Act of the central Force requisite to retain a revolving Body in a Circle, we may proceed to establish the following fundamental *Lemma*.

L E M M A I.

The Space that a Body retained in a Circle, by Virtue of any central Force, would move over in a given Time, by Virtue of that Force only, is equal to the Square of the Arch described in that Time, divided by the Diameter of the Circle.

Dem. The Action of a central Force upon a revolving Body is analogous to that of Gravity upon falling Bodies, and therefore the Space a Body would move over by Virtue of that Force only, is as the Square of the Time that Action continues: as then the Square of the Time the Arch BD is described in (the

Points B and D being contiguous as before) is to $\frac{BD^2}{BF}$ which by what was shewn above, is a Quantity equal to the Space the Body would move over towards the Center in that Time, so is the Square of the Time given to the Space the Body would move over by Virtue of that Force in that Time. If we then put t to

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express the Time the Arch BD would be described in, T for the Time given, and S the Space the Body would move over towards the Center in that Time, we shall have this Proportion.

Viz.

Let the Arch the Body would describe in the Time T, be the Arch BN, then because the Motion of the Body in the Circumference of the Circle is uniform, we have

Squaring the last Proportion, we have

Altering the Position of the Means in the first Proportion, we have

Comparing the third and fourth Step together.

Turning the 5th Step into an Equation.

Dividing the last by BDq

$$1 \quad T^2 : \frac{BD^2}{BF} :: T^2 : S$$

$$2 \quad T : T :: BD : BN$$

$$3 \quad T^2 : T^2 :: BD^2 : BN^2$$

$$4 \quad T^2 : T^2 :: \frac{BD^2}{BF} : S$$

$$5 \quad BD^2 : BN^2 :: \frac{BD^2}{BF} : S$$

$$6 \quad S \times BD^2 = \frac{BN^2 \times BD^2}{BF}$$

$$7 \quad S = \frac{BN^2}{BF} \quad Q. E. D.$$

Coroll. From this *Lemma* it follows, that the central Force requisite to retain a revolving Body in the Circumference of a Circle, must be such as supposing the projectile Motion stopp'd, would cause the Body in the Time it would otherwise describe any Portion of that Circumference, to move towards the Center of the Circle over a Space or Distance of such Length, as would arise from the Division of the Square of that Portion of the Circumference by the Diameter of the Circle. And consequently, it will in all Cases be proportionable to the Square of the Arch the Body would describe in a given Time divided by the Diameter of the Circle: Because the Force exerted in any Time is proportionable to the Space it causes a Body to move over in that Time.

To illustrate this, suppose the Diameter of the Circle 25 Feet, and the Arch BN 10 Feet, and that the revolving Body moves over those 10 Feet in a Minute. Then the Square of BN, which is 100, divided by the Diameter, or 25, gives 4 Feet: The central Force therefore at S, necessary to make the Body revolve in the Circle BNF at the Rate of 10 Feet in a Minute, must be such, as supposing the Body left to itself at B, (that is, without any Motion at all towards C, or any Weight

or

or Gravity of its own) would make it move over 4 Feet towards S in a Minute. Again, supposing Things as before, only that the Body revolves with but half its former Velocity, that is, that it describes but 5 Feet of the Circumference of the Circle in a Minute; then the Square of that 5 Feet, which is 25, divided by 25, the Diameter gives 1; so that the central Force in this Case must be such, that by Virtue thereof a Body destitute both of Weight and projectile Force, shall move over a Space 1 Foot long in a Minute. And conversely, if there be a Circle, whose Diameter is 25 Feet, and there be a Power lodged in the Center thereof capable of causing a Body to move over the Space of 4 Feet in a Minute; then in order that a Body may be made to revolve round in the Circumference of that Circle, it must be thrown forwards in a Direction at right Angles with the Radius of the Circle, with such Velocity as would carry it over the Space of 10 Feet in that Time. And if the central Force be capable of causing it to move over but 1 Foot in a Minute, then it must be thrown forwards with a Velocity that shall make it move at the Rate of only 5 Feet: Or, since the Velocity is uniform, and therefore the Arch described is proportional to the Time it is described in, instead of an Arch as BN to calculate with, we may take the whole Circumference, and say, that the Space a Body would move over by Virtue of the Central Force alone, in the Time it describes the whole Circumference, is equal to that which would arise from the Division of the Square of that whole Circumference by the Diameter of the Circle. And this is that exact Adjustment between the central and projectile Force we have been so long in Search of.

Suppose then we would calculate, how far a Stone would fall by its own Weight, in the Time in which, if thrown forwards with a sufficient Velocity, it would move quite round the Earth in a Circle. Let us call the Diameter of the Earth, that is, the Diameter of the Circle it describes, 1, then the Circumference thereof would be 3 nearly, the Square of that 3, is 9, that divided by the Diameter 1, gives 9; so that a Stone would fall by Virtue of its own Weight through a Space equal to about 9 Diameters of the Earth in the Time it would revolve once round it. But it is here to be observed, that the Stone must be supposed to be of the same Weight during its whole Fall, that it is of at first; and so in all other Cases, we suppose the Energy of the central Force to be the same at all Distances whatever, unless when the contrary is expressed.

We have hitherto been considering the Energy of the central Force, by comparing the Space it would cause a Body to move over in any certain Time, with the Space the Body ought to describe

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describe along the Circumference of the Circle in the same Time: Let us now take it in another View, and compare the Velocity a Body ought to move with to describe a Circle, with the Velocity it would acquire in moving over some certain Space by Virtue of the central Force only; from whence will arise a Proposition very useful in Matters of this kind; and which for Distinction Sake we will call

L E M M A II.

The Velocity a Body ought to revolve with in the Circumference of a Circle, is such as it would acquire in moving by Virtue of the central Force only, over half the Radius of that Circle. To explain this, let the Body be supposed at Rest at B, and let it be acted upon by a central Force at S. Upon this the Body will proceed towards S with a Motion, which will increase every Moment (because as in the Case of falling Bodies we suppose the central Force acting upon it with the same Intenseness, whether the Body moves or not) and the Instant it gets half Way to the Center S, it will have acquired some certain Degree of Velocity; that's the Velocity it ought to move with along the Circumference of the Circle to render its projectile Force an exact Counterbalance to the centripetal one of the Power at S. Thus, suppose there were a Loadstone fixed at S, and a Piece of Steel at B, divested of its own Weight, so that it might be at Liberty to move freely towards the Loadstone, and to make the Instance parallel to the Case before us, let it be supposed that the Loadstone attracts the Steel with the same Degree of Force at all Distances, then whatever Velocity that Steel would have when it had got half way to the Loadstone, that's a Velocity with which if the Steel were thrown from B along the Tangent BC, it would move round in the Path BDFB not approaching towards or receding from the Loadstone. But the Steel must be supposed to meet with no Resistance from the Air, for if it does it will continually lose somewhat of its Velocity, and so after some Revolutions would fall upon the Body of the Loadstone. Again, suppose it were required to find the Velocity a Ball ought to be thrown forwards with at a small Distance from the Surface of the Earth, that it should move quite round in a Circle, keeping the same Distance from the Surface of it all the Way. It must be such, as that Ball would acquire by falling half way from the Place it is thrown from to the Center of the Earth (for that is supposed to be the Seat of the Power that gives it its Weight, and thereby hinders it from moving forwards in the Direction it is thrown in.) And that Velocity is easily calculated from what was demonstrated, Part I. Chap. V. § 4. or 5, supposing as *Hugens* has determined by very accurate Experiments, that a Body near the Surface of the Earth falls $15\frac{1}{2}$

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Paris Feet in the first Second of Time. But enough of this by way of Explication, it is Time to demonstrate what is here affirmed.

Dem. Let the Diameter of the Circle be 16 Feet, in which Case half the Radius will be 4 Feet, then by *Coroll.* of *Lemma* 1. the Arch the Body must run over in the Time it would pass through those 4 Feet by Virtue of the central Force, must be such that the Square of it divided by the Diameter may be 4; that is, it must be 8 Feet, (because the Square of 8, which is 64 divided by 16 the Diameter, gives 4,) but if it describes an Arch of 8 Feet, in the Time it would fall through 4 Feet, it then moves with that very Velocity it might acquire by the Fall; because as has been demonstrated Part I. Chap. 5, § 7. if a Body moves uniformly with the Velocity it might acquire by a Fall, it will move over a Space equal to *twice* the Height of the Fall, in the Time thereof. Since then the Body in the Case before us, describes an Arch of the Circumference of 8 Feet in the Time it would fall through 4 Feet, it moves therein with a *Velocity* it would acquire by falling through 4 Feet, that is, half the Radius. Q. E. D. To avoid a Circumlocution in the midst of the Argument, I call the Space the Body would move over by Virtue of the central Force only, from its Analogy therewith, a *Fall*.

Universally thus, let the Diameter be equal to 4 times some certain Space, which Space let us call d ; the Diameter then will be equal to $4d$, and half the Radius will be equal to the Space d , in which Case, the Arch the Body describes in the Time it would fall through the Space d must by the abovementioned Corollary, be such as that the Square of it divided by 4 times d may be once d , that is, the Arch must be twice d (because the Square of twice d , which is 4 times dd divided by the Diameter 4 times d , gives once d) but as before, if the Body move uniformly through *twice* the Space d , in the time it would fall through *once* that Space, it then moves with that very Velocity it would acquire by falling through that Space, that is, *ex Hypoth.* through half the Length of the Radius. Q. E. D.

From what has been demonstrated above, we may easily learn the Relation which the central Force that is requisite to retain one Body in the Circle it describes, has to that which is required for the same, or another Body revolving in a larger or less Circle. Which Relation is expressed in the following *Lemma*.

L E M M A III.

The central Force whereby one Body is retained in the Circle it describes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained therein, as the Square of the Velocity of the former divided by the

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the Radius of the Circle it describes, to the Square of the Velocity of the latter divided by the Radius of the Circle which it describes. If then we call the central Forces whereby the two revolving Bodies are retain'd in their respective Circles C and c, their Velocities V and v, and the Distances they revolve at, that is, the Radii of the Circles they describe, R and r, we shall always have this Proportion, $C : c :: \frac{VV}{R} : \frac{vv}{r}$.

Dem. By Corollary of Lemma 1, the Energy of the central Force is proportionable to the Square of the Arch the Body describes in a given Time, divided by the Diameter of the Circle. And Arches described in a given Time, are as the Velocities whereby they are described, and the Diameter of a Circle is as its Radius; consequently the Action of the central Force is proportionable to the Square of the Velocity the Body moves with divided by the Radius of the Circle it describes: The central Force therefore whereby one Body is retained in the Circle it describes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained therein, as the Square of the Velocity of the former divided by the Radius of the Circle it describes, to the Square of the Velocity of the latter, divided by the Radius of the Circle which it describes. Q. E. D.

These Things which are the Foundation of what follows being established, we may now proceed to demonstrate each Proposition of this Chapter in the Order they lie in; the first of which is, That when two or more Bodies revolve at *equal* Distances from the Center, but with *unequal* Velocities, the central Forces necessary to retain them will be to each other, as the Squares of their Velocities.

Dem. Let the Circle which one of the revolving Bodies is supposed to describe in this and the following Cases, be BDF, in Fig. 26. and that which the other describes be bdf, and let us call the Distances they revolve at from the common Center, that is, the Radii of the Circles they describe, R and r; the Velocities they revolve with, V and v; and the central Forces necessary to retain them C and c. Then by Lemma 3. we shall have for the first step of this Demonstration,

But by the Proposition R is equal to r, therefore

$$\begin{array}{l} 1 \left| C : c :: \frac{VV}{R} : \frac{vv}{r} \right. \\ 2 \left| C : c :: VV : vv. \right. \end{array} \quad \text{Q. E. D.}$$

PROP. II. When two or more Bodies move with *equal* Velocities, but at *unequal* Distances from the Center they revolve about, their central Forces must be *inversely* as their Distances. That is, by how many times greater the Distance a Body revolves at is from the Center, so many times less Force will retain it (*b*).

PROP. III. When two or more Bodies perform their Revolutions in *equal* Times, but at *different* Distances from the Center they revolve about; the Forces requisite to retain them in their Orbs will be to each other as the Distances they revolve at from the Center. For Instance, if one revolves at twice the Distance the other does, it will require a double Force to retain it; if it describes a Circle three times as large as the other does, then a treble Force is requisite, &c. (*c*)

(*b*) *Dem.* By Lemma 3, as before,
$$1 \left| \begin{array}{l} C : c :: \frac{VV}{R} : \frac{vv}{r} \\ C : c :: \frac{1}{R} : \frac{1}{r} \end{array} \right. \begin{array}{l} \\ Q. E. D. \end{array}$$
 By the Proposition V is equal to $2 \left| \begin{array}{l} C : c :: \frac{1}{R} : \frac{1}{r} \\ C : c :: \frac{1}{R} : \frac{1}{r} \end{array} \right. \begin{array}{l} \\ Q. E. D. \end{array}$ *v*, therefore,

(*c*) In order to demonstrate this the following Lemma will be of use.

LEMMA IV.

The periodical Time of a Body revolving in a Circle, is as the Distance it revolves at from the Center *directly*, and as its Velocity *inversely*. And therefore in comparing the periodical Times of two Bodies together, if we call their periodical Times *T* and *t*, the Distances they revolve at, *R* and *r*; and their Velocities *V* and *v*, as before, we shall have this

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PROP. IV. When two or more Bodies revolving at *different* Distances from the Center, are retained by *equal* centripetal Forces, their Velocities will be such, that their periodical Times will be to each other as the Square Roots of their Distances. That is, if one revolves at four times the Distance another does, it will perform a Revolution in twice the Time the other does; if at nine times the Distance, it will revolve in thrice the Time, &c. (d)

Proportion, viz. $T : t :: \frac{R}{V} : \frac{r}{v}$.

Demonstration of the Lemma. The periodical Time of a revolving Body, *cæteris paribus*, depends on the Distance it revolves at from the Center, because the greater that Distance is, the greater is the Circle it describes, and so the longer it is in coming round. Again, *cæteris paribus*, the greater its Velocity is, the sooner it comes round; and therefore the periodical Time is directly as the former, and inversely as the latter: Or, as the former divided by the latter; and $T : t :: \frac{R}{V} : \frac{r}{v}$ Q. E. D. consequently

<i>Demonst. of the Proposition.</i>	By	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Lemma 3.			
By Lemma 4,		2	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying the 2d Step by the			
Ratio of V to v, we have,		3	$TV : tv :: R : r$
By the Proposition, T is equal to			
t, therefore,		4	$V : v :: R : r$
Comparing the 1st and 4th Steps		5	$C : c :: \frac{VV}{V} : \frac{vv}{v}$
together,			
That is,		6	$C : c :: V : v$ Q. E. D.

(d) Dem. By Lemma 3.	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by R : r	2	$CR : cr :: VV : vv$
By the Proposition, C is equal to		
c, therefore	3	$R : r :: VV : vv$

By

PROP. V. And in general, whatever be the Distances, the Velocities, or the periodical Times of the revolving Bodies, the retaining Forces will be to each other in a Ratio compounded of their Distances directly, and the Squares of their periodical Times inversely. Thus for Instance, if one revolves at twice the Distance another does, and is three times as long in moving round, it will require $\frac{2}{9}$, that is, *two ninths* of the retaining Power the other does. For as, if the retaining Power was in a Ratio compounded of the Distance *directly* and *directly* also of the Square of the periodical Time, the latter Body would require *twice* the retaining Power the other would *multiplied by 9*, the Square of the periodical Time; so, as the latter Ratio is an *inverse* one, it requires twice that Power *divided by 9*; that is, $\frac{2}{9}$ or *two ninths* of it. For in compounding of Ratio's the Way is, if they are both *direct* to *multiply* them both together; if one be *direct* and the other *inverse*, to divide the *direct* by the *inverse*; if both are *inverse*, to multiply them both together, and to divide *Unity* by them: Thus, if the retaining Power had been

By Lemma 4.

Squaring the last Step

Comparing the 3d and 5th,

That is,

Extracting the Roots of the 7th Step,

$$\begin{array}{l|l}
 4 & F : t :: \frac{R}{V} : \frac{r}{v} \\
 5 & FT : tt :: \frac{RR}{VV} : \frac{rr}{vv} \\
 6 & FT : tt :: \frac{RR}{R} : \frac{rr}{r} \\
 7 & FT : tt :: R : r \\
 8 & F : t :: \sqrt{R} : \sqrt{r} \text{ Q.E.D.}
 \end{array}$$

as

as the Distance *inversely* and *inversely* too as the Square of the periodical Time, then in the Case before us, the latter Body would have required a retaining Power, which should have been as the Number 1 divided by 2, multiplied by 9; that is as 1 divided by 18: That is, if we suppose 1 to express the central Force requisite for the former Body, the Fraction $\frac{1}{18}$ would have expressed that which would have been necessary for the latter; or in other Words, their retaining Powers would have been related to each other, as 1 is to $\frac{1}{18}$; that is, the former would have been 18 times greater than the latter (e).

PROP. VI. If several Bodies revolve at different Distances from one common Center, and the retaining Power lodged in that Center decreases as the Squares of the Distances increase; that is, if at a double Distance it be

(e) Dem. By Lemma 3.

Multiplying by R to r,

By Lemma 4,

Multiplying the last by V to v,

Dividing the last by T to t,

Squaring the last,

Comparing the 2d and 6th,

Dividing the last by R to r

1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
2	$CR : cr :: VV : vv$
3	$T : t :: \frac{R}{V} : \frac{r}{v}$
4	$TV : tv :: R : r$
5	$V : v :: \frac{R}{T} : \frac{r}{t}$
6	$VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$
7	$CR : cr :: \frac{RR}{TT} : \frac{rr}{tt}$
8	$C : c :: \frac{R}{TT} : \frac{r}{tt} \text{ Q.E.D.}$

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four times weaker; and at a treble Distance, 9 times weaker, &c. the Squares of the periodical Times of those Bodies will be to each other as the Cubes of their Distances from the common Center. That is, if there be two Bodies, whose Distances when *cubed*, that is, multiplied by themselves twice, are *double* or *treble*, &c. of each other, then their periodical Times will be such, as that when *squared* only, that is, multiplied by themselves *once*, they shall be also *double*, or *treble*, &c. of each other (*f*).

(<i>f</i>) Dem. By Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By the Proposition,	2	$C : c :: \frac{1}{RR} : \frac{1}{rr}$
Comparing the 1st and 2d Step,	3	$\frac{VV}{R} : \frac{vv}{r} :: \frac{1}{RR} : \frac{1}{rr}$
Multiplying by <i>R</i> to <i>r</i> ,	4	$VV : vv :: \frac{1}{R} : \frac{1}{r}$
By Lemma 4,	5	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying by <i>V</i> to <i>v</i> ,	6	$TV : tv :: R : r$
Dividing by <i>T</i> to <i>t</i> ,	7	$V : v :: \frac{R}{T} : \frac{r}{t}$
Squaring the last,	8	$VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$
Comparing the 4th and the 8th,	9	$\frac{1}{R} : \frac{1}{r} :: \frac{RR}{TT} : \frac{rr}{tt}$
Multiplying by <i>TT</i> to <i>tt</i>	10	$\frac{TT}{R} : \frac{tt}{r} :: RR : rr$
Multiplying by <i>R</i> to <i>r</i> .	11	$TT : tt :: RRR : rrr. Q.E.D.$

And after like manner, we may easily demonstrate the seventh Corollary of the fourth Proposition of the first Book of Sir *Isaac Newton's Principia*. Which because it is of more than ordinary Difficulty to Beginners, I here add. It is to this Effect.

When Bodies revolve in such Manner that their periodical Times are to each other as some Power (which call *n*) of their Distances,

Distances, the centripetal Forces requisite to retain them, will be to each other inversely as twice that Power wanting one of their Distances; that is, as their Distances raised to the Power $2n-1$.

Dem. By Lemma 3,

Multiplying by the Ratio of R to r ,

By the Supposition,

By Lemma 4,

Comparing the 3d and 4th,

Multiplying by V to v ,

Dividing by R^n to r^n ,

Dividing the Numerators in the last Step by $R : r$, gives $1 : 1$, and dividing the Denominators $R^n : r^n$ by the same, brings them one Power lower, that is, to $R^{n-1} : r^{n-1}$ therefore the last Step is reducible to

Squaring both Sides of the 8th Step, which in the Denominators R^{n-1} and r^{n-1} is done by doubling their Index $n-1$, we have Comparing the 2d and 9th Steps,

Dividing by R to r , which in the Denominators R^{2n-2} & r^{2n-2} is done by adding 1 to their Index $2n-2$, by which means it becomes $2n-1$, we have

$$1 \quad C : c :: \frac{VV}{R} : \frac{vv}{r}$$

$$2 \quad CR : cr :: VV : vv$$

$$3 \quad T : t :: R^n : r^n$$

$$4 \quad T : t :: \frac{R}{V} : \frac{r}{v}$$

$$5 \quad R^n : r^n :: \frac{R}{V} : \frac{r}{v}$$

$$6 \quad VR^n : vr^n :: R : r$$

$$7 \quad V : v :: \frac{R}{R^n} : \frac{r}{r^n}$$

$$8 \quad V : v :: \frac{1}{R^{n-1}} : \frac{1}{r^{n-1}}$$

$$9 \quad VV : vv :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$$

$$10 \quad CR : cr :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$$

$$11 \quad C : c :: \frac{1}{R^{2n-1}} : \frac{1}{r^{2n-1}}$$

Q. E. D.

Scholium. The foregoing Demonstrations would have proceeded with equal Success, had we only taken the Antecedents,

or the Consequents in each Step; that is, only the large or only the small Letters. Which Method, because I shall make use of it hereafter, I will here give a Specimen of. Let it then be required to demonstrate the first Proposition of this Chapter over again in this latter Method.

By Lemma 3. The central Forces are as the Squares of the Velocities divided by the Radii of the Circles described, which put thus

$$1 \quad C : \frac{VV}{R}$$

By the Proposition, the Circles described have one common Radius, call it 1. Then

$$2 \quad R = 1$$

From the 1st and 2d Step compared, we have

$$3 \quad C : VV.$$

That is, the central Forces of the revolving Bodies are to each other as the Squares of their Velocities, which is the Sense of the Proposition.

Again, let it be required to demonstrate the Corollary taken from Sir Isaac Newton's *Principia*, over again in this Method.

By Lemma 3. —

$$1 \quad C : \frac{VV}{R}$$

By the Supposition —

$$2 \quad T : R^n$$

By Lemma 4. —

$$3 \quad T : \frac{R}{V}$$

Comparing the 2d and 3d Steps

$$4 \quad \frac{R}{V} : R^n$$

Multiplying by V

$$5 \quad R : VR^n$$

Dividing both Sides by R, reduces R to 1, and brings R^n one Power lower, that is to R^{n-1} , so that we shall then have

$$6 \quad 1 : VR^{n-1}$$

Dividing by R^{n-1}

$$7 \quad \frac{1}{R^{n-1}} : V$$

Squaring the last Step, which in the Quantity R^{n-1} is effected by doubling the Index $n-1$, we have

$$8 \quad \frac{1}{R^{2n-2}} : VV$$

Multiplying the first Step by R

$$9 \quad CR : VV$$

Comparing the two last

$$10 \quad CR : \frac{1}{R^{2n-2}}$$

Dividing by R, which in the Quantity R^{2n-2} is done by adding 1 to the Index, we have

$$11 \quad C : \frac{1}{R^{2n-1}}.$$

Q. E. D.
Secondly,

Secondly, Of the Forces necessary to retain Bodies revolving in other Orbs.

PROP. VII. If a Body be turned out of its rectilineal Course by Virtue of a central Force, which decreases as you go from the Seat thereof, as the Squares of the Distances increase; that is, which is inversely as the Square of the Distance, the Figure that Body shall describe, if not a Circle, will be a Parabola, an Ellipse or an Hyperbola, and one of the Foci of the Figure will be at the Seat of the retaining Power. That is, if there be not that exact Adjustment between the projectile Force of the Body and the central Power necessary to cause it to describe a Circle, it will then describe one of those other Figures, one of whose Foci will be where the Seat of the retaining Power is (*g*).

(*g*) In order to shew this, it will be necessary to premise the following Lemma's.

L E M M A V.

The Velocity of a Body describing any Curve, (if its Course is regulated by a retaining Power) is in one Point of the Curve to its Velocity in any other Point thereof, inversely as a Perpendicular drawn from the Seat of the retaining Power to a Tangent of the Curve at the first Point, to a Perpendicular drawn from the same Place to a Tangent at the other Point. That is, in fewer Words, the Velocity of the Body is inversely as a Perpendicular let fall from the Seat of the retaining Power to a Tangent at the Point where the Body is. Thus, if the Body be at D, a Point in Curve DEF, Fig. 27, and PD be a Tangent thereto, and SP a Perpendicular to that Tangent, drawn through S the Seat of the retaining Power, the Velocity of the Body at D, compared with its Velocity in other Parts of the Figure,

F f 2

will

will be inversely as SP, that is, as the Quantity $\frac{1}{SP}$.

Dem. It has been demonstrated, Part I. Chap. VIII. that revolving Bodies describe equal Areas in equal Times; that is, if a Body describes the Curve Line DEF, and the Arches DE, EF, &c. be run over in equal Times, the Areas DES, EFS, &c. will be equal; which Areas if we suppose the Times as small as possible, may be considered as so many right-lined Triangles, because in that Case the Lines DE, EF, &c. lose their Curvity, and being produced are Tangents to the Curve. Produce the Line DE to P, and let fall the Perpendicular SP, then the Area of the Triangle DES is had by multiplying its Base DE into half SP; and the Area of EFS is equal to its Base EF multiplied by half a Perpendicular drawn from S to EF produced, &c. But these Triangles being described in equal Times, are all equal, the longer therefore their Bases, the shorter their Perpendiculars. But the Bases being run over in equal Times by the revolving Body, they are as the Velocity of the Body wherewith they are described; consequently the greater the Velocity of the Body, the shorter is the Perpendicular to the Tangent where the Body is. Which is the Sense of the Lemma.

LEMMA VI.

Whatever Curve a revolving Body describes by Virtue of a centripetal Force, whether the Seat of that Force be within or without the Curve, the Action of that Force upon the Body when at any one Point of the Curve, must be to the same upon the Body when at another Point of the Curve, directly as the Distance of the Body from the Seat of the retaining Power and inversely as the Cube of a Perpendicular let fall from the Seat of the retaining Power to a Tangent to the Curve at the Point where the Body is, multiplied by the Radius of a Circle of equal Curvature with the Figure at that Point. That is, if AX in Fig. 28, be the Curve described, and MDN, whose Center is C, be a Circle of equal Curvature with the Figure at the Point D, and PD be a Tangent to that Point, and SP a Perpendicular thereto drawn thro' S the Seat of the retaining Power, the central

Force requisite to act upon the Body at D must be as $\frac{DS}{SP^3 \times CD}$,

that is, if we put R for the Radius CD, as $\frac{DS}{SP^3 \times R}$.

Dem. By Lemma 5. the Velocity of the revolving Body when at D is as $\frac{1}{SP}$, calling then that Velocity V,

we

we have

Squaring both Sides

$$\begin{array}{l|l} 1 & V : \frac{1}{SP} \\ 2 & VV : \frac{1}{SP^2} \end{array}$$

Call CD the Radius of the Circle MDN, R; then by Lemma 3, the central Force necessary to retain the Body in that Circle, (or which is the same thing, at the Point D in the Curve ADX, because they are both of equal Curvature there) would, if placed at C, be as $\frac{VV}{R}$. But whereas it is placed at S it acts obliquely to DC, and therefore to produce the same Effect, must act more forcibly in the Proportion of SD to DT, supposing ST perpendicular to DC; that is, because SP is equal to DT, in the Proportion of SD to SP.

But by the Golden Rule

$$3 \quad SP : SD :: \frac{VV}{R} : \frac{VV \times SD}{SP \times R}$$

Consequently the central Force necessary if placed not at C but at S, is expressible by

$$4 \quad \frac{VV \times SD}{SP \times R}$$

Putting then C for the Action of the central Force necessary to be placed at S, we have

$$5 \quad C : \frac{VV \times SD}{SP \times R} \text{ or } VV \times \frac{SD}{SP \times R}$$

Comparing the 2d and 4th Steps together

$$6 \quad C : \frac{1}{SP^2} \times \frac{SD}{SP \times R}$$

That is,

$$7 \quad C : \frac{SD}{SP^3 \times R}. \quad Q. E. D.$$

LEMMA VII.

Let AD Fig. 29 be a Parabola, whose Axis is AG, and its Focus S, and let PDF be a Tangent at the Point D, through the Focus S and Point of Contact D draw the Lines PS and DG both perpendicular to the Tangent PD, then will PS be equal to half DG.

Dem. Produce DP till it meets the Axis in X, and draw the Diameter DO, and join the Points D and S: Then because DO, as being a Diameter is parallel to GX, the Angle FDO is equal to DXS; 'tis also equal to the Angle XDS (Miln. Conic. Sect. Part IV. Prop. 4.) the Triangle XSD is therefore an Isosceles one, and SP being perpendicular to the Base XD, XP is equal to half XD; but PS and DG being parallel, the Triangles XPS and XDG

XDG are similar, and therefore PS is also equal to half DG. Q. E. D.

L E M M A VIII.

Let AD Fig. 30, be a Portion of a Parabola, an Ellipse or an Hyperbola, one of whose Foci is S, its longer Axis AG, and its Parameter or Latus rectum L; and let PF be a Tangent at the Point D, join the Points D and S, through the latter of which draw the Line SP perpendicular to the Tangent, and through the former the Line DG perpendicular also to the Tangent, and crossing the longer Axis in G, then will DG be equal to $\frac{L \times SD}{2SP}$.

Dem. Through the Point G draw the Line GT perpendicular to DS, then will the right-angled Triangles DGT and DSP be similar, because the Angles DSP and SDG are alternate, consequently

But by Miln. Conic. Sect. Part IV. Prop. 6.

From the first and 2d Step compared together

Turning the 3d Step into an Equation.

$$\begin{array}{l|l} 1 & DG : DT :: SD : SP \\ 2 & DT = \frac{1}{2} L \\ 3 & DG : \frac{1}{2} L :: SD : SP \\ 4 & DG = \frac{L \times SD}{2SP} . Q. E. D. \end{array}$$

L E M M A IX.

Things remaining as before, produce DG to C in Fig. 31, 32, 33, so that the Line DC may be equal to $\frac{L \times SD^3}{2SP^3}$, then will a Circle whose Center is C, and Radius CD, be of the same Curvature, and coincident with the Figure AD at the Point D, whether it be a Parabola, an Ellipse or an Hyperbola.

Demonstration.

I. *For the Parabola.* The same Lines being drawn as in the foregoing Figure, draw also through D, Fig. 31. the Diameter DY, and take a Portion of it as DI equal in Length to the Parameter of that Diameter, and let fall the Perpendicular IH meeting DG produced in H, and let L be the Latus rectum or Parameter of the Figure, then by the 3d Step of Lemma 8.

The Angle HDI wants IDF to make it a right one, the Angle PSD wants SDP to make it a right one, but IDF and SDP are equal, Miln. Conic. Sect. Part IV. Prop. 4. therefore HDI and PSD are so also, the Triangles therefore HDI and SDP are similar; consequently

$$\begin{array}{l|l} 1 & DG : \frac{1}{2} L :: SD : SP \\ 2 & SD : SP :: DH : DI. \end{array}$$

From

From the 1st and 2d Steps compared together	3	$DG : \frac{1}{2} L :: DH : DI$
By Miln. Conic. Sect. Part IV. Prop. 10. Corol.	4	$DI = 4 DS$
From the 3d and 4th Step compared	5	$DG : \frac{1}{2} L :: DH : 4 DS$
Multiplying the Antecedents of the first Step by 4.	6	$4 DG : \frac{1}{2} L :: 4 SD : SP$
Compounding the fifth and sixth Steps, that is multiplying them together	7	$4 DG^2 : \frac{LL}{4} :: DH : SP$
Lemma 7.	8	$\frac{1}{2} DG = SP$
Comparing the 7th and 8th Steps	9	$4 DG^2 : \frac{LL}{4} :: DH : \frac{1}{2} DG$
Turning the 9th Step into an Equation	10	$2 DG^3 = \frac{LL \times DH}{4}$
Taking half the 10th Step	11	$DG^3 = \frac{LL \times DH}{8}$
From Lemma 8,	12	$DG^3 = \frac{L^3 \times SD^3}{8 SP^3}$
Comparing the 11th and 12th Steps	13	$\frac{DH \times LL}{8} = \frac{LLL \times SD^3}{8 SP^3}$
Dividing the 13th Step by $\frac{LL}{8}$	14	$DH = \frac{L \times SD^3}{SP^3}$
Halving the 14th Step	15	$\frac{1}{2} DH = \frac{L \times SD^3}{2 SP^3}$
But by Supposition	16	$CD = \frac{L \times SD^3}{2 SP^3}$
Comparing the two last Steps	17	$CD = \frac{1}{2} DH$

But DH being perpendicular to PF, and the Angle at I a right one, it is obvious a Circle whose Diameter is DH, (and consequently by the last Step, whose Radius is DC,) will pass through the Point I, 31. El. 3. and will have the Line PF for a Tangent to it at D; but by Miln. Conic. Sect. Part III. Prop. 11. Corol. 1. such a Circle will be of equal Curvature with the Parabola at the Point D; and the Circle and the Parabola will be also coincident with each other at that Point, because the Line PF is a Tangent common to both. Q. E. D.

II. For

II. *For the Ellipsis and Hyperbola:* Let AD in Fig. 32, be a Portion of an Ellipsis, in Fig. 33, a Portion of an Hyperbola, one of whose Foci is S, its Latus Rectum L, and its longer Axis AB; let PF be a Tangent at the Point D, and draw the Lines SP, SD, and DG as in Fig. 30; draw also the Diameter Dd and produce it to I, so that DI may be equal to the Parameter of that Diameter; let fall the Perpendicular IH meeting DG produced in H; through the Center E draw the Semiaxis EQ, and the Diameter Kk conjugate to Dd; and to the longer Axis AB, the Ordinates DM and KN; and in the Hyperbola produce EK and HD till they meet in R. Then by the known Property of the Ellipsis and Hyperbola De L'Hospit. Liv. 2. Prop. 2. and Liv. 3. Prop. 2.

Simson's Conic. Sect.

Lib. 2. Prop. 19. and

Lib. 3. Prop. 43.

Comparing the 1st and 2d Steps together, and extracting their Roots

$$1 \quad EQ^2 CA^2 :: DM^2 : AM \times MB$$

$$2 \quad AM \times MB = EN^2$$

$$3 \quad EQ : EA :: DM : EN$$

The Diameter Kk being conjugate to Dd is by the Definition of such a Diameter, parallel to the Tangent PF, and consequently DH being perpendicular to that Tangent, the Triangle GER is a right-angled one, that Triangle therefore and the right-angled one KEN are similar as having in the Ellipse, their Angles at E vertical; in the Hyperbola, common: The rightangled Triangles GER and DGM are also similar, as having their Angles at G in the Ellipse, vertical; in the Hyperbola, common: And consequently the Triangles KEN and DGM are similar in each Figure.

Therefore

Comparing the 3d and 4th Steps together

The rightangled Triangles DER and DHI in the Ellipse are similar as having their Angles at D common, in the Hyperbola as having their Angles at D vertical, consequently

$$4 \quad DM : EN :: DG : EK$$

$$5 \quad EQ : EA :: DG : EK.$$

$$6 \quad DE : DR :: DH : DI.$$

By the Definition
of the Parameter
DI, for the El-
lipse De L'Hof-
pit. Lib. 2. Defi-
nit. 13. for the
Hyperbola Lib.
3. Definit. 15.

The Extremes of
the two last Steps
being the same,
the Product of the
Means of the one
must be equal to
the Product of the
Means of the o-
ther, that is

Multiplying the 8th
Step by EK

By Simson's Con.
Sect. Lib. 2. Prop.
20. for the Ellipse,
Lib. 3. Prop. 45.
for the Hyperbo-
la

From the 9 and 10.
Dividing the 11 by
DH

By the Definit. of
a principal Para-
meter

Wherefore from
the last by Defi-
nit. 10. Elem. 5.

Multiplying the
two last together

Comparing the 5th
and 15th toge-
ther, we have

Multiplying the
two last Terms of
the 16th Step by

$\frac{2}{DH}$, it becomes

$$7 \quad DE : EK :: 2 EK : DI$$

$$8 \quad 2 EK^2 = DH \times DR$$

$$9 \quad 2 EK^3 = DH \times DR \times EK$$

$$10 \quad DR \times EK = EA \times EQ$$

$$11 \quad 2 EK^3 = DH \times EA \times EQ$$

$$12 \quad \frac{2 EK^3}{DH} = EA \times EQ$$

$$13 \quad \frac{1}{2} L : EQ :: EQ : EA$$

$$14 \quad \frac{1}{2} L : EA :: EQ^2 : EA^2$$

$$15 \quad \frac{LL}{4} : EQ \times EA :: EQ^3 : EA^3$$

$$16 \quad \frac{LL}{4} : EQ \times EA :: DG^3 : EK^3$$

$$17 \quad \frac{LL}{4} : EQ \times EA :: \frac{2 DG^3}{DH} : \frac{2 EK^3}{DH}$$

G g

Comparing

Comparing the 12th Step and the 17th together	18	$\frac{LL}{4} : EQ \times EA :: \frac{2 DG^3}{DH} : EA \times EQ$
The Consequents in the 18th Step being the same, the Antecedents are equal, that is	19	$\frac{2 DG^3}{DH} = \frac{LL}{4}$
Multiplying by DH	20	$2 DG^3 = \frac{LL \times DH}{4}$

But the last Step here is the same with the 10th in the Demonstration for the Parabola, from whence therefore the same Conclusion is to be drawn here as there: for the Proposition in Miln. Conic. Sect. referred to at the End of that Demonstration, relates also to both the Ellipse and the Hyperbola.

Demonstration of the Proposition above, to which this Note refers.

Let ADX, Fig. 28. be a Portion of the Figure the revolving Body describes, whether it be a *Parabola*, an *Ellipse*, or an *Hyperbola*; and let L be the Latus Rectum or Parameter of the Figure; let the Seat of the retaining Power be at S, one of the Foci thereof, and let the revolving Body be supposed at D: draw the Line PD a Tangent to the Curve at that Point, and let fall the Perpendicular SP. Perpendicular to the Tangent draw the Line DC, which let us suppose to be the Radius of a Circle as MDN, of the same Curvature, that the Figure ADX is at the Point D. Then it is obvious, that the same Force which placed at S would retain the revolving Body in the Circumference of the Circle when it came at D, will retain it in the other Figure, at that Point, they being at that Place one and the same Curve. But by Lemma 6, such Force is as $\frac{DS}{SP^3 \times R}$. Calling

then the retaining Power C; we shall have $\left| \begin{array}{l} 1 \\ C : \frac{DS}{SP^3 \times R} \end{array} \right|$ this Proportion, viz.

And since the Circle MDN is of the same Curvature and coincident with the Figure ADX, its Radius DC is by Lemma 9, equal to the Quantity $\frac{L \times SD^3}{2 SP^3}$, putting then R for the Radius as

before we shall have this Equation

$$2 R = \frac{L \times SD^3}{2 SP^3}$$

Comparing the 1st and 2d Step together, we shall have this Proportion

$$3 C : \frac{DS}{SP^3 \times L \times \frac{SD^3}{2 SP^3}}$$

Expunging SP^3 as being both Multiplier and Divisor

$$4 C : \frac{DS}{\frac{1}{2} L \times SD^3} \text{ Dividing}$$

That a Body shall describe Figures so different, when the Power that directs its Course is the same, and acts by one and the same Law, is owing to the Degree of Velocity the Body moves with. Thus supposing the central Power placed at S (in Fig. 34.) and that the revolving Body sets off at D in the Direction DB perpendicular to the Line DS, with such Velocity as it would obtain in falling by Virtue of that central Power only, half way from D to S, it will then, by Lemma 2, describe a Circle as DKM, whose Center is the Point S. If it sets off from D in the same Direction as before, but with a less Velocity, it then revolves within that Circle describing an Ellipse, as DLN, one of whose *Foci* is at S, the Seat of the retaining Power, and the other between that and the Point D, as at F. If it sets off from D with somewhat greater Velocity than what it would acquire by the above-mentioned Fall, it will still describe an Ellipse, as DEO, one of whose *Foci* shall be at S as before, but

Dividing all by DS

But $\frac{1}{2} L$ is the same, whatever Point of the Figure described the revolving Body is at, and may therefore be considered as Unity, consequently

$$\begin{array}{l|l} 5 & C : \frac{1}{\frac{1}{2} L \times DS^2} \\ 6 & C : \frac{1}{DS^2} \end{array}$$

That is, the retaining Power, if it be inversely as the Square of the Distance, and seated in the *Focus* of the Figure, whether it be a *Parabola*, an *Ellipse* or an *Hyperbola*, will direct the Course of the revolving Body in such Manner, as that it shall describe the said Figure. Q. E. D.

G g 2

the

the other shall be beyond it as at G. If it sets off with a Velocity which is greater than that it would acquire by the above-mentioned Fall, in the Proportion of $\sqrt{2}$ to 1, that is, the Square Root of 2 to 1, it still describes a Figure, one of whose *Foci* is S as before, but the other, as G, goes off to an infinite Distance; that is, the Figure described becomes a Parabola, as DPQ, whose Focus is S. If it sets off with a greater Velocity than this last, the Seat of the retaining Power remains still at S, but the other *Focus* G goes yet farther off; that is, it comes on on the other Side the Point D as at H, and the Figure described becomes an Hyperbola, as DRT, whose Foci are S and H.

From hence it is observable, that of the four Figures there are but two wherein an exact Adjustment is requisite between the retaining Power and the projectile Force, *viz.* the Circle and the Parabola. In the former of which, the Velocity must be such as the Body would acquire by falling half way to S, in the latter it must exceed that in the Proportion of the Square Root of 2 to 1; that is, in the Proportion of somewhat less than $1\frac{1}{2}$ to 1. Thus for Instance, if one Degree of Velocity is requisite to cause a Body to revolve in a Circle about the Point S; it will require about one Degree and an half to make it describe a Parabola by Virtue of the same retaining Power seated in the same Point S.

Again, the same Figures may be described, though the Seat of the retaining Power be in the other Foci, F, G, or H; for Instance, if it be at F, in order that the Body may describe the Ellipse DLN, it must set off at D with the same Velocity it had in the former Case when it came to N: If it be at G, it will describe the Ellipse DEO, setting off at D with the Velocity it had before at O: And if G, the Seat of the retaining Power be at an infinite Distance, or in other Words (because to say a determinate Point is at an infinite Distance, is a contradiction in Terms) if the retaining Power act in such manner as it would do if the Seat of it was at an infinite Distance; that is, if it act upon the revolving Body in Lines parallel to DG; and with the same Degree of Force, whether the Body be at D, at P, or at Q, &c. it will describe the Parabola DPQ with whatever Velocity it sets off with from D. For to describe a Circle round the Point G at an infinite Distance; it must set off with an infinite Velocity, because, as observed above, it must set off with the Velocity it would acquire by falling half way to that Point; now a Body will describe an Ellipse round a given Point with any Degree of Velocity less then it will describe a Circle with round the same, as it was observed that the Ellipse DLN was described with less Velocity than the Circle DKM, but a Parabola is no other than an Ellipse, one of whose *Foci* is at an infinite Distance,

Distance, the Body will therefore describe a Parabola when the Seat of the retaining Power is at an infinite Distance, with any Degree of Velocity less than an infinite one.

This is the Reason, that Projectiles upon the Surface of the Earth are said to describe Parabola's, because on Account of the great Distance of the Center of the Earth, compared with the Height Bodies can be thrown to, Gravity does as to Sense, act upon them uniformly and in parallel Lines.

Again, when the Point G the Seat of the retaining Power goes off at an infinite Distance one way; it immediately, or indeed strictly speaking, at the same Instant, becomes the Point H at an infinite Distance the other way; so that those two considered as at an infinite Distance, are as it were one and the same Point; and consequently it is the same Thing whether the retaining Power be at G an infinite Distance to the *Right*, or at H an infinite Distance to the *Left*, and therefore the Figure will still be a Parabola, and may also be described with any Degree of Velocity: only if it be at H, an infinite Distance to the left, the Power must be negative in respect of what it was before; that is, it must be of the *repulsive* Kind, as it is obvious it ought to be, to cause the revolving Body to describe a Figure whose Convexity is turned towards the Seat of the retaining Power.

If

If the Seat of the retaining Power H comes nearer, so as to be at a finite Distance from the other Focus S , the Power must still be negative, or repulsive; and the Figure described will be an Hyperbola, whose Foci are H and S . And the Hyperbola in this Case may be described whatever be the Velocity the Body sets off with; for the central Force being of the repulsive kind, the Figure described will necessarily be convex towards the Seat thereof, that is, towards one of its Foci; but none of the Sections of the Cone have one of their Foci on the convex Side of the Figure, and at a finite Distance from its Vertex, except the Hyperbola (*b*).

Hence

(*b*) In order to evince the Truth of what is here affirmed, the following Lemma will be of Use.

LEMMA X.

Let the Curve Line DPQ in Fig. 35. represent a conic Section, one of whose Foci is S , and let the Circle DKR be of the same Curvature therewith at its Vertex D , then if the Curve be a Parabola, DR the Diameter of the Circle will be equal to, if it be an Ellipse, it will be less, if an Hyperbola it will be greater than four times the Distance DS .

Demonstrat. of the Lemma. (By Milnes Conic. Sect. Part III. Prop. 2. Coroll. 5.) DR the Diameter of the Circle is equal to the Parameter of the Axis of the Curve, whether it be a Parabola, Ellipse or Hyperbola, which let us call, L .

Therefore

1. But in the Parabola, by the Definition of the Parameter, (De L'Hospit. Con. Sect. Livr. I. Def. 5.)

Comparing 1st and 2d Step

1	$L = DR$
2	$L = 4 DS$
3	$DR = 4 DS. Q. E. D.$

2. If

2. If the Curve be an Ellipse, let F be its other Focus, then will $DF + DS$ be equal to its longer Axis, De L'Hosp. Con. Sect. Liv. 2. Cor. 2. of the Definitions; call its shorter Axis, X. Then from the Definition of the Parameter (De L'Hosp. Conic. Sect. Liv. II. Def. 8.) we shall have this Proportion

$$4 \quad L : X :: X : DS + DF$$

Turning the last Step into an Equation

$$5 \quad L = \frac{X^2}{DS + DF}$$

(By De L'Hospit. Conic. Sect. Liv. 2. Corol. 4. of the Definitions) the Square of half X is equal to $DS \times DF$, that is

$$6 \quad \frac{X^2}{4} = DS \times DF$$

Consequently Comparing the fifth and seventh Step

$$7 \quad \frac{X^2}{4} = 4 DS \times DF$$

$$8 \quad L = \frac{4 DS \times DF}{DS + DF}$$

Turning this Equation into a Proportion

$$9 \quad L : 4 DS :: DF : DF + DS$$

Comparing the 1st and the 9th Step

$$10 \quad DR : 4 DS :: DF : DF + DS$$

But DF is less than $DF + DS$, therefore DR is less than $4 DS$. Q. E. D.

3. If the Curve be an Hyperbola, let H be its other Focus, then will $DH - DS$ be equal to its longer Axis, by De L'Hospit. Con. Sect. Liv. 3. Corol. 2. of the Definitions: And call as before its shorter Axis X; then from the Definition of the Parameter (De L'Hospit. Conic. Sect. Liv. 3. Defin. 8.) we shall have this Proportion

$$11 \quad L : X :: X : DH - DS$$

Turning the last Step into an Equation

$$12 \quad L = \frac{X^2}{DH - DS}$$

(By De L'Hospit. Conic. Sect. Liv. 3. Coroll. 4. of the Definitions) the Square of half X is equal to $DS \times DH$, that is

$$13 \quad \frac{X^2}{4} = DS \times DH$$

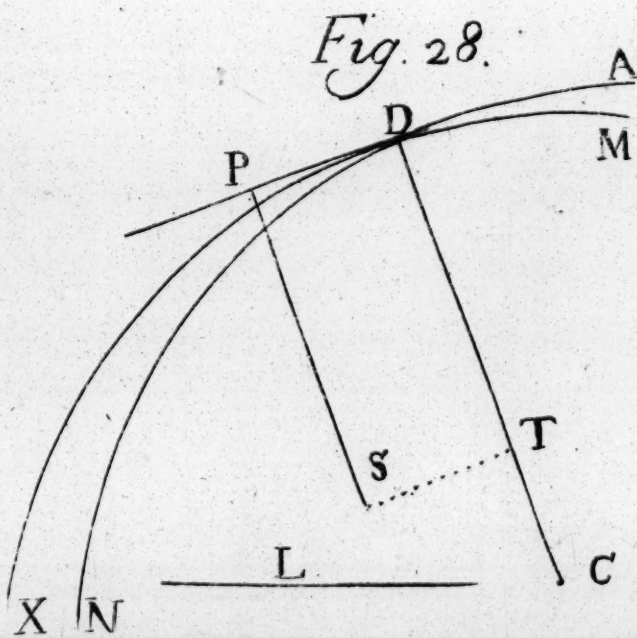
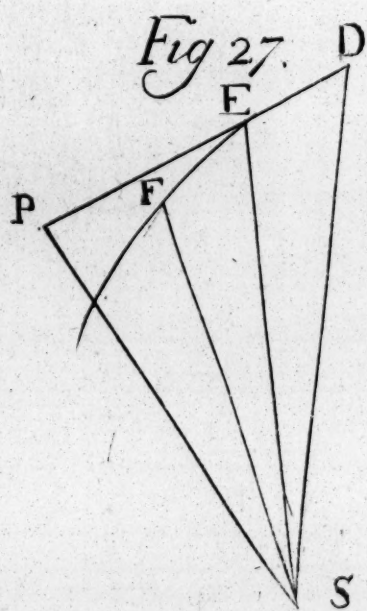
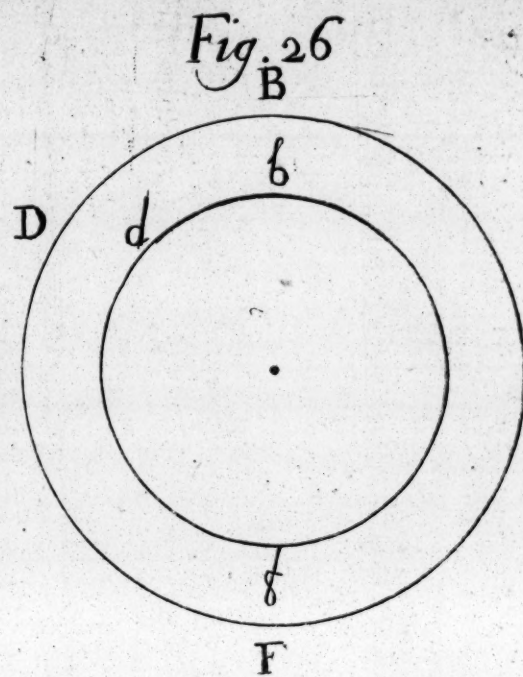
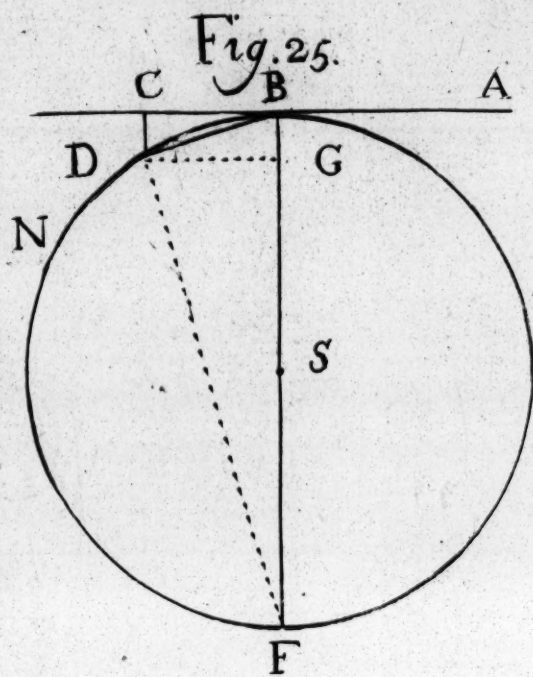
Therefore

$$14 \quad L = 4 DS \times DH$$

Comparing

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Comparing the 11th and 13th Steps

Turning this into a Proportion

Comparing the 1st and 14th Steps

$$14 \quad L = \frac{4 \, DS \times DH}{DH - DS}$$

$$15 \quad L : 4 \, DS :: DH : DH - DS$$

$$16 \quad DR : 4 \, DS :: DH : DH - DS$$

But DH is greater than $DH - DS$, therefore DR is greater also than $4 \, DS$. Q. E. D. This being premised, we may proceed to confirm what was laid down in the Text above.

1. Let the Curve DPQ be a Parabola, and the Circle DKR being of the same Curvature with the Parabola at the Point D , as supposed above, the Velocity a Body ought to set off with from D , to describe the Circle, is the same with that with which it ought to set off, to describe the Parabola: but the Velocity it ought to set off with to describe the Circle, is by Lemma 2. such as it would acquire by falling to S , because DS being (by Lemma 10.) a Quarter of DR , is equal to half the Radius of the Circle; call this Space 2: but the Velocity it ought to set off with to describe a Circle, whose Center is S , is such as it would acquire by falling through half DS , which Space (because the whole Line DS is called 2) must be called one. Now the Velocities Bodies acquire by falling through certain Spaces are to each other as the Square Roots of those Spaces (by Part I. Chap. 5. § 6.) the abovementioned Velocities therefore are to each other as $\sqrt{2}$ to 1. That is, the Velocity a Body ought to set off with from D (the Vertex of the Figure, and in a Direction perpendicular to the Axis DS , which I would always have supposed) in order to revolve in a Parabola whose Focus is S , is to that it ought to have at the same Place (and in the same Direction) to describe a Circle whose Center is S , as $\sqrt{2}$ to 1.

2. Had the Figure DPQ been an Ellipse, a Quarter of DR the Diameter of a Circle of the same Curvature therewith, had been (by Lemma 10.) less than DS ; and so the Velocity the revolving Body must have had to describe that Circle, or (which is the same Thing, because of their equal Curvatures) the Ellipse, would have been acquirable by falling through a less Space than DS , and therefore would have been a less Velocity. That is, it would have exceeded the Velocity requisite for a Circle, whose Center is S the Seat of the retaining Power, in a less Proportion than that of $\sqrt{2}$ to 1.

3. If the Figure DPQ be an Hyperbola, a Quarter of DR is bigger than DS (by Lemma 10.) and so the Space a Body must fall through to get a competent Velocity for that Curve, will be greater than what it must fall through to obtain a competent Velocity for the Parabola: The Velocity therefore requisite for

H h

the

Hence we may observe, that, supposing the Motion of a revolving Body be so adjusted to the Power by which it is retained, that it shall describe a *circular* Orb, and its Velocity be afterwards altered by some Accident, and thereby the Adjustment thereof to the central Force destroyed, that Body shall not immediately fall to the Center: the Form of its Orbit only will be altered, and from being circular, will become elliptical, parabolical, or hyperbolic, according to the Alteration made in its Velocity. Let its Velocity be diminished in any Degree whatever, or let it be increased, so it be in a less Proportion than as $\sqrt{2}$ to 1, its Orbit becomes an Ellipse; if it be increased exactly in that Proportion, a Parabola; if in a greater, an Hyperbola. Hence we have a probable Reason why the Orbits of the Planets are now elliptical; for supposing them to have been circular at first, as it is not unlikely they were, yet upon the first Disturbance in their Motions, whether from their mutual Action upon each other, the Resistance of the Medium they revolve in, the Access of a Comet, or any other Cause whatever; and whether their Motion be increased or decreased thereby, provided it be not increased in a De-

the Hyperbola exceeds the Velocity required for a Circle whose Center is S, in a greater Proportion than that of $\sqrt{2}$ to 1. From which all that is asserted above in the Text to which this Note refers, except what is there otherwise accounted for, may be collected.

gree beyond the Proportion above laid down, their Orbits would immediately become elliptical. Should the Motion of any of them be increased in or above the said Proportion, its Orbit would accordingly be changed into a Parabola, or an Hyperbola, and the Planet would go off, never to return.

PROP. VIII. If the Force of the central Power decreases as the Square of the Distance increases, and several Bodies revolving about the same, describe Orbits that are *elliptical*, the Squares of the periodical Times of those Bodies will be to each other as the Cubes of their middle Distances from the Seat of that Power (k).

(k) To demonstrate this Proposition, let the following Lemma be premised.

LEMMA XI.

The Area a revolving Body describes, is as its Velocity multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is.

Dem. of the Lemma. Let AB in Fig. 36. be the Curve described, DC a Portion of the Curve described in the least Time possible, then will DC represent the Velocity of the Body; produce DC to P, then will DP be a Tangent to the Curve: From S the Seat of the retaining Power let fall the Perpendicular SP, draw DS and CS; then will DCS be the Area the Body describes, but DC by reason of its shortness may be considered as a straight Line, the Area DCS is therefore a Triangle, which is had by multiplying the Base DC into half the Perpendicular SP, therefore it is proportionable to DC multiplied by the whole Line SP, that is, to the Velocity of the revolving Body multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is. Q.E.D.

Dem. of the Proposition. Let ADB and *adb* in Fig. 37 and 38. represent two Ellipses, whose principal Parameters are L and L, and let S be one of the Foci of the one, and *s* one of those of the other, and let the Ellipses be supposed to lie in such manner

H h 2

upon

upon one another, that the Points S and s may be one and the same Point, which let us suppose to be the Seat of the retaining Power; and let the revolving Bodies be at D and d , through which draw the Tangents PD and pd , the Perpendiculars PS and DC , and ps and dc , and the Lines DS and ds ; and let DC and dc be Radii of Curvature to the Points D and d ; and let the Force of the retaining Power at the Distance SD be called F , and at the Distance sd be called f , and let the Velocities of the revolving Bodies when at the Points D and d be called V and v . Then because by the Proposition the Force of the central or retaining Power is supposed to decrease as the Squares of the Distances SD and sd increase, that is, to be reciprocally as the Squares of those Distances, we shall have for the first Step of the following Process, this Analogy, viz. $\left| 1 \right| F : f :: \frac{1}{SD^2} : \frac{1}{sd^2}$

Parallel to the Tangents draw the Lines SH and sh , and let the Force of the retaining Power which acts upon the Body at D in the Direction DS be resolved into two others, viz. DP and DH , the former along the Tangent DP , the latter along the Radius of Curvature DC , then it is obvious that it is by this latter Force only that the revolving Body is retained in the Circle whose Radius is DC , that is, in the Ellipse at the Point D , the other Force along the Tangent only accelerates the Body when moving towards A , and retards it if moving towards B . To compute the Quantity of Force in the Direction DC , say as DS is to DH so is the oblique Force F , or instead thereof $\frac{1}{SD^2}$ (see Step the 1st) to a fourth Number, which fourth Number by the Golden Rule will be $\frac{DH}{SD^3}$, or (because SP is equal to DH) $\frac{SP}{SD^3}$; and in

like manner we shall have $\frac{sp}{sd^3}$ for the Force retaining the other Body in the Circle whose Center is c ; but by Lemma 3. the Force necessary to retain a Body in a Circle whose Radius is CD is expressible by the Square of its Velocity divided by the Radius of the Circle it describes, that is, in the present Case by $\frac{V^2}{CD}$.

And in like manner the Force retaining the other Body in its Circle, will be $\frac{v^2}{cd}$, consequently $\left| 2 \right| \frac{SP}{SD^3} : \frac{sp}{sd^3} :: \frac{V^2}{CD} : \frac{v^2}{cd}$

Multiplying

Multiplying the last Step by CD :

$$3 \quad \frac{SP}{SD^3} \times CD : \frac{sp}{sd^3} \times cd :: V^q : v^q$$

Since CD is supposed to be the Radius of a Circle of equal Curvature with the Figure, we shall have by Lemma 9. these Equations, *viz.*

$$4 \quad CD = \frac{L \times SD^3}{2 SP^3}, \text{ and } cd = \frac{l \times sd^3}{2 sp^3}$$

Comparing the 3d and 4th Steps

$$5 \quad \frac{SP}{SD^3} \times \frac{L \times SD^3}{2 SP^3} : \frac{sp}{sd^3} \times \frac{l \times sd^3}{2 sp^3} :: V^q : v^q$$

That is, by reducing the last Step to lower Terms

$$6 \quad \frac{L}{SP^q} : \frac{l}{sp^q} :: V^q : v^q$$

Extracting the Square Root of each Term

$$7 \quad \frac{\sqrt{L}}{SP} : \frac{\sqrt{l}}{sp} :: V : v$$

Multiplying by SP :

$$8 \quad \sqrt{L} : \sqrt{l} :: V \times SP : v \times sp$$

But by Lemma 2. the Area one Body describes is to that which the other describes in the same time, as $V \times SP$ to $v \times sp$, which Areas call A and a , and we shall have this Proportion, *viz.*

$$9 \quad A : a :: V \times SP : v \times sp$$

Comparing the 8th and 9th Steps

$$10 \quad A : a :: \sqrt{L} : \sqrt{l}$$

Now the Area of the whole Ellipse ADB is to that of the other adb , as AB the longer Axis multiplied by QN the shorter, to $ab \times qn$, those Quantities therefore may be put for the Areas themselves : If then we call the Time in which the Areas A and a are described in, one, (as one Hour, or one Minute suppose) and the Times the whole Ellipses are described in, T and t ; we shall have (since the Area a revolving Body describes is proportional to the Time it is described in) the following Analogies, *Viz.*

$$11 \quad 1 : T :: A : AB \times QN$$

And

$$12 \quad 1 : t :: a : ab \times qn$$

Comparing

Comparing the
3 last Steps to-
gether we have
the two follow-
ing ones, viz.

By De L'Hospital
Conic. Sect. Liv.
2. Def. 8.

Turning the last
Proportion into
an Equation.

Extracting the
Square Roots of
each Side of the
Equation

And in like man-
ner

Comparing the
13th and 17th
Steps together

And comparing the
14th and 18th to-
gether

Dividing the latter
Part of the 19th
Proportion by
 \sqrt{L}

Dividing the latter
Part of the 20th
Proportion by \sqrt{l}

Comparing the
21st and 22d

Squaring each
Term

That is

$$13 \quad 1 : T :: \sqrt{L} : AB \times QN$$

$$14 \quad 1 : t :: \sqrt{l} : ab \times qn$$

$$15 \quad AB : QN :: QN : L$$

$$16 \quad AB \times L = QN^2$$

$$17 \quad \sqrt{AB} \times \sqrt{L} = QN$$

$$18 \quad \sqrt{ab} \times \sqrt{l} = qn$$

$$19 \quad 1 : T :: \sqrt{L} : AB \times \sqrt{AB} \times \sqrt{L}$$

$$20 \quad 1 : t :: \sqrt{l} : ab \times \sqrt{ab} \times \sqrt{l}$$

$$21 \quad 1 : T :: 1 : AB \times \sqrt{AB}$$

$$22 \quad 1 : t :: 1 : ab \times \sqrt{ab}$$

$$23 \quad T : t :: AB \times \sqrt{AB} : ab \times \sqrt{ab}$$

$$24 \quad TT : tt :: AB^2 \times AB : ab^2 \times ab$$

$$25 \quad TT : tt :: AB^3 : ab^3$$

But half the longer Axis AB is equal to the middle Distance SN, (De L'Hospital Conic. Sect. Liv. 2. Corol. 3. of the Definitions) therefore since Halves are proportionable to their Wholes

$$126 \quad TT : tt :: SN^3 : sn^3. \quad Q. E. D.$$

PROB.

PROP. IX. If the retaining Power decreases something faster as you go from the Seat thereof (or which is the same thing, increases something faster, as you come towards it) than in the Proportion mention'd in the last Proposition, and the Figure the revolving Body describes be not a Circle, the Axis of that Figure will turn the same Way the Body revolves; but if the said Power decreases (or increases) somewhat slower than in that Proportion, the Axis of the Figure will turn the contrary Way. Thus, if a revolving Body as D, Fig. 39. passing from A towards B describes the Figure ADB, whose Axis AB at present points towards M and N, and the Power whereby it is retained decreases faster than the Square of the Distance increases, after a Number of Revolutions the Axis of the Figure will point towards O and P, and after that towards Q and R, &c. revolving round the same Way with the Body, and if the retaining Power decreases slower than in that Proportion, the Axis will turn the other Way. (i)

(i) *Dem.* Let S in Fig. 40. represent the Seat of the retaining Power, and let a Body as D, describe the Orb ADB passing from A towards B; and let another as N describe a revolving Orb as MNO equal and similar to the former; and let both the Orbits be described in the same Time; and let the Velocity wherewith the Orb MNO revolves be such, that the Angle ASN shall always be proportionable to the Angle ASD. To explain this a little farther, suppose that when the Bodies N and D set out from M and A, those two Points were then together at A; or that the Orbits did then coincide; but that N advances forwards in such Manner, as that if the Angle ASN is at any time

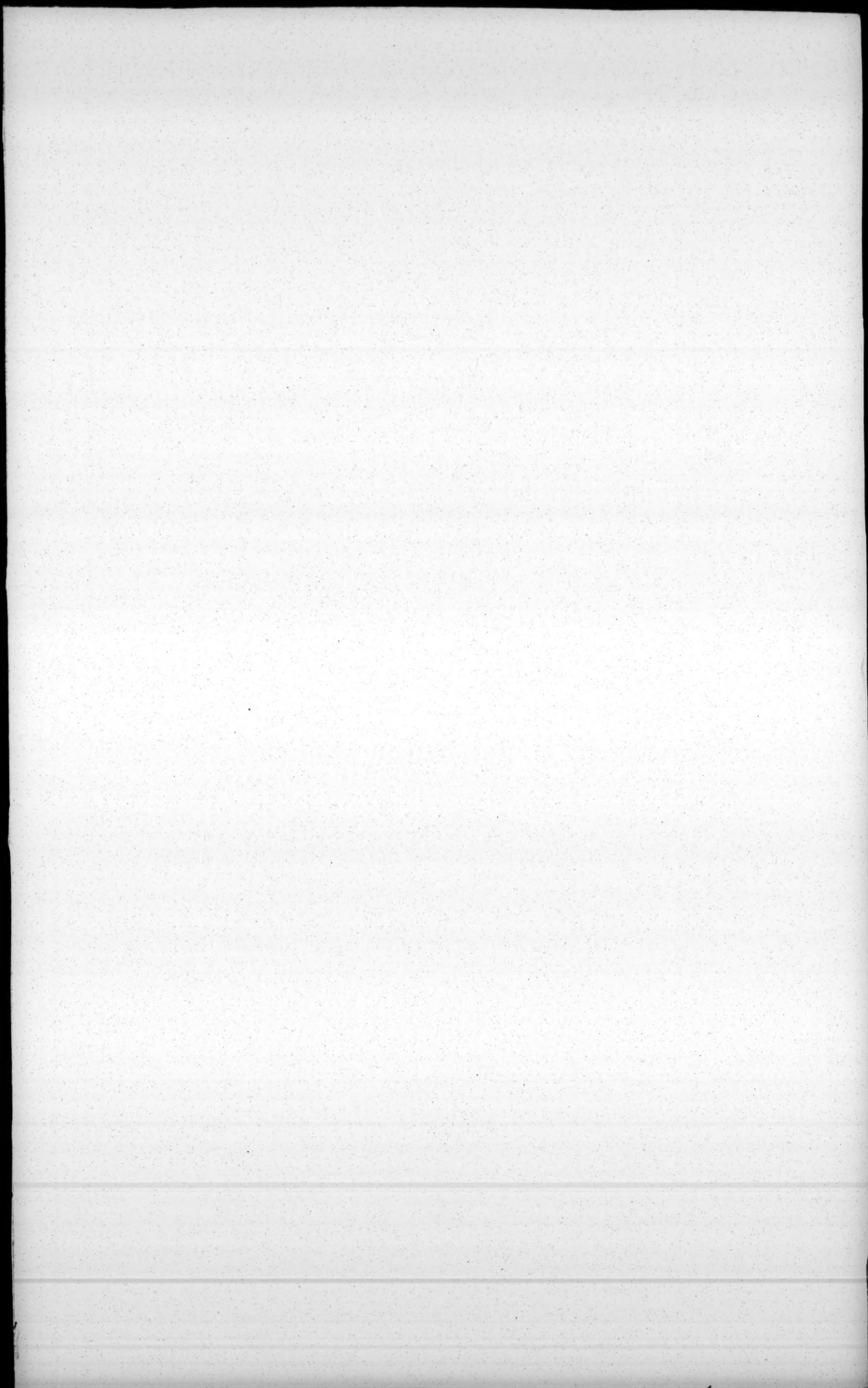
time double, treble, or in any other Proportion of the Angle ASD, it shall always be so: That is, that the Line SN shall always move faster or slower than the Line SD in some certain Proportion; and that the Point M shall follow or go from the Body N, so as that the Arch MN shall always be equal to the Arch AD. And let it farther be supposed that the Line SN is always equal to SD.

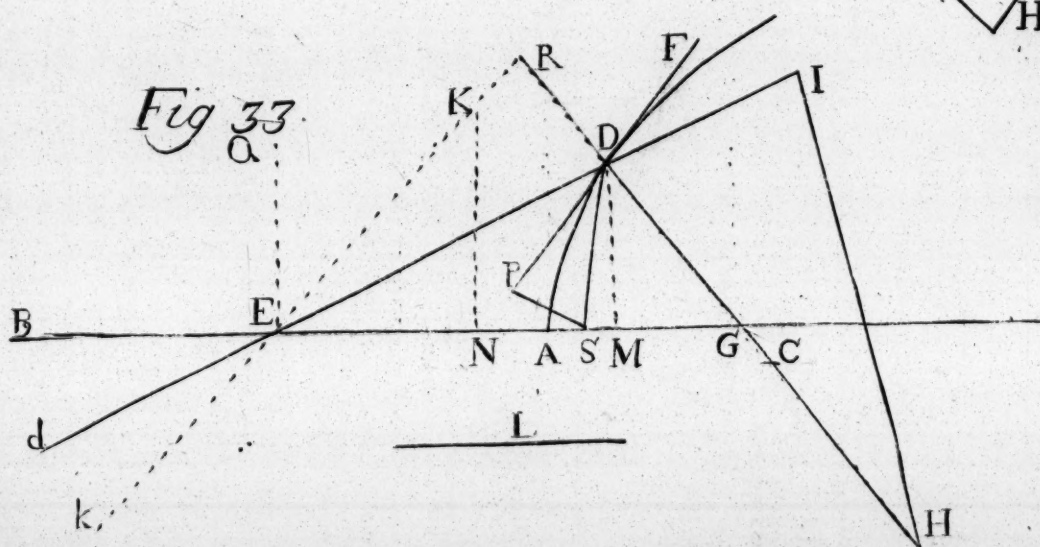
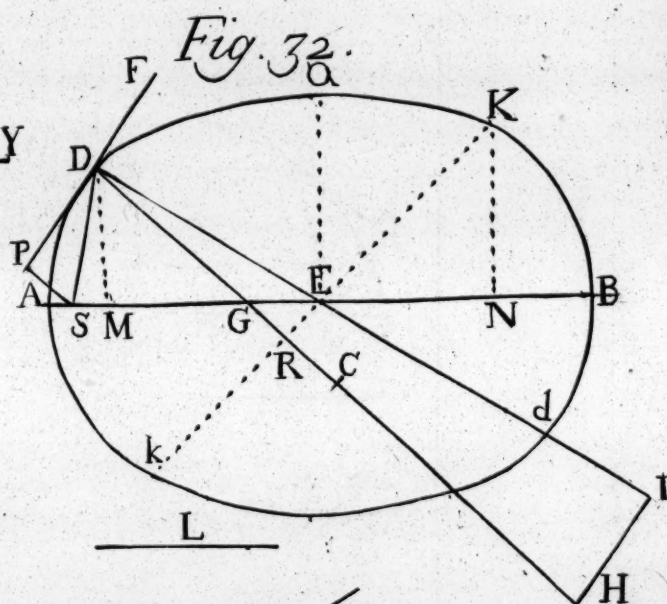
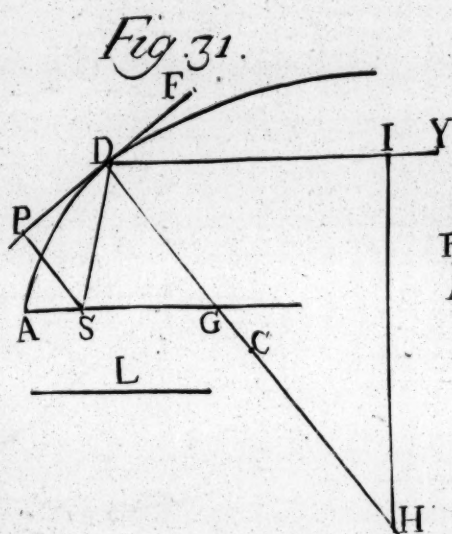
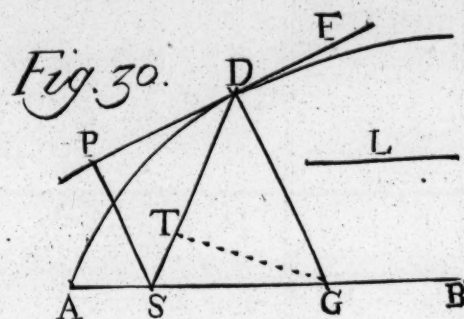
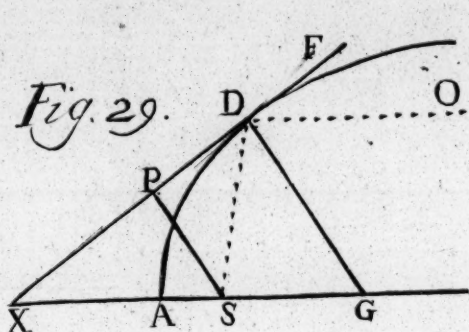
The first Inquiry which arises here is, whether this be a possible Case; that is, whether the Body N moving with the above-mentioned Restrictions, the Line SN can describe Areas proportionable to the Times, because as was shewn, Part. I. Chap. 8. that is always requisite where Bodies revolve round a central Force, and are retained in their Orbits thereby. Let it then be considered in the first Place, that the Body D revolving in the quiescent Orb ADB may do so, its Motion being clogged with none of those Suppositions; secondly, that the Line NS is always equal to DS, and the Velocity wherewith its Extremity N moves forwards, is proportionable to that which D the Extremity of the other moves with; and consequently the Areas which one describes, (for the Areas depend solely on the Length and Velocity of the Lines that describe them) will be proportionable to those which the other describes: Since then those of the former may be proportionable to the Times they are described in, it is possible those of the latter may be so too; the Supposition therefore above laid down is not absurd.

The next Inquiry is, by what Law the Action of the central Force at S must decrease as we recede from it, that a Body as N may describe the revolving Orb MNO in the manner above-mentioned. Or, which will answer our Purpose as well, how the Force necessary to retain the Body N describing the revolving Orb MNO, must differ from that which is requisite to retain the other Body D describing the quiescent Orb ADB equal and similar thereto.

In order to this, let DR, in Fig. 41. represent a Portion of the quiescent Orb described in the least Time possible, this then may be considered as a strait Line; and let the Motion of the Body along this Line be resolved into two, the one towards S along the Line DT, and another along the Line TR at right Angles therewith; then will DT represent the Velocity wherewith the Body descends towards S, and consequently the Action of the central Force whereby it is retained in that Arch; and TR will represent that by which it advances forwards in the mean time. Let it now be supposed that the Orb ADB advances forwards, while the Arch DR is describing; but not by Virtue of any additional Force in the Center, but by some extrinsic one, no matter what: That

is,





Chap. XV

is, suppose that
it would otherwise
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just as it did before
which represents
as before; but
Progress of the
Case. Let it
ing over the
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Q is farther from
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S draw the Line
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QN being the
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that Line, will

Produce C
QN x QG b

Dividing by

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reciprocally

And

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ST perpendicular

is, suppose that the Line SD turns round the Point S faster than it would otherwise have done, carrying the Body with it so much the faster; but that the Body descends towards S along that Line just as it did before: That is, in other Words, that the Line DT which represents the Descent of the Body, is of the same Length as before; but that the other Line TR which represents the Progress of the Body forwards, is longer than it was in the other Case. Let it then become TQ; in which Case, the Body moving over the Line DT and the Line TQ in the same Time, will in reality get to the Point Q: But observe, that the Point Q is farther from S than the Point R is, because the Angle at T is a right one; whereas the Law which we laid down above for the Motion of a Body describing a revolving Orb was such, that at the End of the Time in which DR would be described in the quiescent Orb, the Body in the revolving Orb, should be at the same Distance from S as if its Orb had not revolved; if then round the Point S, we describe a Circle as RFG, and thro' S draw the Line QG, N must be the Point the Body must come to. Since then the Body in describing the revolving Orb without any Addition to the central Force above what would cause it to describe the quiescent one, would come to Q; but with such an Addition as is necessary to make it revolve in the manner above laid down, it comes to N, it is obvious that the Line QN being the Distance between those two Points will aptly represent that Addition. It remains then to get the Measure of that Line, which may be done in the following manner.

Produce QT to F, then (by 35. El. 3.) will the Rectangle QN x QG be equal to QR x QF. Therefore

$$\begin{array}{l} \text{Dividing by QG} \end{array} \quad \begin{array}{l} 1 \mid \text{QN} \times \text{QG} = \text{QR} \times \text{QF} \\ 2 \mid \text{QN} = \frac{\text{QR} \times \text{QF}}{\text{QG}} \end{array}$$

Let it be observed then in the first Place, that as the Motion of revolving Bodies is such that they describe equal Areas in equal Times, the longer the Line DS is (that is the nearer D is to A, suppose) the shorter the Line TR must be, and for the same Reason also the Line TQ: That is, both TR and TQ are reciprocally as SD, which let us put thus

$$\begin{array}{l} \text{And} \\ \text{Observe secondly, that since RF} \\ \text{is a Chord of a Circle, and} \\ \text{ST perpendicular to it} \end{array} \quad \begin{array}{l} 3 \mid \text{TR} : \frac{1}{\text{SD}} \\ 4 \mid \text{TQ} : \frac{1}{\text{SD}} \\ 5 \mid \text{RT} = \text{TF} \end{array}$$

I i

From

From the Figure

Because RT and TF are equal

$$\begin{array}{l|l} 6 & QR = TQ - TR \\ 7 & QF = QT + TR \end{array}$$

Since then by the 3d and 4th Steps, TR and TQ are both reciprocally as SD, and by the 6th and 7th Steps, QF is equal to the Sum of those Lines, and QR is equal to their Difference, it is obvious that QF and QR are also each reciprocally as SD,

which put thus

$$8 \quad QF : \frac{1}{SD}$$

And

$$9 \quad QR : \frac{1}{SD}$$

Multiplying the 8th and 9th Steps together

$$10 \quad QR \times QF : \frac{1}{SD^2}$$

Observe also that when the Points D, T, R, Q and N coincide, which must ever be supposed, because we are all this Time considering only what is done at one and the same Point of the Curve ADB, QN is nothing with respect to NG, therefore QG and NG are the same Line; therefore QG may be said to be as NG, or as half of it, *viz.* RS, or which will be the same Thing, DS; therefore instead of QG in the 2d Step, we may put DS,

and then it will stand thus

$$11 \quad QN = \frac{QR \times QF}{DS}$$

The 11th Step without altering the Value of it, may be set thus

$$12 \quad QN = QR \times QF \times \frac{1}{DS}$$

Comparing the 10th and 12th Steps together we have

$$13 \quad QN = \frac{1}{DS^2} \times \frac{1}{DS}$$

That is

$$14 \quad QN = \frac{1}{DS^3}.$$

That is, the Line QN, or the additional central Force requisite to cause a Body to move in a revolving Orb must be reciprocally as the Cube of the Distance of the revolving Body from the Seat of the retaining Power. Whereas then, when the centripetal Force decreases as the Square of the Distance increases, the revolving Body describes a Parabola, an Ellipse, or an Hyperbola; if to that centripetal Force be superadded another, that shall decrease as the Cube of the said Distance increases; those two Forces acting conjointly upon a revolving Body shall cause it to describe the same Figure as before, but the Axis thereof shall revolve the same Way that the Body does.

But observe here, that if a Force decreases faster in any Degree whatever than the Square of the Distance increases, but slower than the Cube, that Force is the Sum of two Forces, one of which decreases as the Square, the other as the Cube of the Distance

Thus it is the heavenly Bodies, *viz.* the Planets both primary and secondary, and also the Comets, perform their respective Revolutions. The Figures in which the primary Planets and the Comets revolve, are Ellipses, one of whose Foci are at the Sun. The Areas they describe by Lines drawn to the Center of the Sun, are in each proportionable to the Times in which they are described. The Squares of their periodical Times are to each other as the Cubes of their middle Distances from the Sun. The secondary Planets describe also Circles or

Distance increases. Thus for Instance, let the Distances be as 2 to 1, the Squares of them will be 4 to 1, their Cubes 8 to 1; and let the Forces be as 6 to 1; I say, this Proportion will arise from adding 4 to 1 to 8 to 1; for 4 to 1 added to 8 to 1, is 12 to 2, that is 6 to 1. Again, let the Forces be as 5 to 1; I say, this also will arise from adding 4 to 1 to 8 to 1; for instead of 4 to 1 let us take 12 to 3 which is the same Thing, this superadded to 8 to 1 makes 20 to 4, that is, 5 to 1. Again, let the Forces be as 7 to 1; this also is the Sum of 4 to 1 added to 8 to 1; for instead of 8 to 1 we may take 24 to 3, which added to 4 to 1 gives 28 to 4, or 7 to 1. And the same for Fractions: so that let a Proportion be what it will between 4 to 1 and 8 to 1, it may be considered as arising from 8 to 1 added to 4 to 1. When the retaining Power therefore by which a revolving Body is kept in its Orbit, decreases faster than the Square of the Distance increases, but not so fast as the Cube thereof does; it is a Power decreasing as the Square, having another Power decreasing as the Cube of the Distance superadded to it; and therefore as the Proposition in the Text asserts, if the retaining Power decreases something faster, &c. Q E. D. As to the Figures turning the contrary Way, when the centripetal Forces decreases somewhat slower than as the Square of the Distance increases; that is but the Converse of this, and therefore needs no particular Demonstration.

Ellipses, one of whose Foci are in the Center of their Primary ones. The Area that each describes by Lines drawn to the Center of its Primary, is proportionable to the Time it is described in. The Squares of the periodical Times of the Secondaries belonging to the same Primary, are to each other as the Cubes of their Distances from their Primary. And above all, that very Force by which Stones or heavy Bodies fall to the Surface of the Earth with us, is at the Distance of the Moon, just sufficient to retain her in her Orbit, supposing it to decrease as we go from the Center of the Earth in that Proportion, in which the Cause, that retains her and the rest of the Planets in their Orbs, whatever it be, must do to make them elliptical (1). And farther, which also is a necessary

(1) To find out whether this be so or not, let us calculate what Velocity the Moon would acquire, were she to fall half way to the Center of the Earth by Virtue of that Gravity, whereby heavy Bodies tend to the Earth; and compare it with the Velocity she moves with in her Orbit; because by Lemma 2 of this Chapter, if she be retained in her Orb by that Gravity, those Velocities ought to be the same. The mean Distance of the Moon from the Earth in round Numbers is 60 Semidiameters of the Earth, therefore the Force of Gravity at the Distance of the Moon is the Square of 60 times less than it is at the Surface of the Earth; therefore the Fall of a Body at that Distance in a Second of Time, will be so many times less than it is here; but the Fall of a Body here in a Second, is about 15 *Paris* Feet, consequently at the Moon it is but ,004166 Feet. The Space therefore it would move over in a Second, with the Velocity acquired by that Fall, is, by Part I. Chap. 5. § 7. twice that Quantity, viz. ,008332 Feet, which Number therefore may be put to express its Velocity. Now because the Velocities Bodies acquire

necessary Consequence of what we are about to lay down, the Motion of the Planet *Saturn* is observed to be disturbed by *Jupiter*; and the Secondaries of *Jupiter* to be disturbed in their Motions upon the nearer Approach of *Saturn*; and the Course of the Moon is incessantly altered by the Action of the Sun, in such manner as to cause all those Irregularities mentioned in Chapter the 8th of this Part. All which, together with the Precession of the Equinoctial Points, the Nutation of the Poles of the Earth, and the Phænomena of the Tides, which naturally flow from it, make it extremely probable that there is a Virtue diffused about the Sun and Planets, not unlike that of Attraction, which decreases as the Squares of the Distances from the Centers of those Bodies increase, and

acquire by falling, are as the Square Roots of the Spaces they fall through, Part I. Chap. 5. § 6. say, as the Square Root of ,004166, is to the Square Root of 590866170, which is half the Semidiameter of the Moon's Orbit, (supposing the Semidiameter of the Earth to be 19695539 Feet, which is *Cassini's* Measure, and the Distance of the Moon from the Earth to be 60 Semidiameters as above) so is ,008332 to 3135 Feet, which is therefore the Space the Moon would move over in a Second with the Velocity she would acquire by falling half way to the Center of the Earth. But this comes within an hundredth Part of her Velocity in her Orbit, as may easily be calculated from the Time she revolves in, *viz.* 27 Days, 7 Hours and 43 Minutes, and the Semidiameter of her Orbit, which according to the Measures we have taken above, is 1181732340. That Force therefore by which Bodies fall to the Ground with us, is at the Distance of the Moon such as is requisite to direct her Course.

N. B. The Proportion of a *Paris* Foot to that of *England*, is as 367196 to 343800.

acts

acts upon Bodies in Proportion to the Quantity of Matter they contain.

This being allowed, it will follow, that as the Sun attracts the Planets, and thereby retains them in their Orbs, they in like manner attract the Sun, though with Forces proportionable only to the Quantities of Matter they contain; so that strictly speaking, each primary Planet revolves not about the Center of the Sun, but about a Point which is the Center of Gravity between the Sun and that; and that the Sun moves also round that Point, and is always opposite to the Planet with respect thereto: And likewise that the Center of the solar System is not in any one Body, but in the common Center of Gravity of all the Bodies of which it consists. But then the Sun is so immensely large in respect of any one, or all of those Bodies put together, that that Center is very near the Center of the Sun. In like manner, the Moon does not revolve about the Center of the Earth as a Point at rest, but the Earth and Moon revolve each about the Center of Gravity common to them both, which Center of Gravity it is, and not the Center of the Earth, that describes the Orbit the Earth is commonly said to revolve in. And so of the other Planets which are attended by Secondaries.

The Irregularities in the Motion of the Planet *Saturn*, and those of the Secondaries of *Jupiter*, owing to the Cause abovementioned, are

are so exceeding small, that it shall suffice to have just mentioned them.

The more remarkable Effects of the disturbing Force of the Sun, are the lunar Irregularities, the Precession of the Equinoctial Points, the Nutation of the Poles of the Earth, and the ebbing and flowing of the Sea, which shall be particularly considered in the following Chapter.

C H A P. XIX.

Of the Lunar Irregularities, the Precession of the Equinoctial Points, the Nutation of the Poles of the Earth, and the ebbing and flowing of the Sea.

I. **T**O account for the lunar Irregularities, let S in Fig. 42. represent the Sun, T the Earth, and LMNO the Orbit of the Moon, and let the Moon be in one of its Quadratures at L, and let the Lines LS and TS be drawn. It is obvious, that the Tendency the Moon has towards the Sun is along the Line LS, and that which the Earth has, is along the Line TS: Let then the former of these be resolved into two others, the one along LA parallel and equal to TS, the other from L to T along

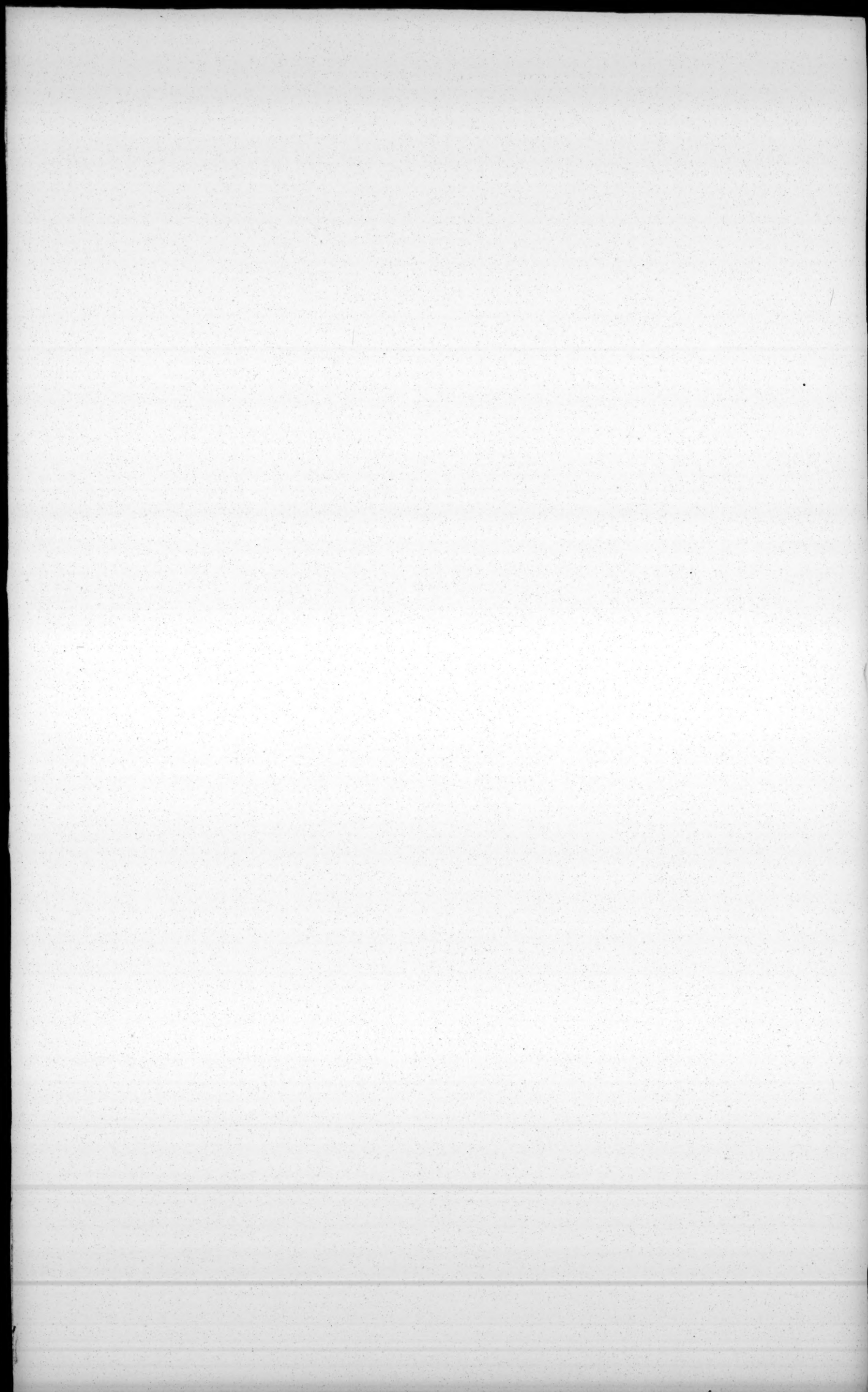
along the Line LT . The former of these Tendencies being parallel and equal to that whereby the Earth tends along the Line TS , alters not the Situation of the two Bodies L and T with respect to each other; that is, it disturbs not the Motion of the Body L ; but the other along LT increases its Tendency towards T .

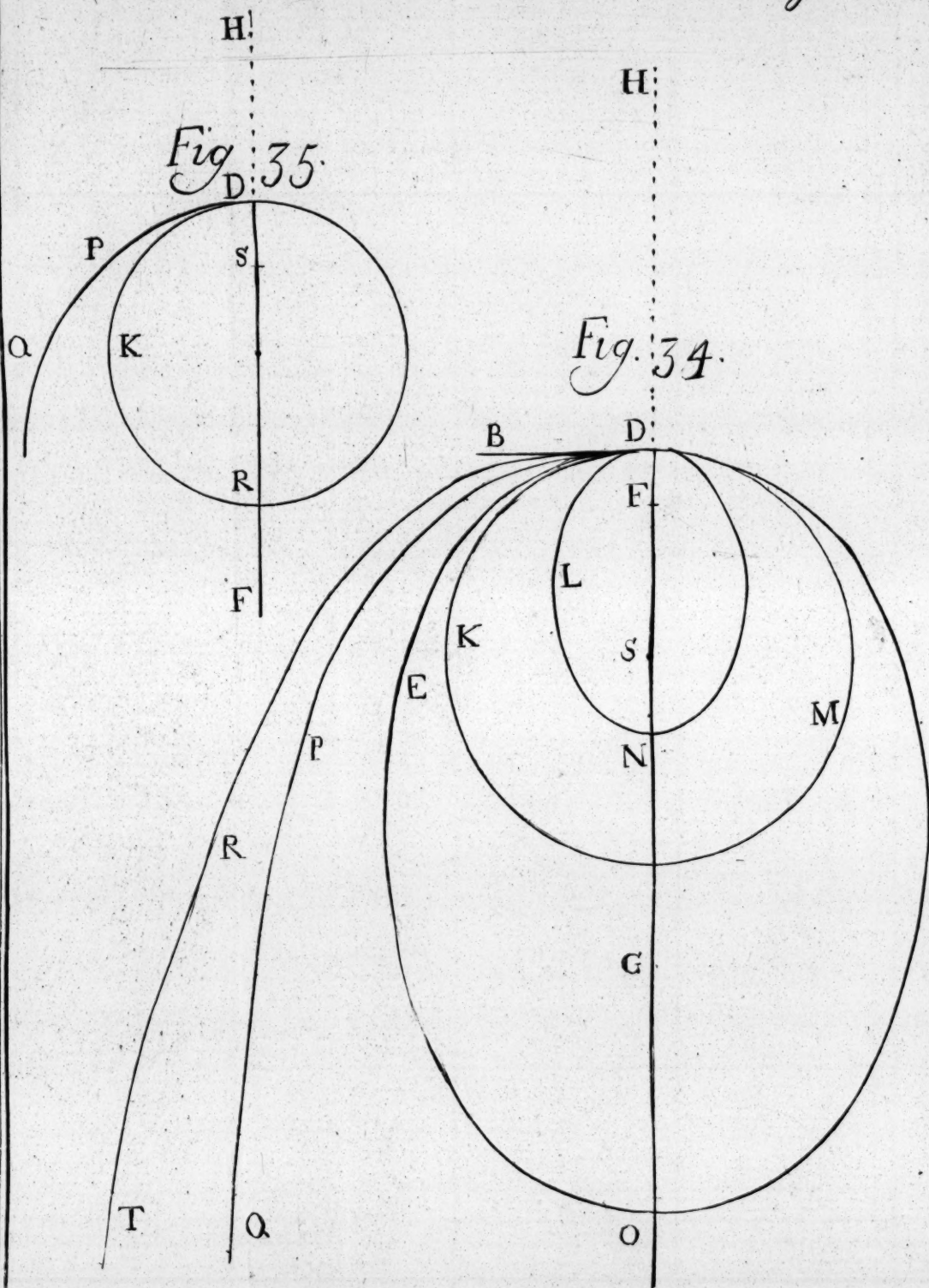
And this Increase will be to the Tendency the Moon has to A , which is the same the Earth has to S , as the Distance LT to LA , or TS . Or in other Words, the Gravity of the Moon towards the Earth in the Quadratures is augmented by the Action of the Sun; and that Augmentation is to the Tendency the Earth has to the Sun, as the Length of the Line LT , or the Distance of the Moon from the Earth, to TS the Distance of the Earth from the Sun.

So that the greater the Moon's Distance is from the Earth, the Distance of the Sun remaining the same, the greater will this increase of the Moon's Gravity towards the Earth be. But if the Distance of the Moon from the Earth remains the same, and the Distance of the Sun be augmented, this additional Increase will be the less in Proportion to the Cube of that Distance (*a*).

Let

(*a*) For, if TS be increased, while LT remains the same, LT will be so much the less with respect to TS , that is the Increase will be diminished in Proportion to the Sun's Distance:
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Let now the Moon be in one of its Syzygies at M, then will the Tendency she has to the Sun over and above what the Earth has, which is farther off at T, be to that which the Earth has, as the Difference of the Squares of SM and ST is to the Square of SM: but the Difference between the Squares of SM and ST bears such Proportion to the Square of SM as twice MT, that is MO, does to SM; because the Difference between the Squares of two Numbers that are nearly equal (as SM and ST are on account of the great Distance of S) bears double the Proportion to the Square of the lesser Number, that the Difference between the Numbers themselves bears to the lesser Number (*m*). The Tendency therefore the

But when TS the Distance of the Sun is increased, the absolute Force of the Sun, and therewith the abovementioned Increase, will be diminished also in proportion to the Square of that Distance, consequently taking in both the Accounts, it will upon the whole, be diminished in Proportion to the Cube of that Distance.

(*m*) *Dem.* Let *a* be the lesser Number, and *a* + *b* the larger, and let their Difference *b* bear no sensible Proportion to the lesser *a*.

Then the Square of the lesser is	_____	<i>aa</i>
The Square of the larger is	_____	<i>aa + 2ab + bb</i>
The Difference between these is	_____	<i>2ab + bb</i>
And <i>bb</i> being rejected as inconsiderable, the	}	<i>2ab</i>
Difference is only		
Now the Proportion of <i>2ab</i> to <i>aa</i> is (taking <i>a</i>	}	<i>2b</i> to <i>a</i> .
out of each Term)		

But *2b* to *a*, is double the Proportion that once *b* has to *a*, and therefore the Difference between the Squares of the two Numbers bears double the Proportion to the Square of the lesser Number, that the Difference between the Numbers themselves bears to the lesser Number. Q. E. D.

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Moon

Moon when at M , has to the Sun, over and above what the Earth has, is to that which the Earth has, as MO , or twice LT , to SM , or because of the Sun's great Distance, as twice LT to TS . Her Tendency therefore to the Earth is now diminish'd in that Proportion: But as was shewn above, it was augmented in the Quadratures in the Proportion only of LT to TS . The Diminution here is therefore double of the Augmentation there.

And whereas that Augmentation, when the Distance of the Sun remains the same, was shewn to increase with the Distance of the Moon; but when the Distance of the Moon remains the same, to decrease with the Cube of the Sun's Distance; this Diminution being always double of that, will do the same.

When the Moon is in the other Syzygy at O , she is attracted towards the Sun less than the Earth is by the Difference of the Squares of SO and ST ; which as to the Effect, is the same Thing as though the Earth was not attracted at all towards S , and the Moon were attracted the contrary Way, so that her Tendency to the Earth is here also diminished, as well as when she was at M , and almost in the same Degree; for on Account of the Sun's great Distance, the Difference between the Squares of SO and ST is nearly the same as between ST and SM .

Or because this way of accounting for the Diminution of the Gravity of the Moon towards

wards the Earth in the Syzygies may not be sufficiently clear, it may be considered otherwise, thus. The annual Course of the Moon round the Sun being performed in the same Time that the Earth's is, she ought to be retained in that Course by the same Force that the Earth is, whereas when she comes to M, the Action of the Sun upon her is greater than it is upon the Earth, by the Difference of the Squares of SM and ST; and when she is at O, it is less than it is upon the Earth by the Difference between the Squares of ST and SO: So that in the former Case she is drawn too much towards the Sun, and in the latter too little; and therefore in both Cases her Tendency towards the Earth is diminished; and almost in the same Degree; because, as was observed above, the Difference of the abovementioned Squares is nearly the same in either Case.

Let the Moon be in a Point of her Orbit between the Quadrature and the Syzygy, as at L in Fig. 43. Then being nearer the Sun than the Earth is, she will be attracted with a stronger Force: Let it be expressed by LS produced to D 'till LD is of such Length, that TS being put to express the Action of the Sun upon the Earth, LD may be long enough to express the stronger Force of the Sun upon the Moon: And let LD be resolved into two others, one of which let be LA equal and parallel to TS, then will the other be AD, or its equal and parallel LG. This LG

K k 2

is

is the only disturbing Force upon the Moon at *L*, the other *LA* being parallel and equal to *TS*, affects the Moon just as the Sun does the Earth; and so alters not their Situations with respect to each other. Let then, to avoid a Confusion of Lines, this Figure with the Line *LG*, be removed to the 44th. This Force *LG* may be resolved into *LI* and *LH*, the one a Tangent to the Orbit of the Moon, and the other perpendicular thereto: The former accelerates the Motion of the Moon when going from the Quadrature at *Q* to the Syzygy at *B*; and will retard it when going from *B* to *R*. The other when *H* falls upon *TL* produced, as in this Figure, diminishes the Tendency of the Moon towards the Earth, and when it falls between *L* and *T*, it augments it.

When the Moon is at *L* between the Quadrature *R* and the opposite Syzygy *O*, in Fig. 45. the Tendency of the Moon towards the Sun is less than that of the Earth; it may then be expressed by *LD* a Line shorter than *TS*, and is resolvable into *LA* a Line equal and parallel to *TS*, and *DA*, or its equal and parallel *LG*. Which *LG* is the only disturbing Force, and may, as in Fig. 44. be resolved into two others, one of which shall draw the Moon towards *O*, the other to or from *T*, as the Case may happen. So that in the first Place, the nearer the Moon is to its Syzygies, the greater will be its Velocity; and the nearer it is to the Quadratures, the slower it will move;
because

because one of the Forces into which LG is resolvable (as LI in Fig. 44.) accelerates its Motion from the Quadratures to the Syzygies; and retards it as much from thence to the Quadratures. *Which is the first Irregularity (n).*

2. When the Moon is in the Quadratures as at L or N, or in the Syzygies as at M or O, see Fig. 42. the disturbing Force is directed to or from the Center of the Earth; and therefore when the Moon is passing those Points it is no Impediment to her describing Areas proportionable to the Times; but when she is at L in Fig. 44, or 45, where it is expressed by LG, only one of the Lines into which it is resolved, as LH, points to or from the Center of the Earth, the other, as LI, pointing another Way, prevents her describing Areas proportionable to the Times. So that it is only in the Quadratures and Conjunctions, that the Areas are proportionable to the Times. *Which is the second Irregularity.*

3. The Motion of the Moon being accelerated during her Progress from the Quadratures to the Syzygies, and retarded from thence to the Quadratures, her Motion in the Syzygies is too quick, in the Quadratures too slow: add to this, her Tendency to the Earth is in the former Situation too small, in the latter too large: Both which conspire to render her Orbit more curved in the Quadratures than in the Syzygies, so that she runs off farther from

(n) See the Lunar Irregularities enumerated in Chap. VIII.
the

the Earth in the Quadratures, and comes nearer in the Syzygies than she would otherwise do, describing an Orbit, one of whose Axes, *viz.* that which passes through the Quadratures, is longer, than that which passes through the Syzygies. *Which is a third Irregularity.*

4. The Gravity of the Moon towards the Earth in the Syzygies, being twice as much diminished by the Action of the Sun, as it is augmented in the Quadratures; if we take a whole Revolution together, it may be considered as diminished only. In the Perihelion therefore, at which Time the Earth and Moon are nearest the Sun, it will be diminished the most of all; that Diminution (as shewn above) being inversely as the Cube of the Sun's Distance, and so the Gravity or Tendency of the Moon to the Earth will be the least. On which Account she will run out into a greater Orbit; and so her periodical Time will be greater, than when the Earth is in its Aphelion. *Which is a fourth Irregularity.*

5. When the Moon is in the Quadratures, the Action of the Sun (as shewn above) increases the Tendency of the Moon to the Earth in Proportion to her Distance from thence, this Force superadded to the Action of the Earth upon the Moon, which decreases as the Square of the Distance increases, occasions that Force to decrease as you go from the Earth, *slower* than it would otherwise do; or which is the same Thing to increase slower as you go the
other

other way. But by Prop. 9. of the foregoing Chapter, when a retaining Power decreases or increases slower than the Square of the Distance increases or decreases, and the Planet describes an Ellipse, the *Linea Apsidum* of that Planet will go backwards; the *Linea Apsidum* therefore of the Lunar Orbit, when she is in the Quadratures, goes backwards: When the Moon is in the Syzygies the Action of the Sun diminishes her Tendency towards the Earth, and thereby make it decrease as you go from, or increase as you go to the Earth, too fast; and so the *Linea Apsidum* at that Time goes forwards. But the Diminution in the Syzygies exceeds the Augmentation in the Quadratures, and so the *Linea Apsidum* goes farther forwards than backwards every Time, till at length it revolves quite round according to the Order of the Signs. *Which is a fifth Irregularity.*

6. When the Gravity by which a Planet is retained in an Orbit that is excentrical, decreases or increases *too fast*, the Planet when going off from the Seat of the retaining Power, that is, towards its upper *Apsis*, will go off too far; and when it is coming to its lower *Apsis*, it will approach too near; and so the Excentricity of its Orbit will be increased. When its Gravity decreases or increases *too slow*, the Planet will not in the former Case go off far enough, nor come so near to the Seat of the retaining Power in the latter, as it ought to do; in this Case therefore, the Excentricity of its Orbit

Orbit will be diminished. But the Tendency of the Moon to the Earth when in the Syzygies, decreases or increases too fast, the Excentricity of her Orbit is therefore at that time the greatest; and on the contrary it is the least when she is in the Quadratures. And if we compare several Revolutions of the Moon together, we shall find, then when the *Linea Apsidum* is in the Quadratures, the Excentricity of the lunar Orbit will be the least of all; because in that Situation of the *Linea Apsidum*, the Difference between the Tendency the Moon has to the Earth in one of the Apes, and that which it has in the opposite one, is the least of all: Whereas when the *Linea Apsidum* is in the Syzygies, that Difference will be the greatest; and therefore the lunar Excentricity will be so too. *Which is a sixth Irregularity.*

7. We have hitherto been considering such Irregularities in the Course of the Moon as would happen if its Orbit were coincident with the Plane of the Ecliptic: But as it is not so, there will arise others; in order to account for which, it will be proper to premise the following Considerations.

First, That when the Line of the Nodes is in the Syzygies, the Plane of the Moon's Orbit passes through the Center of the Sun as well as through that of the Earth, and so the Moon in that Situation of the Nodes, is not drawn

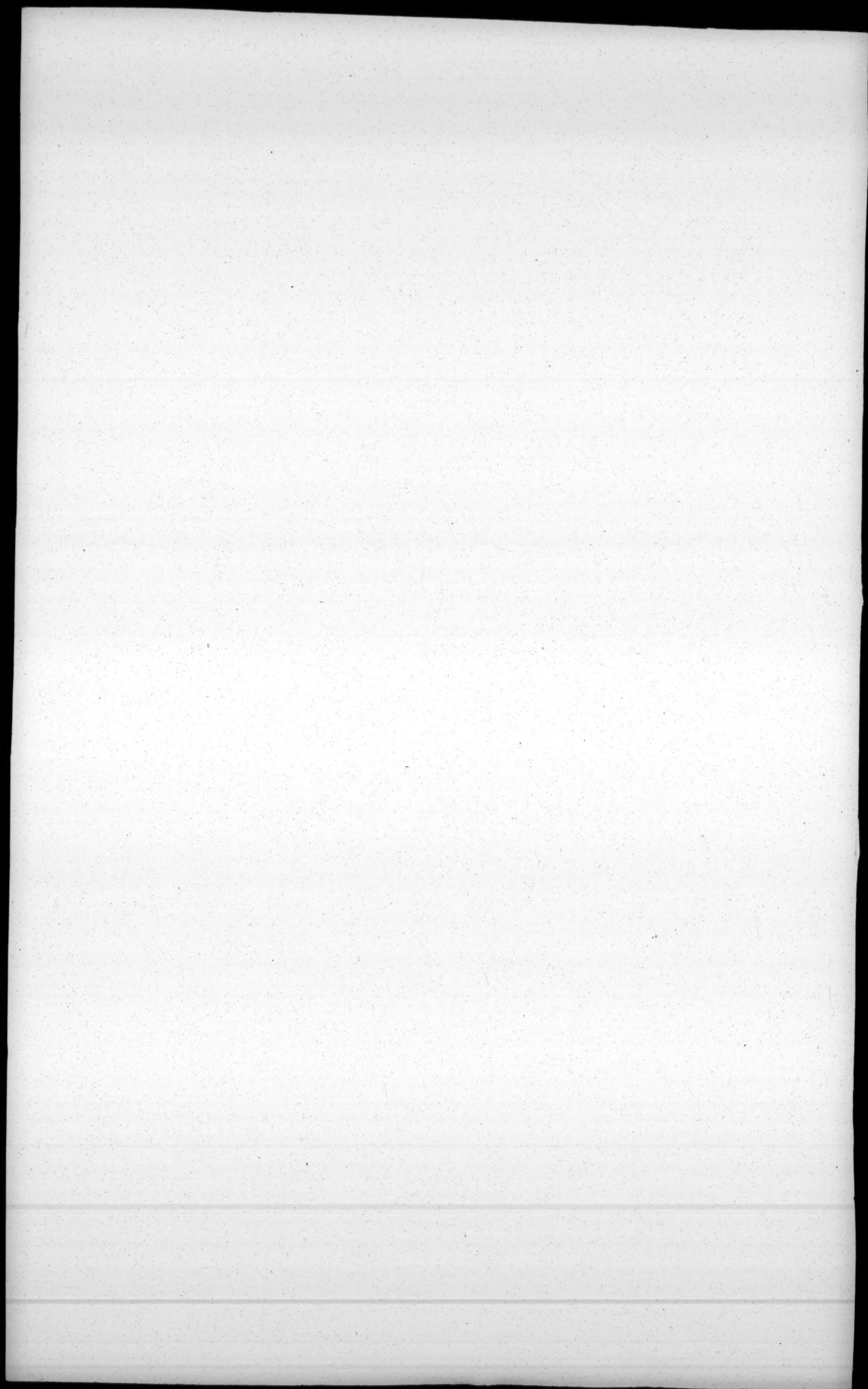


Fig. 36.

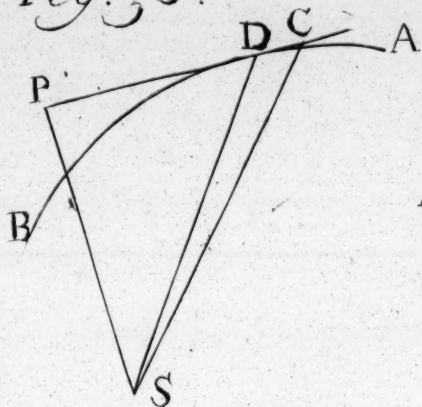


Fig. 37.

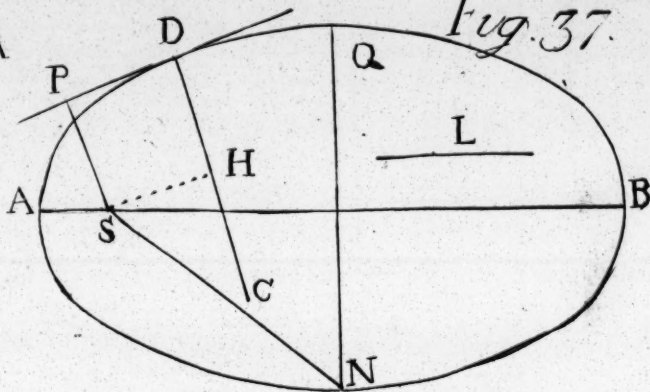


Fig. 39. R P

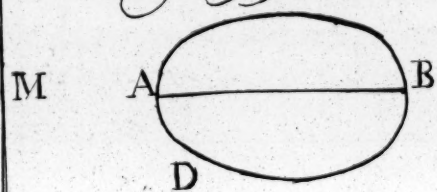


Fig. 38.

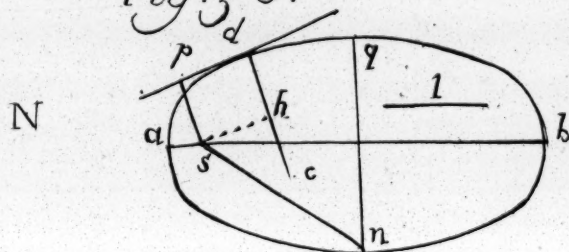


Fig. 41.

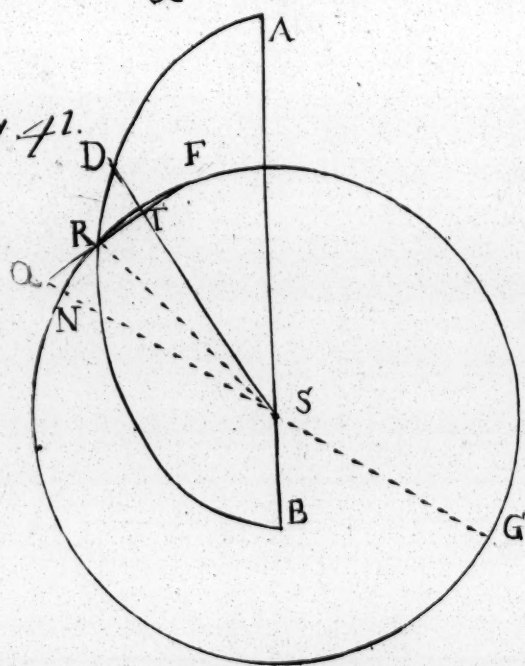
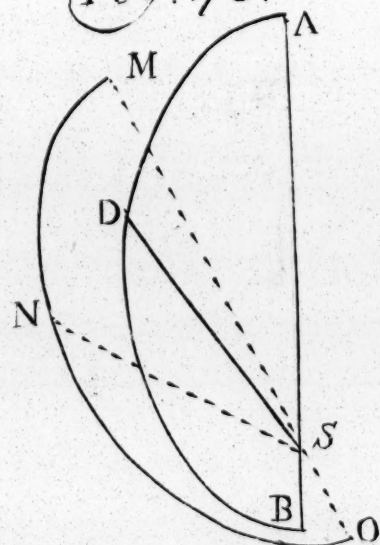


Fig. 40.



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Secondly, That when the Line of the Nodes is in any other Situation, and the Moon not in one of the Nodes, she is continually drawn out of the Plane of her own Orbit, on that Side on which the Sun lies. For instance, if the Plane of her Orbit produced passes above the Sun, the Sun draws her downwards; if on the contrary, the Plane of her Orbit produced passes below the Sun, it draws her upwards.

From these two Considerations it follows, that when the Line of the Nodes is not in the Syzygies, and the Moon having passed either of the Nodes, has got out of the Plane of the Ecliptic on either Side, the Action of the Sun occasions the Moon to return back to the Plane of the Ecliptic sooner than she otherwise would do; but where the Moon enters that Plane, there is the next Node; so that each Node does as it were come towards the Moon, meeting her part of the Way. And the nearer the Line of the Nodes is to the Quadratures, the greater is this Effect, because in that Case the Sun is the farthest of all from the Plane of the lunar Orbit produced. So that the Line of the Nodes goes backwards the fastest of all, when it is in the Quadratures; and not at all in the Syzygies. *Which is the seventh Irregularity.*

8. Again, when the Nodes are in the Quadratures, and the Moon has lately passed one

of them, and is approaching that Syzygy that is next the Sun, the Action of the Sun upon the Moon prevents her ascending so far; that is, departing so far from the Ecliptic as she otherwise would do; and so diminishes the Inclination of her Orbit to the Ecliptic: And as she goes on to the next Quadrature, by hastening her Descent thither, it occasions the Moon to cross it in a larger Angle than she would otherwise do; and so increases the Inclination of the Orbit as much as it diminished it before. And, for the same Reason, while the Moon passes from that Quadrature to the opposite Syzygy, the Action of the Sun decreases the Inclination of her Orbit, and increases it again in her Passage from thence to the next Quadrature. All which needs no farther Illustration, unless the following Instance may be of Use. If you toss a Stone up into the Air, the Action of the Earth upon the Stone prevents it from rising so high as it would otherwise do; and if a Stone be thrown down obliquely, the same Action by bending its Course towards the Earth all the Way, makes it strike the Earth in a larger Angle than it would otherwise do.

When the Nodes are in the Syzygies, the Inclination of the lunar Orbit to the Plane of the Ecliptic is neither increased nor diminished: The Sun being then in the Plane of the Moon's Orbit produced.

But while the Nodes are passing from the Syzygies to the Quadratures, the Inclination of
of

of the Moon's Orbit to the Plane of the Ecliptic, is diminished in every Revolution of the Moon; and while they are passing from thence to the Syzygies, it is continually increasing. So that the Inclination of the lunar Orbit is the greatest of all when the Nodes are in the Syzygies, and least when they are in the Quadratures. *Which is an eighth Irregularity.* But this requires a particular Explanation.

Let then S in Fig. 46. represent the Sun, NFDG the Plane of the Ecliptic, QR the Quadratures, and MO the Syzygies. And let NLDI be the Orbit of the Moon; and suppose the Nodes at N and D in the middle between the Syzygies and the Quadratures. Farther, let there be a Point H in the Ecliptic opposite to the Point S, and let the Orbit NLDI be so inclined to the Plane of the Ecliptic, that if it were extended every Way, it would pass above S and below H. Then because when the Moon is nearer the Sun than the Earth is, she is attracted towards the Sun more than the Earth is; and when she is farther off, the Earth is attracted more than she is, in which case she is therefore as it were attracted the other way; let us imagine a Sun at S, and another at H; and let it be remembered that the Orbit of the Moon produced, passes above S and below H: And let the Moon be passing from N towards L. Then the Attraction here being towards S, and the Orbit produced being above S, it is obvious that the Moon will not pass to L

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but

but to A, a Point between L and F, describing the Curve NA; so that the Inclination of the lunar Orbit is perpetually diminished, while the Moon is passing over 90 Degrees from the Node N, it being a quarter of a Circle from N to A. To avoid Confusion in the Figure, let us suppose that the Moon came to L. In going from thence to the next Quadrature at B, which is 45 Degrees, the Attraction of S prevails still, because the Moon is as yet nearer to S than the Earth is; and therefore as the Orbit produced is above S, and the Moon going downwards, the Attraction of S hastens her Descent, and so makes her describe the Curve LC instead of LB, which if produced would make with the Plane of the Ecliptic, a larger Angle than her Orbit LB does at D; in going over this 45 Degrees therefore the Inclination of her Orbit is increased. Let us now suppose her going from B towards D; the Attraction here lies towards H, because she is now past the Quadrature; and she is tending to a Point below H; H therefore attracts her upwards, making her describe the Curve BE, instead of BD, which is about 45 Degrees more, and makes a less Angle with the Plane of the Ecliptic, than BD does: Not that E falls beyond D, it only happens so in the Figure, because we supposed the Moon to move from L and B instead of A and C. Upon the whole therefore, while she moves from N to D, the Inclination of her Orbit is diminished during three Parts
out

out of four of her Passage. In like manner it is diminished by the Attraction of H, while she goes from D to I, and augmented by the same Attraction in going from thence to K, and diminished again between K and N. Add to this, that while the Moon moves from N to L, or from D to I, the disturbing Force, whether of S or H, is much more considerable than it is when she is between L and D, or I and N; because in the former Case, the Difference between her Distance from the Sun, and that of the Earth from the same, is greater than it is in the latter. While the Nodes therefore are between M and R, and O and Q; that is, while they are passing from the Syzygies to the Quadratures, the Inclination of the lunar Orbit is continually diminishing.

For though we have supposed the Nodes equally distant from the Quadratures and Syzygies, the like Effects will happen, though different in Degree, when they are nearer to the one than to the other; as is easy to imagine.

Let now the Nodes be in the mid Way between R and O, and Q and M, as in Fig. 47. and let the lunar Orbit produced pass above S and below H as before, and let the Moon be coming from the Node D towards B. She being there in the Power of H, and moving in a Plane which if produced passes below H, will be attracted upwards thereby, so as to describe the Line DC instead of DB; by which means the Inclination of her Orbit will be

be increased. In going from thence to I, she is in the Power of S, which lies below her Orbit produced, and so her Ascent will be diminished, and she will go to A, instead of I; whereby the Inclination of her Orbit will be lessened; and afterwards as she goes towards N she will be attracted downwards all the Way coming to E instead of N; by which means the Inclination of her Orbit is again increased. So that upon the whole, it is increased three Parts out of four of her Passage from Node to Node; for the like will hold in her Passage through the other Part of her Orbit, and as well when the Nodes are not in the middle between the Quadratures and the Syzygies, as when they are, except in Point of Degree. And for the Reason mentioned in the other Case, the Force which augments the Inclination of the Orbit, is superior for the Time being, to that which diminishes it. While the Nodes therefore are passing from the Quadratures to the Syzygies, the Inclination of the Moon's Orbit to the Plane of the Ecliptic is continually increasing. Which is what remained to be made out.

All these Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion, because as was observed above, the Effect of the Sun's Action whereby they are produced, is inversely as the Cube of its Distance from the Earth. They are also greater when the Moon is in Conjunction with the Sun,

Sun, than in Opposition for the same Reason; for the Earth and Moon taken together, are nearer the Sun in the former Situation of the Moon, than they are in the latter.

II. Let us now imagine that the Circle NLDI is a solid Ring like that of *Saturn*, and that it moves round its Center T the same Way the Moon does round the Earth; it is obvious that every Point of this Ring, will endeavour to put on the same Motion that we have shewn the Moon to do: That is, that every Point in its Passage from N to L, will endeavour to move in the Line NA, (see Fig. 46.) every Point between L and B, will endeavour to describe the Curve LC, and every one between B and D, the Curve BE; and the like for Fig. 47. So that the Ring as to the Motion of its Nodes, and its Inclination to the Plane in which its Center moves, will be affected in the same manner that the Orbit of the Moon is; and therefore its Nodes, when in the Syzygies, will stand still, and its Inclination to the Plane of the Ecliptic will be the greatest: In all other Situations the Nodes will go backwards, and fastest of all when in the Quadratures, at which Time the Inclination of the Ring will be the least.

Let us now suppose that there is a Redundancy of Matter surrounding the Earth in the equatoreal Parts thereof; or in other Words, that the Earth is an oblate Spheroid, having its equatoreal Diameters longer than that which
passes

passes through the Poles, as we shall shew it to be in the next Chapter. This Redundancy of Matter, will, like a Ring surrounding the Earth and fixed to it, endeavour to put on the abovementioned Motions, and thereby communicate them to the Earth itself. The equinoctial Points therefore which answer to the Nodes of the Ring, when they are in the Syzygies, that is, at the Equinoxes, will stand still, and the Inclination of the Equator to the Plane of the Ecliptic will be the greatest; in all other Situations they will go backwards, and fastest of all at the Summer and Winter Solstices, at which Times they are as it were in the Quadratures with the Sun; and then the Inclination of the abovementioned Plane will be the least.

III. From hence it follows, that the Axis of the Earth, being perpendicular to the Plane of the Equator, changes therewith its Inclination to the Plane of the Ecliptic twice in every Revolution of the Earth about the Sun. For instance, it increases while the Earth is moving from the Solstitial to the Equinoctial, and diminishes as much in its Passage from the Equinoctial to the Solstitial Points. Which Phænomenon is called the Nutation of the Poles.

IV. Another Phænomenon and of the same kind with the lunar Irregularities, is the ebbing and flowing of the Sea; only, as those arise from

Fig. 42.

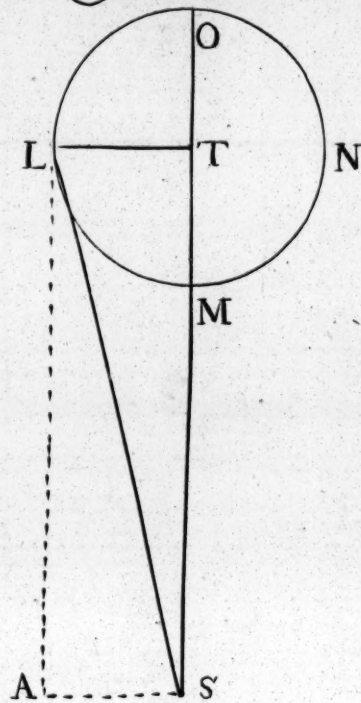


Fig. 43.

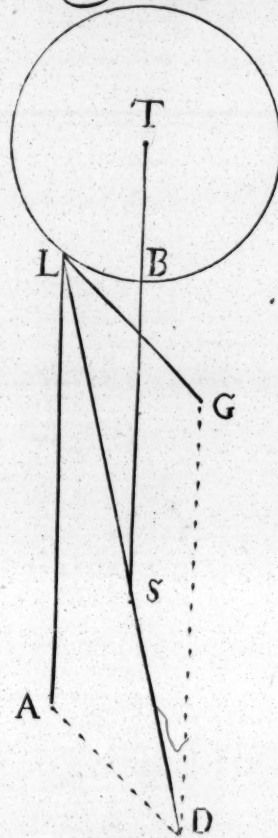


Fig. 45.

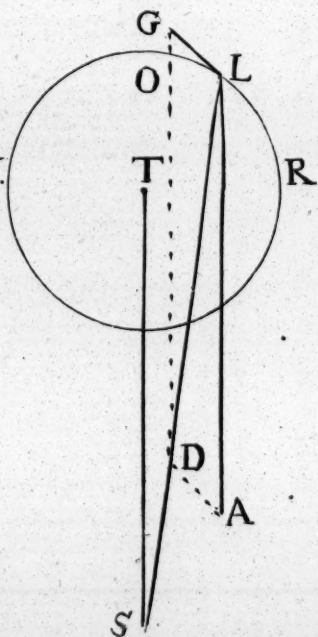
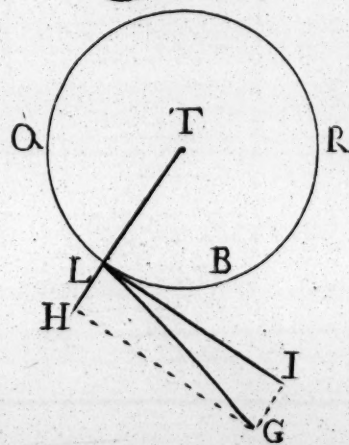


Fig. 44.



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from the Action of the Sun upon the Moon, this is owing to the Influence both of the Sun and Moon upon the Waters of the Ocean; and is to be accounted for upon the same Principles, after the following Manner.

Let the Point *L* in Fig. 48. represent the Moon, *MNOP*, the Earth, whose Center is *C*. And let *G* be the common Center of Gravity of the Moon and the Earth. Which latter let us suppose surrounded with Water to a great Depth. Then, according to what was observed at the End of the last Chapter, these two Bodies continually revolve about the Point *G*, the Point *M* describing the Circle *AB*; the Point *C*, the Circle *NP*; and the Point *O*, the Circle *EF*; and all in the same periodical Time; consequently by Proposition the third of the foregoing Chapter, the Forces they require to retain them in those Circles, ought to be to each other as their Distances from the Point *G*: That is, as *GM*, *GC* and *GO*. Consequently the Point *O*, which for Distinction sake we will call the *Nadir*, requires a greater Force than the Center *C*; and the Center, a greater Force than the Point *M*, which we will call the *Zenith*. Now these Points are retained in those Circles by the Moon at *L*, consequently the *Nadir* which requires the most, is attracted the least, as being farthest off; and the *Zenith* which requires the least, being the nearest, is attracted the most; that is, the *Nadir* is attracted too little, and the *Zenith* too much:

M m

The

The obvious Consequence of which is, that the Water both in the *Zenith* and *Nadir*, will endeavour to leave the Center C; or, in other Words, will lose part of its Weight. But the Water at N and P will have its Weight augmented, just as the Tendency of the Moon at L (in Fig. 42.) towards T was shewn to be augmented by the Action of the Sun at S. So that the Water at N and P will be heavier than an equal Quantity at M or O. And consequently the Surface of the Waters at N and P will subside, and that at M and O will rise, 'till the Equilibrium be restored. On which Account, the Form of the Earth, or rather the Sea, will become an oblong Spheroid or Oval, as represented by NKM^H, in Fig. 49. whose longer Axis produced passes through the Moon at L. As therefore the Moon turns round the Earth once a Day, this Oval of Waters turns with her, occasioning thereby the two Floods and Ebbs observable in each 25 Hours: or to speak more accurately, the Oval of Waters keeps pace with the Moon in her Monthly Course, while the Earth in the mean Time, by its Rotation about its Axis, carries each part of its Surface from Ebb to Flood, and from Flood to Ebb continually.

And as the Moon thus raises and depresses the Water, the Sun does the same; but in a much less Degree, on Account of the small Proportion the Semidiameter of the Earth bears to the vast Distance of the Sun; for, as was shewn

shewn of the Moon, the Force of the Sun whereby it disturbs her Motion, was proportionable to the Relation the Distance of the Moon from the Earth bears to that of the Sun from the same, which in the Case before us, is the Relation the Semidiameter of the Earth bears to the Distance of the Sun, which Relation is very small.

When the Moon is in Conjunction or Opposition with the Sun, the Tides which each of them endeavours to raise are in the same Place, which is the Reason they are so large at those Times. Whereas when the Moon is in the first or last Quarter, the Sun being in the Meridian when the Moon is in the Horizon, depresses the Water where the Moon raises it, on which Account, the Tides are then (*ceteris paribus*) the least of all.

On the full and new Moons, which happen about the Equinoxes, at which Time the Luminaries are both in the Equator or near it, the Tides are the greatest of all, on the three following Accounts; in the first Place, the two Eminences of Water are at the greatest Distance from the Poles, and so the Difference between Ebb and Flood is more sensible; for if those Eminences were at the Poles, its obvious we should not perceive any Tide at all: Secondly, the equatoreal Diameter of the Earth produced passes through the Moon, which Diameter is longer than others, and so there is a greater Disproportion between the Distances of the

Zenith, Center and Nadir, from the Center of Gravity of the Earth and Moon, than at other Times: Thirdly, the Water rising higher in the open Seas, it rushes to the Shores with greater Force, where being stopped, it rises higher still; for it not only rises at the Shores in Proportion to the Height it rises to in the open Seas, but also according to the Velocity it flows with from thence against the Shore. The Reason why the Spring Tides, which happen a little before the vernal and after the autumnal Equinox, *viz.* in *February* and *October* are the greatest, is because the Sun is nearer the Earth in the Winter than in the Summer, and so the Tides, which otherwise would be the greatest at the Equinoxes, are so, a little before the former, and as much after the latter.

We have hitherto considered the Tides in general; we must now see what happens as to Places of different Latitudes. All which will be easily understood by the help of the 49th Figure: In which let AFD represent the Earth whose Center is T, the Poles P and O, this the South, the other the North Pole, EQ the Equator, and the Circles FH and KD two Parallels of it, the last on the North Side of it, and the first on the South Side of it. Let the Fluid surrounding the Earth, form itself into an oblong Spheroid, whose longer Axis HK produced, passes through the Moon at L. The right Lines TK or TH, reckoning from the
Center,

Center, will represent the greatest Height of the Water, and supposing NM perpendicular to KH, TN or TM will denote the least, and will represent the Height of the Water in all Parts of the Globe through which that Circle NM passes. The right Lines TE, TF, TQ and TD, supposing them drawn, will shew the Height of the Water in the respective Places E, F, Q and D. Let us now consider a Place, which by the diurnal Motion of the Earth, describes the Parallel KD: When this Place is at K, the Height of the Water TK, is the greatest; that is, it is high Tide or Flood when the Moon L is in the Meridian; but afterwards in the same Place, the Height of the Water is the least, when the Place is come to X, and again it is Flood when the Place is come to D. But because TK is greater than TD, in the present Case when the Moon is on the North Side the Equator, the Height of the Sea will be greater, when the Moon is in that Part of the Meridian which is above the Horizon, than when it is in that which is below it. Likewise TH is greater than TF, and therefore in a Place which lies under the Parallel FH, or on the other Side the *Line*, the greatest Height of Water that happens when the Moon is on that Side, is also when she is in that Part of the Meridian that is above the Horizon of that Place. Hence it is, that the Moon in the Northern Signs makes the greatest Tides on our Side the *Line*, when she is above the Earth; and

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and in the Southern Signs when she is below
it. But on Account of that libratory Motion
of the Waters, by which they thus rise and
fall alternately, and which would continue some
Time, although the Sun and Moon should cease
to act, the Difference between those Tides
which happen when the Moon is above and
below the Horizon, is not so great as it would
otherwise be; and the highest Spring Tides
are not exactly in the new and full Moons, but
happen generally three or four Tides after
them, and sometimes later: Because when the
Luminaries come to act more forcibly than
ordinary, as being in Conjunction or Opposition,
the Waters will librate backwards and forwards
several Times, before they arrive at their great-
est Height.

Things would be thus, if the Globe of the
Earth were covered with Water of a sufficient
Depth, but the Continents which stop the Tide;
the Straights between them and the Islands,
and the Shoalness of the Sea in some Places,
which are Impediments to the free Course of
the Water, cause many Exceptions to what has
been laid down; and in particular, that even in
the open Ocean the Time of high Water is
not, when the Moon comes to the Meridian,
but always some Hours after it. But to be
particular in such Circumstances is not the De-
sign of this Treatise. See more in *Philosoph.
Transact.* No. 226. However, there is one
Thing, which because I don't find it taken
Notice

Notice of by others, I shall just mention; and that is, that at both the Summer and Winter Solstices, there ought to be but one Tide in 24 Hours within the arctic and antarctic Circles; to shew this, let KD be the Tropic of Cancer, and let the Sun and Moon be perpendicular over a Point of that Tropic, as at L: Then as observed before, there will be an Ebb at N, but because NPK is a quarter of a Circle, N will be a Point of the arctic Circle, and the high Water under that Circle will be at A, the Point opposite to N; so there will be but one Flood and one Ebb in 24 Hours Time: And as is obvious enough, the same will happen in all Places within that Circle, except at the Pole itself, where there is no Tide at all. The same Things will also happen at the other Solstice, when the Luminaries are in the Tropic of Capricorn. And in those Parallels which lie between the abovementioned Circles and the Equator, each Ebb will be nearer the Time of the lesser Flood, than it will be to that of the greater. Thus, in the Parallel KD, the Ebbs will be at X, and in the Parallel FH at Y, which Points are nearer to F and D the lesser Heights of the Water, than to H and K the greater ones. The Reason it does not happen exactly in this Manner, at least not in Point of Degree, is that Rising and Falling or Libration of Waters mentioned above, by which the Ebbs and Floods of each Place are rendered less

less unequal than they ought, according to the Theory, to be.

C H A P. XX.

Of the Figures of the Heavenly Bodies.

WH E R E A S the Heavenly Bodies do not consist wholly of solid Matter, but are in all probability partly fluid and partly solid, like our Earth, or at least were so at first; those which have no Motion about their Axes, if such there be, will, from the mutual Gravity and Attraction of their Parts among themselves, settle into a spherical Form. But as to such as revolve about their Axes, all their Parts will endeavour to recede from the Axis of their Motion, and thereby the equatoreal Parts where the Motion is the quickest, will tend less towards the Center than the rest; their Endeavour to fly off from the Axis about which they revolve, taking off part of their Tendency that Way; so that those Parts will become lighter than such are nearer the Poles. The polar Parts therefore will press in towards the Center, and raise the equatoreal Parts, till the Quantity of Matter in the latter is so far increased, as to compensate for its Lightness, and an Equilibrium be restored. On which Account, the
Form

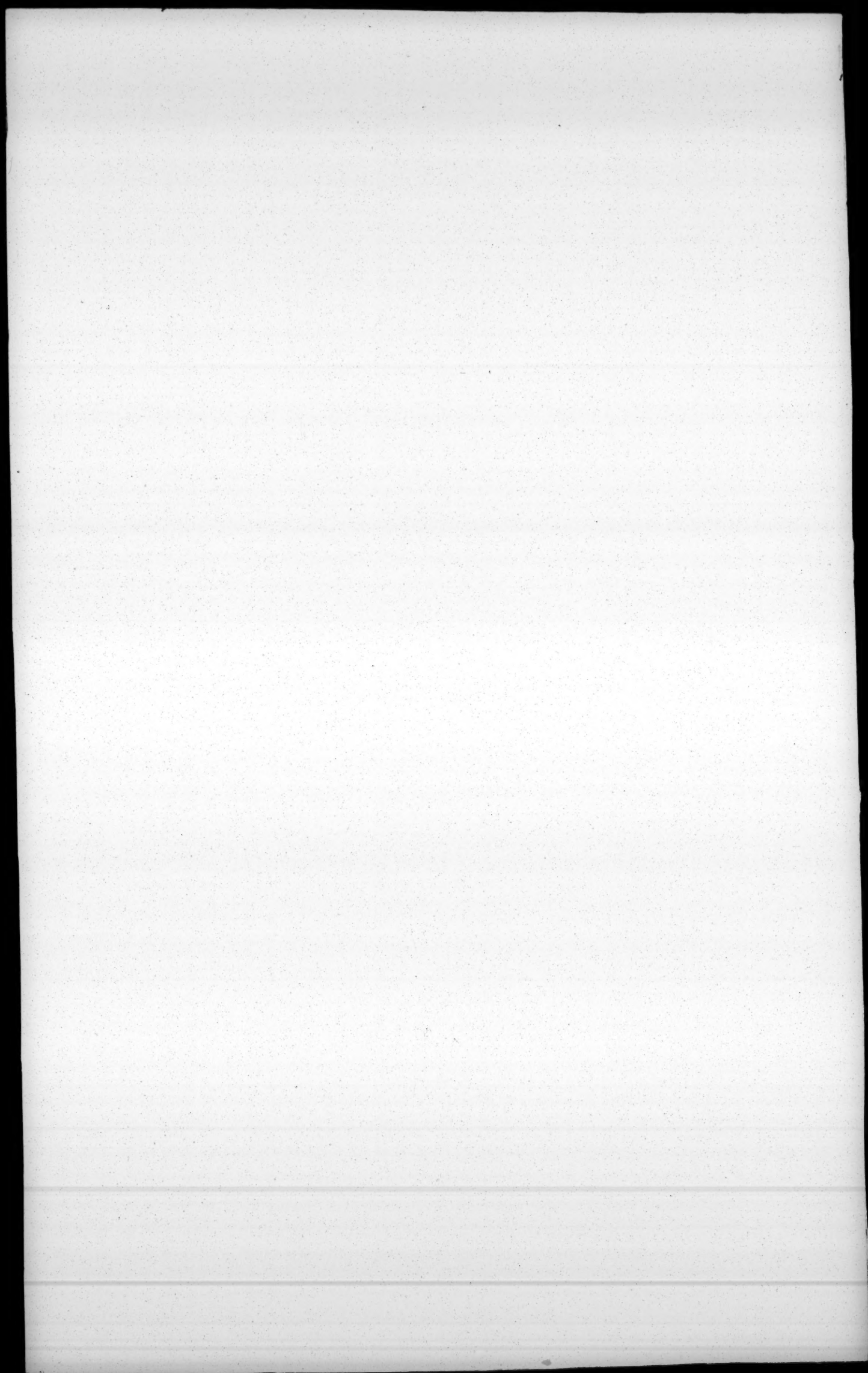
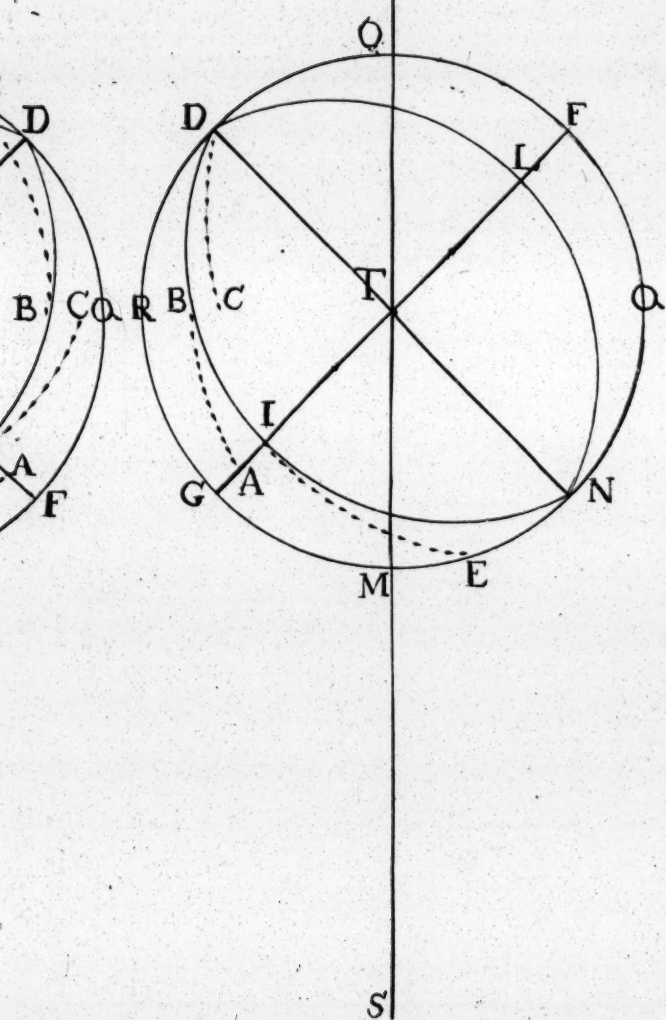


Fig. 47.



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Form they will assume, will be that of an oblate Spheroid, whose shorter Axis passes through the Poles. And other Circumstances remaining the same, the faster the Bodies revolve, the more oblate or flat will their Form be; accordingly the Axis of the Planet *Jupiter*, which Planet turns round its Axis in less than 10 Hours, is, as appears from the Observations of Mr. *Flamsteed* and Mons. *Cassini*, no greater with respect to the Diameter of its Equator, than in the Proportion of seven to eight.

By Virtue only of the Rotation of the Earth about its Axis, the Weight of Bodies at the Equator is less than at the Poles, in the Proportion of 288 to 289; that is, a Body which at the Poles would weigh 289 Pounds, would, if carried to the Equator, lose by Virtue of the centrifugal Force alone, one Pound of its Weight, and so weigh but 288 Pounds. From hence arises, as observed above, a spheroidical Form of the Earth, and from that spheroidical Form arises another Diminution of Gravity at the Equator, by which, if the Earth were homogeneous throughout, Bodies at the Equator would lose one Pound in 1121, and so on both Accounts taken together, the Gravity of Bodies at the Poles would be to the same at the Equator as 230 to 229 (*n*). From whence, if we suppose

(*n*) To calculate this, let us suppose the Semidiameter of the Earth to be 19695539 *Paris* Feet; which on Account of the vast Distance on the Surface of the Earth *Cassini* took in measuring

During it, Sir *Isaac Newton* makes use of. Since then the Earth turns round its Axis in 23 Hours 56 Minutes and 4 Seconds, a Body at the Equator moves through 1436,223 Feet in a Second: But the centrifugal Force of a Body revolving in a Circle is equal to the centripetal Force, which would be requisite to retain it in that Circle, and the Space a Body would fall through in a Second, by Virtue of that centripetal Force, is by Chap. XVIII. Lemma 1. equal to the Square of the Arch described in a Second, divided by the Diameter of the Circle, that is, in the present Case to ,0523 Feet, or 7,54064 Lines. Now the Space a Body falls through at *Paris*, by Virtue of the Gravity there, is 15 Feet 1 Inch and $2\frac{1}{8}$ Lines, or 2174,055 Lines. Therefore the centrifugal Force at the Equator is to the Gravity at *Paris* as 7,54064 to 2174,055. But Bodies do not fall at *Paris*, by the whole Force of Gravity, or as they would do at the Poles of the Earth, because they are in some Measure prevented by the centrifugal Force there. We must therefore compute how much that centrifugal Force is, and add it to their Gravity at *Paris*, to find what it is at the Poles. Which may thus be performed.

Let EPQ in Fig. 50. represent the Earth, PP its Poles, EQ its equatoreal Diameter. Let A represent any Point between the Equator and the Poles, and parallel to the equatoreal Diameter EQ draw the Line AH, then will the centrifugal Force at E be to that at A as the Radius EC, or which is very nearly the same, AC, to AH, Chap. XVIII. Prop. 3. for AH is the Radius of the Circle the Point A describes, while the Earth revolves about the Axis PP. But a Body at A is not thrown off by Virtue of this centrifugal Force directly from the Center of the Earth, but from H along the Line HA produced. Let then AB represent that Force, which because it does not tend directly from the Center, let it be resolved into two others, *viz.* BN and AN; the former perpendicular to the Radius, the latter coincident therewith: It is by this latter Force only that the Gravity of a Body at A is diminished. But the centrifugal Force at the Equator was shewn to be to that other Force along the Line AB, as AC to AH, and that other is to the Force along AN as AB to AN, which because of the similar Triangles ABN and ACH is also as AC to AH, therefore the centripetal Force at the Equator is to that at A, so far as it diminishes the Gravity of Bodies there, as AC² to AH²; that is, because AH is the Co-sine of Latitude of the Place A, as the Square of the Radius to the Square of the Co-sine of the Latitude of the Place where the Body is. Now the Latitude of *Paris* is 48 Degrees and 50 Minutes, the centrifugal Force therefore at the Equator is to that

at

suppose the Gravity of Bodies within the Earth to be directly as their Distance from the Center, as it was shewn to be in Part I. Chap. 3. § 8. those Numbers will also express the Relation between its polar and equatoreal Diameter (6).
This

at *Paris*, as the Square of the Radius to the Square of the Co-
fine of that Latitude, that is, as 7,540 64 Lines to 3,267. Add
this to 2174,055 the Gravity by which Bodies descend at *Paris*,
and we have 2177,322 for the Gravity they fall with at the
Poles: Which Number is to 7,540 the centrifugal Force at the
Equator, as 289 to 1. So that the Gravity at the Poles is to
the Gravity at the Equator, so far as it is diminished in this lat-
ter Place by the Rotation of the Earth about its Axis, in the
Proportion of 289 to 288. But this is not all: For it may be
gathered, from what Sir *Isaac Newton* has demonstrated in his
Principia, Book I. Sect. 13. where he treats of the attractive
Forces of Bodies not spherical, (though by a tedious and intricate
Calculation too long to be inserted here) that supposing the Earth
to be an oblate Spheroid, such as we shall determine by and by,
and homogeneous throughout, a Body at the Poles even when the
Earth is at Rest, would be heavier than the same at the Equator
in the Proportion of about 1121 to 1120. The Weight of a
Body therefore at the Poles, when the Earth revolves about its
Axis, is to the Weight of the same at the Equator, in a Pro-
portion compounded of 289 to 288, and 1121 to 1120; that
is, in the Proportion of 230 to 229.

N. B. A *Line* is the 12th Part of an Inch.

(6) To shew this, call an equatoreal Column extended from
the Surface to the Center, r ; and a polar Column, x ; and call
the Force of Gravity at the Equator, p : then from what was just
observed in the foregoing Note, supposing the Earth to be at

Rest, the Force of Gravity at the Poles will be $\frac{1121p}{1120}$, because

as $1120 : 1121 :: p : \frac{1121p}{1120}$: and because the Gravity of a Co-
lumn if it be equally heavy in all Parts, is equal to the Force
of Gravity multiplied by the Contents of the Column, the
Weight of an equatoreal Column of Matter, if as heavy in all
Parts as at the Surface, would, supposing the Earth to be at Rest,
be equal to p multiplied by r , that is, pr ; and the Gravity of a

N n 2

polar

This is upon a Supposition that the Earth was at first fluid, or a Chaos, having its solid and fluid Parts confusedly mixed together; but if we suppose it at first partly fluid and partly dry, as it now is, since we find that the Land

polar Column would for the same Reason be $\frac{1121px}{1120}$; but where-

as if we suppose the Force of Attraction inversely as the Squares of the Distances, the Gravity of Bodies within the Earth will decrease as we go to the Center, Part I. Chap. 3. § 8. where it terminates in Nothing, the Weights of the above-mentioned Columns decreasing uniformly therewith, will be but half what we made them before, that is, $\frac{pr}{2}$ and $\frac{1121px}{1120 \times 2}$. Call the centrifugal Force at the Equator, n ; then since the centrifugal Force decreases also as we approach the Center, and there terminates in Nothing, the centrifugal Force of a whole Column of Matter will be $\frac{nr}{2}$. Take this away from $\frac{pr}{2}$, the Weight of that Co-

lumn when the Earth is at Rest, and the remainder $\frac{pr}{2} - \frac{nr}{2}$ will be the Weight of the same when it moves. But to preserve an Equilibrium of Parts, this Weight must be equal to $\frac{1121px}{1120 \times 2}$ which was shewn to be the Weight of a Column at the Poles.

Which gives us this Equation $\frac{pr}{2} - \frac{nr}{2} = \frac{1121px}{1120 \times 2}$

Multiplying by 2, we have $pr - nr = \frac{1121px}{1120}$

Multiplying by 1120, we have $1120pr - 1120nr = 1121px$

Which gives us this Equation $r : x :: 1121p : 1120p - 1120n$.

But as was determined in the

foregoing Note, p is to n as

289 to 1. Putting there-

fore those Numbers for p

and n , in the last Step, we

shall have

$$r : x :: 230 : 229.$$

That is, the equatoreal Semidiameter is to the polar one, as 230 : 229. Which was to be shewn.

is

is very nearly of the same Figure with the Sea, except raised a little to prevent its being overflowed, the Earth must still be of the same Form; for otherwise the major Part of the Water would flow towards the Equator, and spread itself like an Inundation over all the Land in those Parts.

The spheroidical Figure of the Earth is greatly confirmed by Observations made with Pendulum Clocks, at different Distances from the Equator. First of all *Monf. Richer* in the Year 1672, when at the Island *Cayenne*, found that his Clock, which at *Paris* kept true Time, now lost 2 Minutes and 28 Seconds every Day. *Dr. Halley* going to the Island *St. Helena* in the Year 1677, was obliged to shorten the Pendulum of his Clock one eighth part of an Inch. With many others, all which compared together make it appear that a Pendulum at the Equator that swings Seconds, ought to be about one sixth Part of an Inch shorter than at *Paris*; from whence it appears that the Difference between the Gravity at the Poles and at the Equator, is almost twice as great as what arises from the Rotation of the Earth about its Axis, and its spheroidical Form put together (*p*). And therefore the Difference between the equatorial and polar Diameter must be so too.

From

(*p*) For the Length of a Pendulum, swinging Seconds is proportionable to the Space a Body would fall through in a Second; as being equal to an eighth Part of that Space, Part I. Chap. 6. Prop. 10. And the Space a Body would fall through in a Second, is proportionable to the Force by which it falls; consequently

From hence it is probable, that the Parts of the Earth which lie near the Center, may be denser than such as lie nearer the Surface. For on this Supposition, and not otherwise, is it accountable, that the Gravity at the Equator discoverable by Pendulums, and that which arises from the above-mentioned Causes, should be so different (*q*). So

quently the Length of a Pendulum swinging Seconds under the Equator ought to be to the Length of one that shall swing Seconds under the Pole, as the Gravities in those Places are to each other: That is, as 229 to 230. The Length of a Pendulum therefore under the Equator, being 438,5 Lines, (as being two Lines shorter than one at *Paris*) the Length of one at the Poles should be 440,4 Lines; that is, it ought to be but 1,9 Line longer than the former. Whereas it is found by the Observations abovementioned, that the Difference between a Pendulum in the Latitude of *Paris*, and one under the Equator, is as much as that, or rather more; and consequently the Difference between one at the Equator and one at the Poles would be almost twice as much; there being almost as much Difference between a Pendulum at the Poles and at *Paris*, as between one at *Paris* and the Equator: And consequently the Diminution of Gravity is almost twice as much as that which arises from the Rotation of the Earth about its Axis, and its spheroidical Form put together.

(*q*) If the Earth be more dense at the Center than near the Surface, we may consider that Redundancy of Matter near the Center over and above what there would be there, was the Earth of uniform Density, apart, and as a separate Body from the rest, then upon Account of the spheroidical Form of the Earth, a Body at the Poles is nearer to this redundant Matter than at the Equator, therefore in removing a Body from the Poles to the Equator, its Gravity so far as it depends on the Attraction of this redundant Matter, that is, that Part of its Gravity which is owing to that Attraction, is diminished: The whole Gravity of the Body is less therefore at the Equator than at the Poles. As therefore the Pendulum discovers a greater Difference of Gravity than arises from the Rotation of the Earth about its Axis, and the spheroidical Form of it considered as homogeneous, put together, it is very probable that the Earth is more dense at the Center than at the Surface.

So that upon the whole, Bodies are lighter at the Equator than at the Poles, on a three-fold Account. First, by the centrifugal Force there, in the Proportion of 289 to 288: Secondly, on Account of the spheroidical Form of the Earth, or its Flatness at the Poles, in the Proportion of 1121 to 1120; and lastly on Account of the greater Density of the Earth at the Center, but in a Proportion not yet sufficiently determined, for want of knowing the exact Length of a Pendulum that swings Seconds at the Poles.

The greater warmth of the Air near the Equator, increases the Length of a Pendulum by Rarefaction, on which Account alone, it would be necessary to shorten it at the Equator; but this, as may be gathered from the Observations of *Pitcairn* and *De la Hire*, cannot be between this Latitude and the Equator above one fortieth part of an Inch; for as *De la Hire* observed, there was but two third Parts of a Line Difference between an Iron Rod six Feet long, exposed to the Frost and the Summer's Sun. Besides, this is allowed for above, in saying that a Pendulum ought to be shortened but one sixth part of an Inch.

It follows from hence, that the Tendency of heavy Bodies upon the Surface of the Earth, except at the Equator and the Poles, is not towards the Center of the Earth, but towards a Point between that and the Equator. Thus,
let

let EPQ in Fig. 50. represent the Earth, PP its Poles, EQ its equatoreal Diameter. The Tendency of a Body as A, upon the Surface of the Earth, between E and P, will not be towards C, but along the Line AD which crosses the equatoreal Diameter in a Point nearer the Body than the Point C is (r).

Another Consequence of the spheroidical Form of the Earth is, that as you go from the polar Parts to the equatoreal, the Degrees upon the Surface of the Earth grow less and less;

(r) To determine the Point D, say as EC is to AH; that is, as the Radius is to the Co-sine of the Angle of the Latitude of the Place A, so is the centrifugal Force at E, to a fourth Number, which will express the centrifugal Force at A in the Direction AB. Produce therefore the Line HA to B, till AB be of such Length, that it may bear the same Proportion to AC, that the Quantity last found has to Gravity upon the Surface of the Earth. Compleat the Parallelogram ABCD, and D will be the Point sought, and the Tendency of an heavy Body will be along the Line AD. Thus suppose it required to find the Direction in which heavy Bodies tend towards the Earth in at the Latitude of 51 Degrees and 46 Minutes. Since the centrifugal Force at the Equator, as shewn above, bears that Proportion to the Force of Gravity, which 1 does to 289, let us take those Numbers (or which will do as well, any other two that are proportionable to them) the former to express the centrifugal Force of a Body at the Equator, and the latter the Force of Gravity. Then say, as the Radius is to the Co-sine of 51 Degrees 46 Minutes, so is 1 to a fourth Number, which will be found ,618. This Quantity is therefore to Gravity as ,618 to 289. Therefore AB or its equal CD is to AC in that Proportion. But the Proportion between CD and AC being known, and the Angle at C the Latitude of the Place, viz. 51 Degrees 46 Minutes being given, the Angle CAD will be found about 5 Minutes, which is the Measure of the Deviation of the Line of Direction of heavy Bodies in that Latitude, from a Line drawn to the Center of the Earth.

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Chap.

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that is, a Degree measured upon the Meridian near the Poles, contains more Miles than the same measured near the Equator. The Reason of this is, that, as appears by Inspection of the Figure, an Arch near the Poles comes nearer to a straight Line; that is, it is less curved than one near the Equator; the former is therefore an Arch of a larger Circle than the latter; but the larger the Circle, the larger are the Degrees measured upon its Circumference. For in going from P towards E, we are not to imagine ourselves upon the Circumference of a Circle whose Center is C, but to pass continually from one Arch to another, which Arches are Portions of different Circles, the Centers of which approach, as the Arches themselves become more curved.

As the Waters of the Earth by Virtue of the Attraction of the Moon, and the Revolution of the Earth about the common Center of Gravity of that and the Moon, were shewn in Chapter the last, to put on the Form of an oblong Spheroid, whose Axis produced passes through the Moon; so in like manner, if we consider the Moon as we have now done the Earth, we shall find, that as she turns round her Axis in the same Time she turns round the Earth, and therefore has nearly the same Side always towards the Earth, her Figure is that of an oblong Spheroid, whose longer Axis points to the Earth; those Parts which lie next

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the Earth being attracted too much, and also having too small a centrifugal Force; and those on the contrary, which lie on the opposite Side of her being attracted too little, and having a centrifugal Force too great; which for the Reasons given in the above-mentioned Place, will necessarily give her that Form.

Her Revolution about her Axis would, as in the Earth, give her a contrary Figure, but it is so very slow, that it is without any sensible Effect.



Material ERRATA in Part III and IV.

PART III.

Page 164 Line 2 for supposed, read disposed.

PART IV.

Page	Line	Instead of	Read
12	21	ADME	ADBE
22	23	{ neither the Sun nor the Earth	{ both the Sun and the Earth, or round neither of them
34	18	Fig. 2.	Fig. 3.
34	32	XV.	XVI.
45	2	Side	Side of the Equator
48	29	XIV.	XVI.
95	13	can-	cannot
99	26	(d)	(b)
114	4	Horizon	Heavens
114	22	Semidiameter	a Semidiameter
119	29	VIII.	VII.
122	27	Refraction	Reflection
140	14	when	from the Place where
141	29	Juvinum	Juvifum.

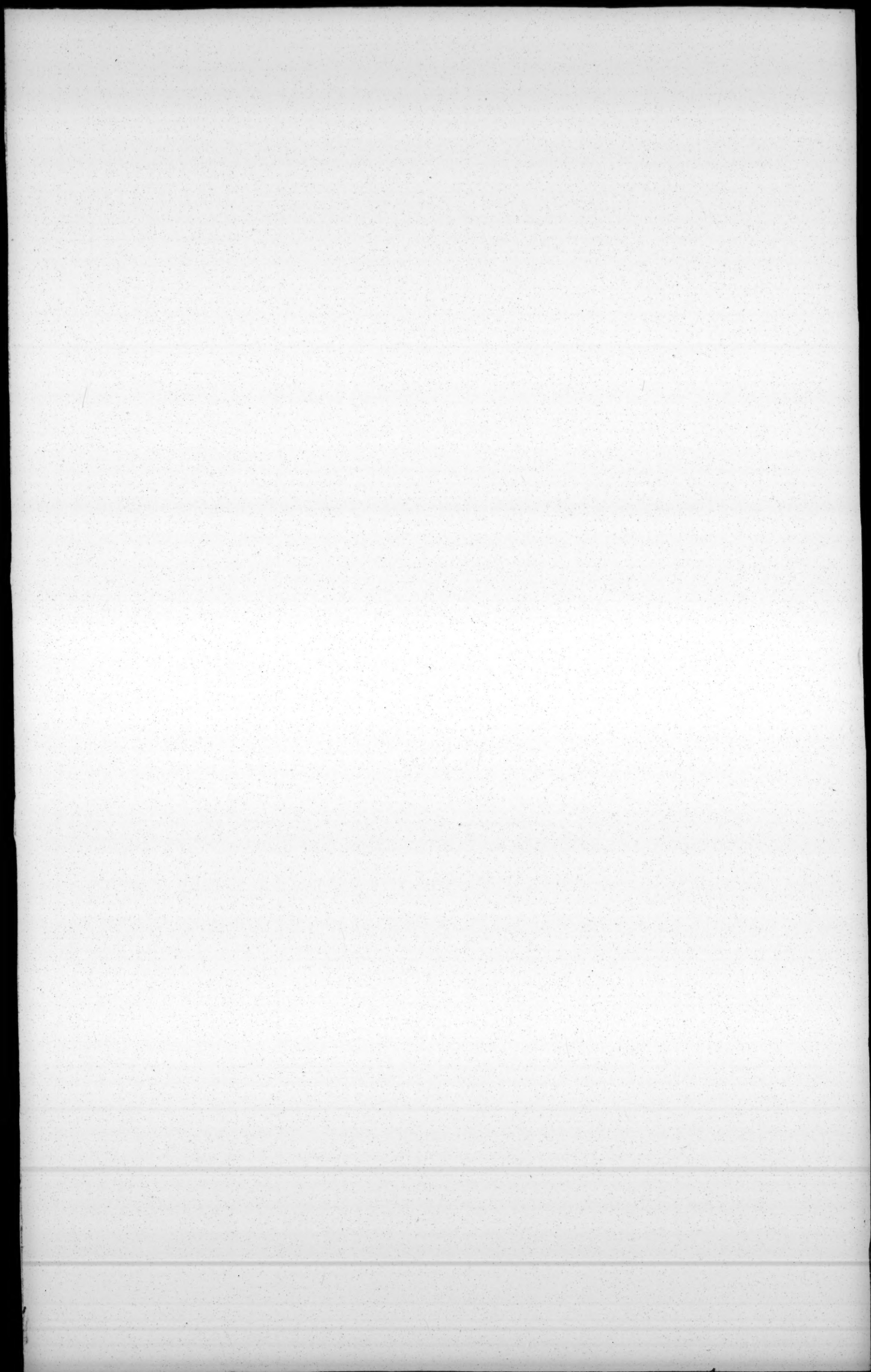


Fig. 49.

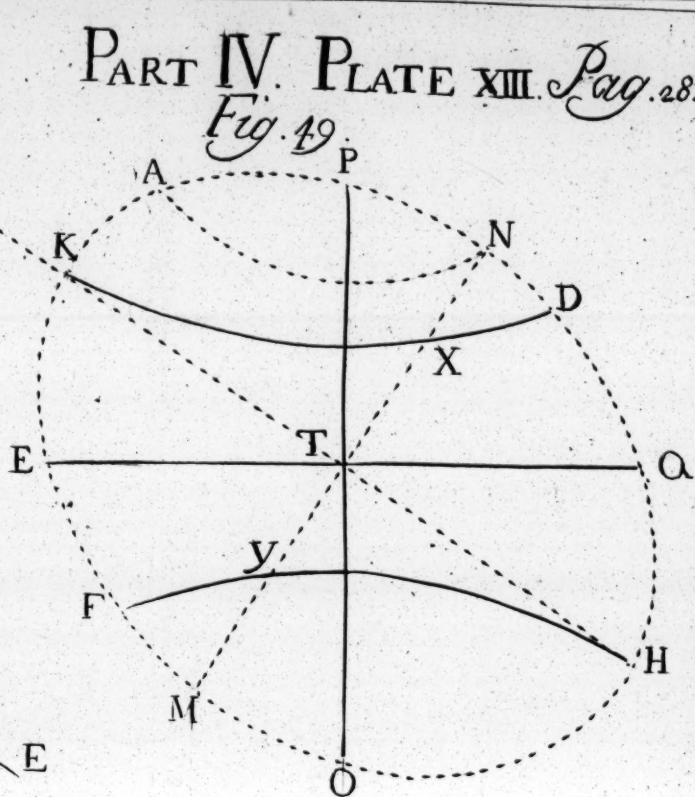


Fig. 48.

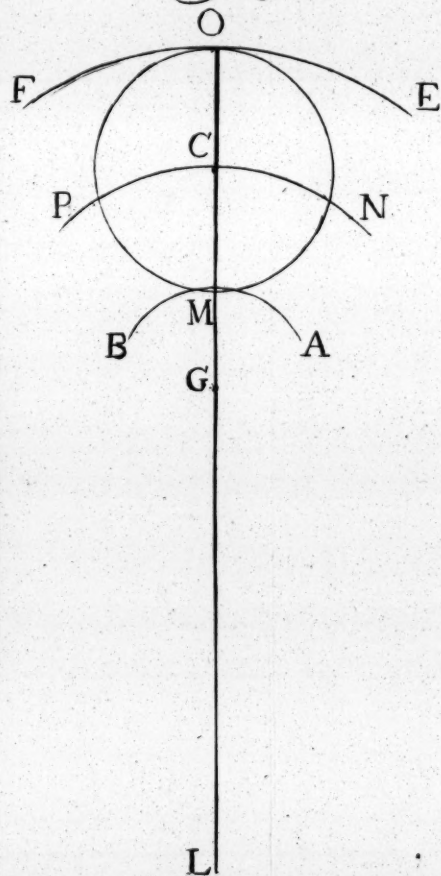


Fig. 50.

